Muography and Geology: Does It Matter Which Continent You Stand on?

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Abstract

The present work has one aim and one aim only: to increase the geological credibility of simulations of muon propagation in real-world rocks. We accomplish this by introducing five different sets of real-world geological systems. Our approach contrasts with the so-called "standard rock" approach, which uses a simplified rock composition as a proxy for geological materials. However, while the conventional approach relies on an assumed average geological composition, it fails to appreciate the complexity of real-world rocks, which indeed are extremely varied in both density and chemical composition. In contrast, each of the five geological systems we have used in our simulations is statistical in nature and represent an average composition of a massive number of similar type of rocks from around the world. The studied real-world geological systems were (1) upper continental crust, (2) bulk continental crust, (3) lower continental crust, (4) oceanic crust, and (5) oceanic upper mantle. Furthermore, water and standard rock were used as references as those are more familiar materials among astroparticle physicists. The simulations were conducted using the standard tools of Geant4 (muon attenuation in materials) and CORSIKA (muon energy in intensity distributions on the ground level), while the parametrized estimates were based on the works of Guan et al. (modified from the Gaisser formula) and Chirkin and Rhode (MMC code). The muon rates were compared to the experimental data of Enqvist et al. extracted in the Pyhäsalmi mine, Finland.

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1. INTRODUCTION

The literature concerning muography has steadily increased in recent years. Still, very few publications have considered the true complexity of rocks in various continents. This is a problem as a real-life muography survey carried out in any part of the world deals with real-world rocks that are as highly varied in terms of composition and densities as their hosting continents. Indeed, each continent has a different history and hence different geological features, not even mentioning the variations in rock compositions and densities from one place to another.

In brief, the present-day convention to use the standard rock model as a basis for experimenting with muon propagation codes does not grasp the true nature of real-world rocks as the latter rarely coincide with the density and composition of the said standard rock. To demonstrate the broad spectrum of different types of geological domains in the layered earth structure, we have conducted a series of extensive simulations to understand the differences in conducting muographic measurements in various parts of the world. Our analyses were based on the fact that different geological environments and geographic areas have different bulk rock compositions and density variation profiles. The present work introduces five real-world geological systems that differ from each other in terms of their density and rock chemistry. Clearly, the density and, to a minor extent, the chemistry have a clear impact on the muon survival as it is known well that the attenuation of muons depends mostly on the density of the material the muons pass through before ceasing to exist. In this respect, rock compositions also always play a role.

The studied real-world geological systems were (1) upper continental crust [1], (2) bulk continental crust [2], (3) lower continental crust [3], (4) oceanic crust [2], and (5) oceanic upper mantle [4]. The oceanic rocks are tectonically thrust on the continental crust in the latter two models. Furthermore, water and standard rock were used as references as those are more familiar materials among astroparticle physicists. The simulation tools were Geant4 [5] (attenuation) while the muon rate estimates were based on CORSIKA [6], Guan et al. [7] (modified from Gaisser [8], Chirkin and Rhode [9] (MMC code) and on the experimental data of Enqvist et al. [10] extracted in the Pyhäsalmi mine, Finland.

2. GEOLOGICAL BACKGROUND

The variations in physical and compositional parameters of separate geological domains and the different rock types within owe to the rock cycle process. The rock cycle is a fundamental concept in geology that describes how the three main groups of rocks—sedimentary, igneous (i.e., rocks of magmatic origin), and metamorphic rocks—change through geologic time. Any rock type can transform into any other rock type by passing through one or more processes like crystallization, metamorphism, erosion, and sedimentation. Metamorphism is a process in which a volume of rocks experiences a solid-state change due to heat, pressure, and/or hydrothermal fluids. Metamorphism can also include the partial melting of a rock mass. Metamorphism can occur on a local scale (e.g., due to ascending of a magmatic melt intrusion through the crust) or on a regional scale, in which case metamorphism occurs over a large region (even over thousands of kilometers wide zones) and over an extensive vertical scale within the continent. In general, the older the continent and its rocks are, the more significant role metamorphism plays on rock densities and compositions. Densities may change due to metamorphism-related compaction, volume loss, and loss of water pore water as well as the water locked in hydrous minerals. However, densities may also alter due to chemical and structural changes in minerals and the formation of new minerals during metamorphism (different minerals have different densities). Yet, another source of density contrasts in rocks is weathering of the upper continental crust, a phenomenon that competes with erosion in deciding whether or not a given area has a thick, shallow, or nonexisting weathering blanket above the crystalline basement rocks. As a rule of thumb, weathering drastically lowers the average density of any rock type.

It must also be emphasized that the real-world continents are not composed of rocks of similar origins and characteristics or even age. Indeed, continents are a complex amalgam of geologically distinct terranes from which each typically comprises a complex assemblage of different types of sedimentary, igneous, and metamorphic rocks. Moreover, the inherited complexity of a continent and each of its tectonically or magmatically attached terrane continues both laterally and vertically in various scales. Further complexity is derived from porosity, fractures, cavities, and other physical rock properties that impact the density of rocks. Such features are filled with air, water, gas, or brine. Instead of focusing on such local phenomena, the present work concentrates solely on muon penetration in continental-scale rock bodies of different origins. Both the herein used densities and rock chemical datasets are statistical in nature, and the provided simulations do not consider such phenomena as local porosity or fracture patterns. This is not to say that features like porosity and fractures are not important on a local scale since they obviously are (see, e.g., Lechmann et al. [11] for details). In its place, this work provides geoscientists with a practical instrument for evaluating one's project region against the data provided herein. The presented results are expected to provide a geologist or geophysicist who considers applying muography a first approximation of what it may take to reach beneficial results from a muography survey in the given project terrane. While the rock porosities and fractures are beyond the scope of the present study, it is still recommendable that the results shown herein are further corrected by local assumptions or data if such information is available.

For the above reasons, it is obvious that the standard rock model can hardly be considered a particularly successful representation of real-world rocks. To correct this shortcoming in the present-day muon propagation simulations, which are a relatively important part of muography, we have applied five geological systems in our simulations. However, this number does not play any particular importance (we could have chosen more) except that it is high enough for giving a decent improvement. The five geological systems are not necessarily perfectly fitting mirrors to most real-world rocks, but they nonetheless offer useful standard rock alternatives. Each of these geological systems is statistical in nature and represents an average composition of a massive number of similar type of rocks from around the world. Figure 1 shows the basement age of the continental crust, distribution of mid-ocean ridges, oceanic crust, and continental shelf according to Mooney [12]. In this case, the crust is subdivided by age and pre-Cambrian shields and platforms cover approximately 70% of the continental crust. The figure also demonstrates that even within continents or tectonic plates the rock formations may change significantly and within rather short distances.

In addition to carrying out simulations of the aforementioned five geological systems (the densities and chemical compositions are listed in Table 1), we also put efforts into answering the question of how deep underground muography can be, at least theoretically, applied. However, this is not straightforward because of the rapidly reducing muon rate. In the present work, we focus on those studies keeping in mind that most people working in these fields are geologists, geochemists, geophysicists, and mining engineers and are likely interested in this open question. We have attempted to answer this need by continuing our simulations to a depth of 5 km, which should be more than enough for any future application of muography in depth. Another reason to continue the simulations to such a great depth was to offer food for thoughts for astroparticle physicists interested in muon propagation.

3. PHYSICS BACKGROUND

Simulations in astroparticle physics are often performed using the Monte Carlo method, which entails the random nature of particles produced and propagates in and through the media. Therefore, the models are often based on an interaction-by-interaction analysis rather than collective models. However, the collective models are often helpful while introducing mechanisms determining the basic principles behind rather complex simulation codes.

Perhaps the most famous example of this kind of simulation code is the CORSIKA software package [6] which is a standard tool to simulate extensive air showers (EAS) in detail. This is also a good approach in muography as practically all single muons (interesting from the muography point of view) result from EAS. However, somewhat surprisingly, only very few models describing atmospheric muons can estimate the total muon flux (in muons per m^2 per second) within the given period of time at, for example, sea level. This is because this number cannot be extracted directly from the models describing the muon production of primary particles in the extensive air showers, but it depends on the all-particle primary cosmic-ray spectrum that is not completely known.

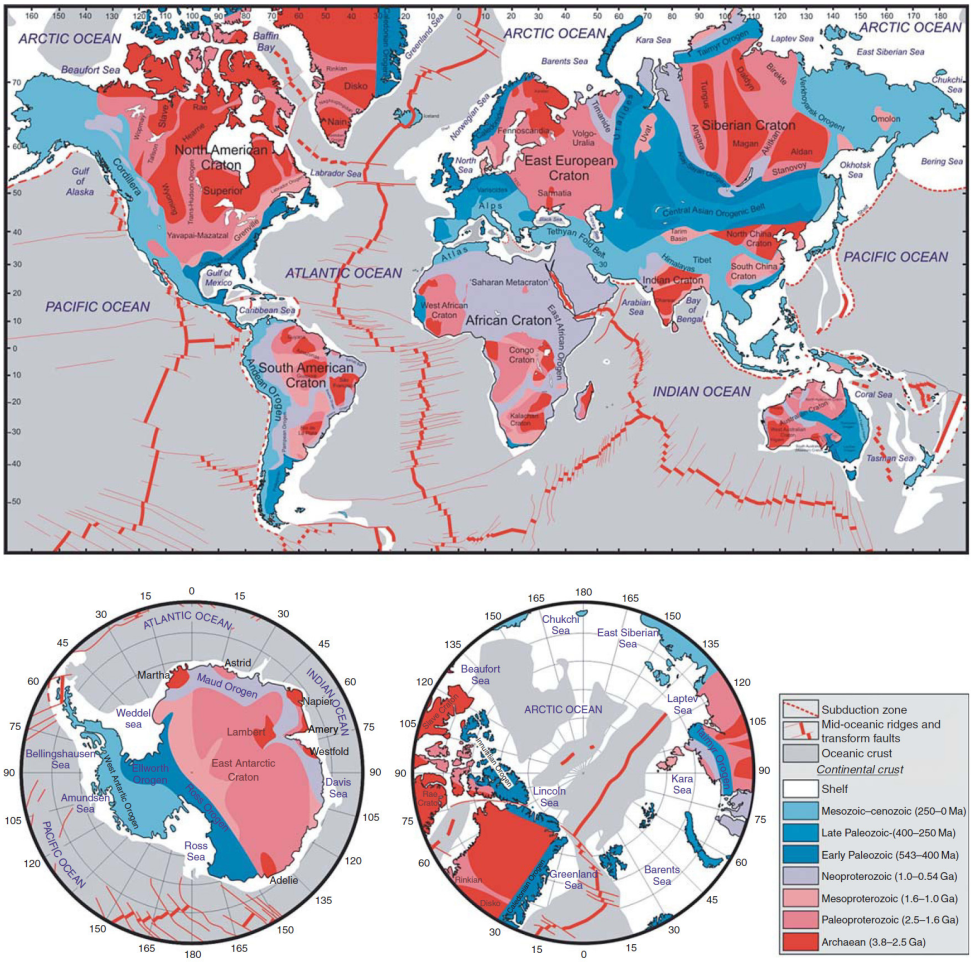

FIGURE 1: Basement age of the continental crust, distribution of mid-ocean ridges, oceanic crust, and continental shelf. The crust is subdivided by age. Pre-Cambrian shields and platforms comprise 69% of the continental crust by area. From Mooney [12].

One way to overcome the problem is to use CORSIKA and the known muon rate at some level underground and then normalize the single muon rate to that number. For example, (i) find the muon flux at the known depth and calculate the energy threshold for it (the minimum muon energy, including the angular corrections to reach the depth) using, e.g., CORSIKA for muon energy and angular distributions and Geant4 for the muon transport in the matter, and (ii) use CORSIKA to reproduce the muon flux using the energy threshold extracted in (i) and the number of muons produced in extensive air showers to match the numbers. The resulting number of muons (through $1\,\mathrm{m}^2$ per second) having energies in excess of the given energy threshold can be translated into the duration, which answers the question of how long it takes to record the given number of muons or the muon flux.

Another more straightforward way is to use the Gaisser formula [8] to estimate the muon rate at sea level. However, today it is well known that the Gaisser formula overestimates the flux at the lower part of the muon energy spectrum. Therefore it is better to use some version modified from that of Gaisser. However, in the present work, this was carried out using the approach of Guan et al. [7] (modified from Gaisser [8]).

Once the total muon flux is extracted (one way or another), the next step is to understand the muon propagation in the material of interest. There, the main interest is to understand the rate muons lose energy while passing through the material. This rate is often called the stopping power (usually given in units MeV/cm or MeV/m). From the literature, it is relatively straightforward to conclude that in materials (such as the standard rock), the muon stopping power is close to constant to approximately 100 GeV, where it is heavily dominated by ionization, after which pair production, bremsstrahlung, and photoelectric interactions, respectively, with increasing energy, are entering to interactions and above 10 TeV ionization is the minor contribution while pair production dominates.

In addition to continuous energy loss, muons (like all other charged particles) also suffer from other interactions such as collisions that may stop them entirely long before they have lost enough of their initial (kinetic) energy being no longer relativistic particles. Those muons decay to other particles and thus cease to exist. Therefore, the probability for a muon to survive to a certain depth in rocks (or other material) not only depends on its kinetic energy but also is a stochastic (i.e., random) process that relies on a collection of random variables. As a result, the detailed muon survival distributions cannot be calculated using a simple equation;

TABLE 1: Average chemical compositions of five different types of real-world continental and oceanic crustal and mantle materials (models) as they occur on the present-day continents. Major elements are shown as weight percent oxides. Trace elements were omitted for their negligible effect on the total compositions. Original data were recalculated by normalizing the total sums of the oxides to 100%, except for the lower continental crust whose data are already presented in this way [3, Table 11]. These five crustal models and the standard rock model were used as chemical input in our simulations. Note that the crustal models are all dissimilar for both composition and density. [a](juvenile 2.5–1.8 Ga) [1], Table 4, normalized to 100%. [b](irrespective of age) [2], data adopted from [13]. [c](irrespective of age) [3], Table 11. [d][2]. [e][4], Table 1, the average composition of Type I and Type II lherzolites normalized to 100%. [f]Average proton number Z and the average ratio of proton number Z (f_1) and mass number A (f_2). [g]Density ρ is in SI units [kg/m^3]. [h][14]. [i][2]. [j][15]. [k][16]. [l][17].

Element	Upper continental crust[a]	Bulk continental crust[b]	Lower continental crust[c]	Oceanic crust[d]	Oceanic upper mantle[e]
SiO_2	66.86	57.24	53.4	49.77	45.61
TiO_2	0.64	0.90	0.82	1.51	0.06
Al_2O_3	15.26	15.88	16.9	16.09	2.65
FeO	4.90	9.09	8.57	10.56	—
Fe_2O_3	—	—	—	—	8.01
MnO	—	—	0.10	—	0.13
MgO	2.26	5.29	7.24	7.74	41.13
CaO	3.57	7.39	9.59	11.36	2.34
Na_2O	3.34	3.10	2.65	2.82	0.06
K_2O	3.02	1.10	0.61	0.15	—
P_2O_5	0.14	—	0.10	—	0.01
$\langle A\rangle^{f_0}$	21.55355	22.33100	22.37880	22.81891	21.25421
$\langle Z\rangle^{f_1}$	10.66177	11.01357	11.04157	11.24187	10.52009
$\langle Z/A\rangle^{f_2}$	0.496736	0.496088	0.496100	0.495858	0.497143
$\langle Z^2/A\rangle^{f_3}$	0.789615	0.762895	0.759367	0.743782	0.684634
Density ρ[g]	2660[h]	2700[i]	2940[j]	3000[k]	3300[l]

instead, the procedure is much more complex and requires sophisticated simulation software and, sometimes, a significant amount of computing power. In the present work, the muon transport was conducted employing the MMC code by Chirkin and Rhode [9].

Another relevant parameter in muography is the so-called range. The range is the distance that a particle travels in a medium, and it depends on different interactions, collisions, scatterings, etc.; the particle endures while passing through the material. The range is closely connected with the stopping power. Muons as heavy and fast particles also experience collisions and scatterings but to a smaller extent and are mostly only slightly deviating from their original direction. Muon ranges are shown in Figure 2 for common earth's crustal materials as a function of muon energy. It is worth noting that the range can be defined in many different ways, but the basic idea is to answer how deep muons can penetrate the matter with the given energy. As this is a statistical problem, just one number or curve is seldom enough for estimating the number of muons surviving at the given depth with the given muon energy. Therefore, we have chosen to use the midpoint in this estimation in the present work. The midpoint is the depth a half of muons could reach with the given initial energy, i.e., corresponding to the muon survival probability of 50% to the given depth.

Another definition for the range is the so-called CSDA range (or the Continuous Slowing Down Approximation range) which assumes that the energy loss at each point along the muon track is equal to that resulting from the total stopping power. Yet, another possibility could be, for example, to use numbers like 90% or similar. The differences are as they are. However, as all these limits behave qualitatively in an identical way for the present work's scope, the midpoint approach seems justified.

Generally speaking, the muon flux on the surface varies between 150 and 200 muons per square meter per second (muons/m^2 s), depending on the given energy threshold, location, and air pressure. The latter is, however, important only if the muon energy is not very high, say less than 100 GeV. The flux is heavily dominated by low-energy muons since at the depth of 100 m of rock (as a distance corresponding to approximately 60 GeV for vertical muons), the muon flux is approximately 1 muon/m^2 s while at the depth of 400 m (corresponding to approximately 300 GeV for vertical muons) it is only approximately 0.02 muons/m^2 s. Those numbers are still high enough to perform high-statistics muon-based measurements on density variations in a relatively short period of time.

For example, the approach of Guan et al. [7] predicts the muon flux of 187 muons per square meter per second for the integrated muon flux at sea level. This number is used for the simulations, and it is consistent with the measured flux of 180 ± 20 muons/m^2 s on the ground in the Pyhäsalmi mine by Enqvist et al. [10]. However, it is worth noting that this number is necessarily not independent on the set muon energy threshold (in the latter around 4-5 MeV because in 5 cm thick plastics scintillation detectors, the muon energy loss peaks at approximately 10 MeV) as low-energy muons always dominate the energy spectrum of muons.

Lipari and Stanev [18] proposed a simple equation for the muon intensity as a function of depth (see also Formaggio and Martoff [19] and references therein). The relation between the muon flux (F_μ in unit muons/m^2 s) and the depth (x in unit meter

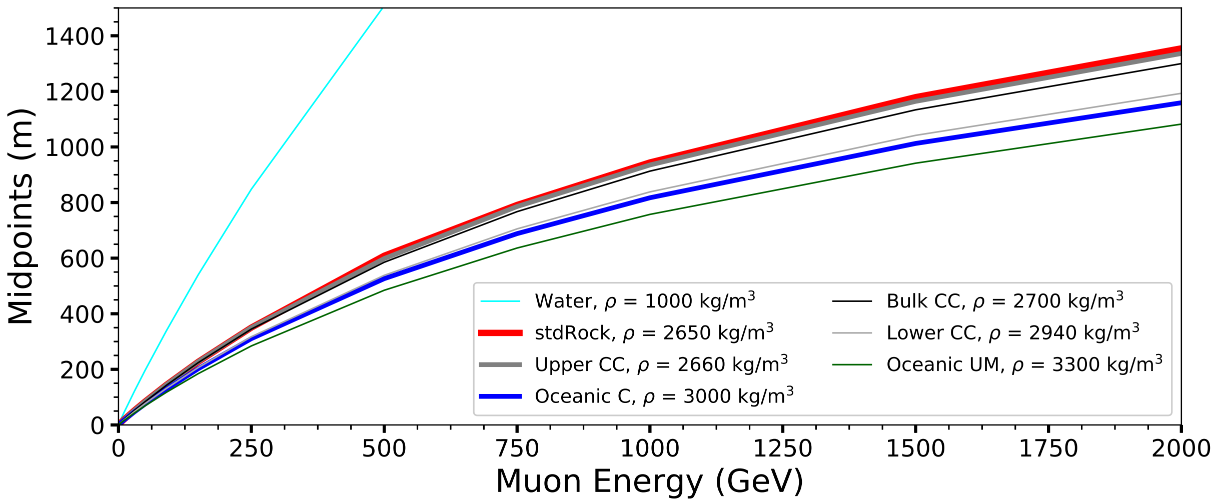

FIGURE 2: Muon ranges (midpoints) for five common earth continental materials and those of water and the standard rock for comparisons, simulated with the Geant4 [5] software package. One notes that the range in water is significantly longer than that of any given solid material.

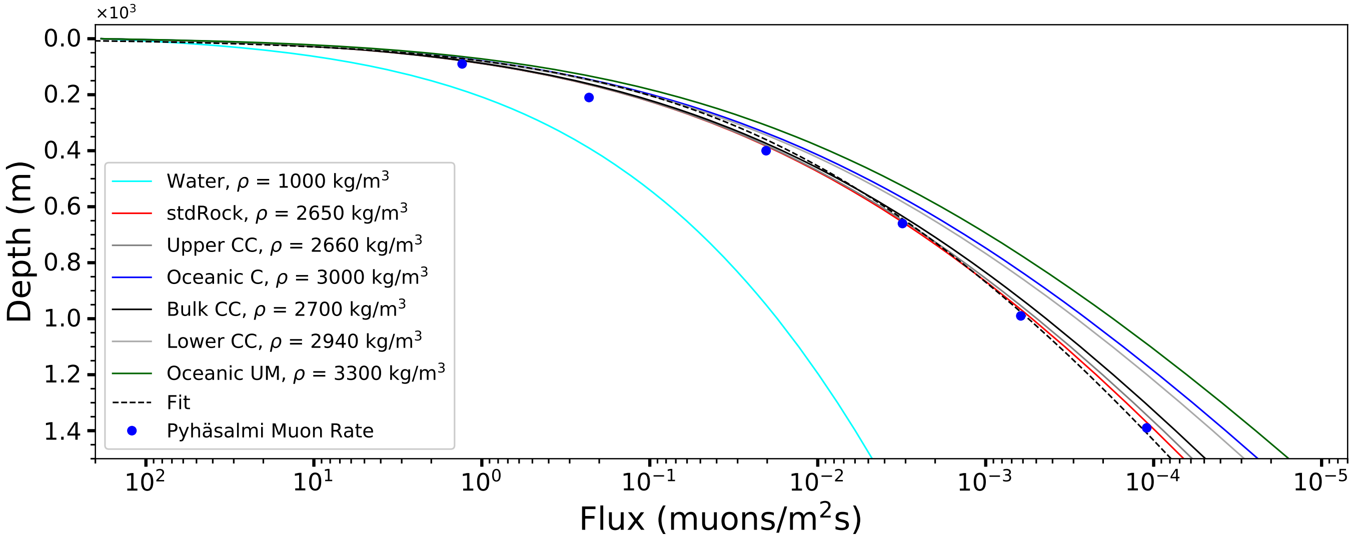

FIGURE 3: Simulated and measured muon rates at different depths (in meters) in five earth continental materials together with water and standard rock for comparison. Simulations were performed using the approach by Chirkin and Rhode [9] (MMC code), while the measured data for the Pyhäsalmi mine are from Enqvist et al. [10] (see also Kuusiniemi et al. [20]).

water equivalent, or m.w.e.) can be written as

$$F_\mu(x) \approx A \left(\frac{x_0}{x}\right)^\eta e^{-\frac{x}{x_0}}, \tag{1}$$

where A, x_0, and η are constants. For example, Enqvist et al. extracted the values of $A = 0.025(4)\,\mathrm{m}^{-2}\,\mathrm{s}^{-1}$, $x_0 = 1330(140)$ m.w.e. and $\eta = 2.18(12)$ assuming the mean rock density of $2.85\,\mathrm{g/cm}^3$ down to 1.4 km in the Pyhäsalmi mine, Central Finland [10]. Uncertainties are given in parentheses referring to the last digit(s). With $\chi^2 \approx 0.007$, the fit can be considered good.

The simulated (MMC code) muon flux curves through the materials of five different geological models are shown in Figure 3. Furthermore, the figure also shows the flux curves of water and standard rock for comparison together with the measured data of Enqvist et al. while the fit (dashed curve) is based on equation (1).

The reason for Pyhäsalmi data being lower than those of the standard rock is likely due to underestimating the effects of the 150 m deep open pit which surely has a large impact on the muon rates, especially at shallow depths. It is also interesting to note that even if the densities of continental materials are rather similar (ranging between 2660 and 3300 $\mathrm{kg/m}^3$), the number of muons passing through just 300 meters changes the number of muons by a factor of two, which can be considered a significant difference and justifies the old rule of thumb: 10% difference in density results in 30% difference in the muon flux. Furthermore, these numbers can be translated into the durations of measurements required to detect the given number of muons.

4. CONCLUDING REMARKS

On the basis of Figures 2 and 3, it seems evident that different rock domains found on the present-day continents result in significantly different muon rates, which cannot be neglected in muographic surveys. Therefore, it evidently does matter which continent you stand on while conducting muography surveys. This is a particularly important aspect to notice if muographic studies are conducted deep underground (say, below some hundreds of meters).

Another interesting observation is that the standard rock is not the best possible example of a typical ("standard") rock. The reason is that it underestimates the density of many typical rocks. This is not, however, a problem if the results are used, for example, to estimate the background rate of cosmic-ray induced muons, as is the case in many underground laboratories; in such cases, the rates are overestimated rather than underestimated. Still, many studies benefit from more realistic numbers than those systematically overestimated.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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