

# $U(1)_\chi$ , Seesaw Dark Matter, and Higgs Decay

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## Abstract

It has recently been pointed out that the underlying symmetry of dark matter may well be  $U(1)_\chi$  (coming from  $SO(10) \rightarrow SU(5) \times U(1)_\chi$ ) with the dark parity of any given particle determined by  $(-1)^{Q_\chi+2j}$ , where  $Q_\chi$  is its  $U(1)_\chi$  charge and  $j$  its spin angular momentum. Armed with this new insight, previous simple models of dark matter are reinterpreted, and a novel idea is proposed that light seesaw dark matter exists in analogy to light neutrinos and is produced by the rare decay of the standard-model Higgs boson.

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## INTRODUCTION

In the decomposition of  $SO(10) \rightarrow SU(5) \times U(1)_\chi$ , the fermions of the standard model (SM) are organized into

$$\underline{16} = (5^*, 3) + (10, -1) + (1, -5), \quad (1)$$

where

$$(5^*, 3) = d^c [3^*, 1, 1/3, 3] + (\nu, e) [1, 2, -1/2, 3],$$

$$(1, -5) = \nu^c [1, 1, 0, -5], \quad (2)$$

$$(10, -1) = u^c [3^*, 1, -2/3, -1] + (u, d) [3, 2, 1/6, -1]$$

$$+ e^c [1, 1, 1, -1], \quad (3)$$

under  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$ . This symmetry breaking pattern is accomplished using the scalar  $\underline{45}$  of  $SO(10)$ . Since

$$\underline{45} = (24, 0) + (10, 4) + (10^*, -4) + (1, 0) \quad (4)$$

under  $SU(5) \times U(1)_\chi$ , a nonzero vacuum expectation value (VEV) of the  $(1, 0)$  component will work. Under  $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\chi$ ,

$$(24, 0) = (8, 1, 0, 0) + (1, 3, 0, 0) + (3, 2, 1/6, 0)$$

$$+ (3^*, 2, -1/6, 0) + (1, 1, 0, 0). \quad (5)$$

Hence a nonzero VEV of the  $(1, 1, 0, 0)$  component will break  $SU(5) \times U(1)_\chi$  to the SM gauge symmetry without breaking  $U(1)_\chi$ . The Higgs scalars of the  $\underline{45}$  are all superheavy and will not affect the phenomenology of the SM Higgs boson to be discussed below. To allow the quarks and leptons to acquire mass, the scalar  $\underline{10}$  representation, which contains the necessary Higgs doublets, i.e.

$$\underline{10} = (5^*, -2) + (5, 2), \quad (6)$$

with

$$\Phi_1 = (\phi_1^0, \phi_1^-) [1, 2, -1/2, -2], \quad \Phi_2 = (\phi_2^+, \phi_2^0) [1, 2, 1/2, 2] \quad (7)$$

from  $(5^*, -2)$ ,  $(5, 2)$  respectively, is required, resulting in the allowed Yukawa couplings

$$d^c(u\phi_1^- - d\phi_1^0), \quad u^c(u\phi_2^0 - d\phi_2^+),$$

$$e^c(\nu\phi_1^- - e\phi_1^0), \quad \nu^c(\nu\phi_2^0 - e\phi_2^+), \quad (8)$$

as desired. The nonzero vacuum expectation values  $\langle \phi_{1,2}^0 \rangle = v_{1,2}$  also break electroweak  $SU(2)_L \times U(1)_Y$  to electrodynamic  $U(1)$ . Since the SM gauge bosons all have  $Q_\chi = 0$ , it is obvious (but not recognized for its importance until recently [1]) that all SM fermions have odd  $Q_\chi$  and all SM bosons have even  $Q_\chi$ . This means that each SM particle has even  $(-1)^{Q_\chi+2j}$  where  $j$  is its spin angular momentum. It is thus a short step to realizing that any scalar with odd  $Q_\chi$  and any fermion with even  $Q_\chi$  would have odd  $(-1)^{Q_\chi+2j}$ , making it a natural stabilizing symmetry for dark matter. Indeed, all previous simple models of dark matter based on an  $Z_2$  discrete symmetry may be incorporated into such a framework.

The scalar  $\underline{126}$  representation of  $SO(10)$  contains a singlet  $\zeta \sim (1, -10)$  under  $SU(5) \times U(1)_\chi$ , which may be used to break  $U(1)_\chi$  at the TeV scale and would allow  $\nu^c$  (the right-handed neutrino) to obtain a large Majorana mass, thereby triggering the canonical seesaw mechanism for small Majorana neutrino masses. This is usually described as lepton number  $L$  breaking to lepton parity  $(-1)^L$  [2], but here it is clear that it has to do with the breaking of gauge  $U(1)_\chi$  to  $(-1)^{Q_\chi}$ .

In the minimal supersymmetric standard model (MSSM),  $R = (-1)^{3B+L+2j}$  is used to distinguish the SM particles from their superpartners, which belong thus to the dark sector if  $R$  is assumed conserved. Since  $R$  is identical to  $(-1)^{3(B-L)+2j}$ , it has long been recognized [3] that a theory with gauge  $B - L$ , broken by two units, would be a natural framework for dark matter. For an incomplete list of papers on this topic and discussions on their relevance, see Ref. [1]. In particular, the decomposition

$$SO(10) \rightarrow SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)/2} \quad (9)$$

shows that the fermionic  $\underline{16}$  of  $SO(10)$  contains

$$(u, d) \sim (3, 2, 1, 1/6), \quad (d^c, u^c) \sim (3^*, 1, 2, -1/6), \quad (10)$$

$$(\nu, e) \sim (1, 2, 1, -1/2), \quad (e^c, \nu^c) \sim (1, 1, 2, 1/2). \quad (11)$$

Hence  $3(B - L)$  is odd for all quarks and leptons. As for the scalar sector, the  $\underline{10}$  representation contains the bidoublet  $\Phi \sim (1, 2, 2, 0)$ . Hence its  $3(B - L)$  is even. In other words,  $(-1)^{3(B-L)}$  coincides with  $(-1)^{Q_\chi}$ . However, the former requires a left-right intermediate scale, whereas the latter does not. They are thus conceptually and phenomenologically distinct. In this study,  $U(1)_\chi$  separates from  $SU(3)_C \times SU(2)_L \times U(1)_Y$  at the unification scale [1], and its symmetry breaking scale is independent of the electroweak scale.

It should also be pointed out that in  $E_6 \rightarrow SO(10) \times U(1)_\psi$ , the decomposition  $27 = (16, -1) + (10, 2) + (1, -4)$  shows that  $Q_\psi$  may be invoked as the underlying dark symmetry as well.

## REAPPRAISAL OF $Z_2$ DARK MATTER

It has been remarked that it is very easy to invent a model of dark matter. The first step is to introduce a new  $Z_2$  symmetry under which all SM particles are even and a new neutral particle of your choice is odd. It should then have the appropriate mass and interaction to account for the relic abundance of dark matter in the Universe, but not excluded by direct or indirect search experiments.

The simplest such model [4] assumes a real scalar singlet, odd under  $Z_2$ . It has been studied extensively [5] and is still a viable explanation of dark matter. In the framework of  $U(1)_\chi$ , a scalar with odd  $(-1)^{Q_\chi+2j}$  requires it to have odd  $Q_\chi$ . The scalar singlet  $s \sim (1, -5)$  of  $\underline{16}$  is such a particle. It is in fact the scalar analog of  $\nu^c$ . They have the same  $Q_\chi$ , but differ in spin. Hence one is dark matter and the other is not. In Ref. [2], they are both assigned odd lepton parity, which is now replaced by odd  $\chi$  parity. If  $U(1)_\chi$  is indeed the origin of  $s$ , then it should be complex and it should have  $Z_\chi$  interactions. However, from the allowed  $\zeta^*ss$  trilinear scalar coupling,  $s$  splits into two real scalars with a large mass gap. The lighter is dark matter and the heavier decays into the lighter plus a physical or virtual  $Z_\chi$  gauge boson. This would not affect the lighter scalar's suitability as dark matter, but would predict possible verifiable signatures involving  $Z_\chi$ . Note that the elastic scattering of a real scalar singlet off nuclei through  $Z_\chi$  is forbidden because both real and imaginary parts of a complex scalar are needed to construct a vector current. The present experimental bound on  $M_{Z_\chi}$  is about 4.1 TeV from LHC (Large Hadron Collider) data [6, 7], which may be improved [8] with further study.

Instead of choosing  $s \sim (1, -5)$  from the  $\underline{16}$  of  $SO(10)$ , the scalar doublet  $(\eta^0, \eta^-) \sim (1, 2, -1/2, 3)$  may also be considered [9, 10, 11]. In that case, it is distinguished from  $(\phi_1^0, \phi_1^-) \sim (1, 2, -1/2, -2)$  and  $(\phi_2^+, \phi_2^0) \sim (1, 2, 1/2, 2)$  by their  $Q_\chi$  charge. Hence  $\Phi_{1,2}$  are even but  $\eta$  is odd under  $(-1)^{Q_\chi+2j}$ . This  $Z_2$  discrete symmetry [12] allows  $\eta^0$  to be dark matter [13], at least in principle. However, its interaction with quarks through the  $Z$  boson rules it out by direct-search experiments. In the SM, the allowed quartic coupling  $(\Phi^\dagger \eta)^2$  serves to split  $Re(\eta^0)$  from  $Im(\eta^0)$ , and since  $Z$  only couples one to the other, the offending interaction with quarks is avoided kinematically in elastic nuclear recoil with a mass gap larger than a few hundred keV. This is known as the inert Higgs doublet model. In the case of  $Q_\chi$ , such a quartic coupling is forbidden, so if  $\eta$  originates from  $U(1)_\chi$ , other particles are needed for it to be dark matter. They turn out to be exactly  $s \sim (1, -5)$  and  $\zeta \sim (1, -10)$ , already discussed. The allowed couplings  $(\eta^0 \phi_2^0 - \eta^- \phi_2^+)s$ ,  $(\bar{\phi}_1^0 \eta^0 + \phi_1^+ \eta^-)s$  combined with  $\zeta^*ss$  form the necessary effective quartic coupling as shown in Fig. 1. In this scenario, a linear combination of  $\eta^0$  and  $s$  is dark matter.

Another possible simple model of dark matter is to have a singlet fermion  $N_L \sim (1, 1, 0, 0)$  from the  $SU(5) \times U(1)_\chi$  (24, 0) or (1, 0) of the  $\underline{45}$  of  $SO(10)$ . Since  $N$  has even  $Q_\chi$ , it is odd under  $(-1)^{Q_\chi+2j}$ . However, it has no renormalizable interaction with the particles of the SM and thus not a good dark-matter candidate without some additional fundamental particle such as a singlet scalar [14, 15] which has  $Q_\chi = 0$ , i.e. the

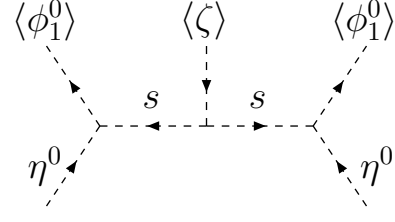


FIGURE 1: Effective quartic  $(\Phi_1^\dagger \eta)^2$  coupling.

scalar counterpart of  $N$ . A more interesting option is to combine  $N$  with the scalar doublet  $\eta$  discussed in the previous paragraph because there is now an allowed Yukawa coupling between the left-handed lepton doublet  $(\nu, l)_L$  with  $N$  through  $\eta$ , i.e.  $(\bar{\eta}^0 \nu_L + \eta^+ l_L)N_L$ . This forms the basis of the scotogenic model [16] of radiative neutrino mass. Whereas the original model assumes the  $(\Phi^\dagger \eta)^2$  quartic scalar coupling, it must now be replaced by the effective operator of Fig. 1. The resulting diagram [17] for generating a radiative Majorana neutrino mass is then given by Fig. 2.

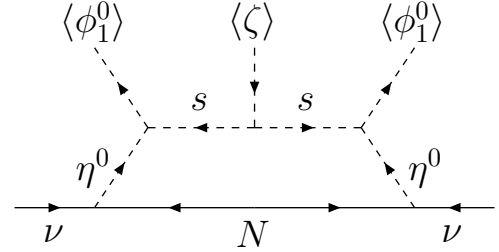


FIGURE 2: Scotogenic Neutrino Mass.

Whereas  $N$  could be dark matter, its only interaction with the particles of the SM is through the left-handed lepton doublet, and is known [18] to be restricted phenomenologically, thus limiting its viability as thermal dark matter. Hence a linear combination of  $s$  and  $\eta^0$  is again the likely dark-matter candidate in this case. They both couple to  $Z_\chi$  but differently. Further study is then needed to reappraise this  $U(1)_\chi$  interpretation of the scotogenic model.

Once both  $s$  and  $N$  are present, the coupling  $s^* N \nu^c$  is allowed. This has also recently been considered [19, 20] with the assumption that it is very small so that a freeze-in mechanism applies to the decay of  $\nu^c$  to  $s$  and  $N$ .

## SEESAW DARK MATTER

In the  $U(1)_\chi$  model, the singlet neutrino  $\nu^c \sim (1, -5)$  gets a large Majorana mass from the scalar  $\zeta \sim (1, -10)$ , both of which have even  $(-1)^{Q_\chi+2j}$ . This realizes the scenario of seesaw neutrino mass at the scale  $\langle \zeta \rangle = u$  which may be TeV or higher. Suppose the fermion singlets

$$N \sim (1, 0), \quad D_\chi \sim (1, -10), \quad (12)$$

from the  $\underline{45}$ ,  $\underline{126}$  representations of  $SO(10)$  are added, then the allowed Yukawa coupling  $f_D \zeta^* D_\chi N$  combined with a large Majorana mass for  $N$  would induce a small seesaw mass for  $D_\chi$ . Note that both  $N$  and  $D_\chi$  have odd  $(-1)^{Q_\chi+2j}$ . Hence  $D_\chi$  could be naturally light dark matter, i.e.  $m_{D_\chi} = f_D^2 u^2 / m_N$ , in parallel with the seesaw neutrino mass, i.e.  $m_\nu = f_\nu^2 v^2 / f_\nu u$ .

As for gauge  $U(1)_\chi$  anomaly cancellation, the fermion  $D_\chi^c \sim (1, 10)$  from the  $126^*$  of  $SO(10)$  should be added. It may combine with  $D_\chi$  to form a Dirac fermion, as proposed recently [21]. Here  $D_\chi^c$  is assumed to have an extra symmetry shared by the counterpart singlet  $N^c \sim (1, 0)$ . This separate system is also assumed to be heavy and annihilate efficiently to SM particles through  $Z_\chi$  in the early Universe. Another possible but different connection between seesaw neutrino mass and dark matter has also been proposed [22], based on an imposed  $Z_4$  discrete symmetry and a nonrenormalizable dimension-five coupling.

Consider now the interaction of  $D_\chi$ . It interacts mainly with  $Z_\chi$ . This is in analogy with  $\nu$  which interacts mainly with  $Z$  and  $W^\pm$ . Just as  $\nu$  decouples at a temperature of order 1 MeV,  $D_\chi$  would decouple at a temperature of order  $T \sim 1 \text{ MeV} (m_{Z_\chi}/m_Z)^{4/3}$ . There remains however a suppressed Yukawa coupling to  $\zeta_R = \sqrt{2}(\text{Re}(\zeta) - u)$ , i.e.

$$\frac{f_D}{\sqrt{2}} \frac{f_{Du}}{m_N} \zeta_R D_\chi D_\chi = \frac{m_{D_\chi}}{\sqrt{2}u} \zeta_R D_\chi D_\chi. \quad (13)$$

Since  $m_{\zeta_R}$  is heavy, the above interaction is only realized through  $HD_\chi D_\chi$ , coming from the mixing of the SM Higgs boson  $H$  with  $\zeta$ , which is itself also suppressed, i.e. of order  $v/u$ . With these two suppressions, the resulting interaction strength will be very small, as shown below.

## HIGGS DECAY TO DARK MATTER

The particles beyond the SM are the  $Z_\chi$  gauge boson, the complex scalar  $\zeta$  which breaks  $U(1)_\chi$  and couples to  $\nu^c \nu^c$ , together with the  $N$  and  $D_\chi$  fermion singlets of Eq. (12) which belong to the dark sector. Whereas there are two Higgs doublets, i.e.  $\Phi_{1,2}$  of Eq. (7), one linear combination with the vacuum expectation value  $v = \sqrt{v_1^2 + v_2^2}$  is the SM analog and corresponds to the observed 125 GeV boson at the LHC; the other is heavier and is not relevant to the discussion below.

The scalar interactions between the SM Higgs  $H$  and  $\zeta$  is given by

$$V = \mu_1^2 \Phi^\dagger \Phi + \mu_2^2 \zeta^* \zeta + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_2 (\zeta^* \zeta)^2 + \lambda_3 (\Phi^\dagger \Phi) (\zeta^* \zeta), \quad (14)$$

where  $\Phi = (0, v + H/\sqrt{2})$  and  $\zeta = u + \zeta_R/\sqrt{2}$ . The mass-squared matrix spanning  $(H, \zeta_R)$  is then

$$\mathcal{M}_{H\zeta}^2 = \begin{pmatrix} 2\lambda_1 v^2 & 2\lambda_3 v u \\ 2\lambda_3 v u & 2\lambda_2 u^2 \end{pmatrix}. \quad (15)$$

The  $H - \zeta_R$  mixing is then given by  $\lambda_3 v / \lambda_2 u$ . Hence the  $HD_\chi D_\chi$  coupling is

$$f_H = \frac{m_{D_\chi}}{\sqrt{2}u} \frac{\lambda_3 v}{\lambda_2 u} = \frac{\sqrt{2} \lambda_3 v m_{D_\chi}}{m_{\zeta_R}^2}. \quad (16)$$

The decay rate  $\Gamma_H$  of  $H \rightarrow D_\chi D_\chi$  is then

$$\Gamma_H = \frac{f_H^2 m_H}{8\pi} \sqrt{1 - 4x^2} (1 - 2x^2), \quad (17)$$

where  $x = m_{D_\chi}/m_H$ . If the reheating temperature of the Universe after inflation is below the decoupling temperature of

$D_\chi$  for thermal equilibrium and above  $m_H$ , its only production mechanism is freeze-in through  $H$  decay before the latter decouples from the thermal bath. The correct relic abundance is possible if  $f_H$  is very small. Hence  $D_\chi$  could be FIMP (Feebly Interacting Massive Particle) dark matter [23], and for  $x \ll 1$ , the right number density is obtained for [24]

$$f_H \sim 10^{-12} x^{-1/2}. \quad (18)$$

As a numerical example which satisfies all the above conditions, let  $m_{D_\chi} = 5 \text{ GeV}$ , then  $x = 0.04$ . Assuming  $\lambda_3 = 0.4$ , then  $f_H = 5 \times 10^{-12}$  in Eq. (16) is obtained with  $m_{\zeta_R} = 10^7 \text{ GeV}$ . Assuming that this is also the value of  $m_{Z_\chi}$ , then the decoupling temperature of  $D_\chi$  is about 5.2 TeV.

Since the  $U(1)_\chi$  breaking scale is about  $10^7 \text{ GeV}$  in this example of seesaw dark matter, the  $Z_\chi$  gauge boson is much too heavy to be discovered at the LHC. It is also not relevant in the thermal interactions of the SM particles with the dark sector below 5.2 TeV. Similarly, the elastic scattering of  $D_\chi$  with nuclei through  $Z_\chi$  exchange is very much suppressed, so that it is not detectable in direct-search experiments.

## CONCLUDING REMARKS

Using  $Q_\chi$  as a marker in  $SO(10) \rightarrow SU(5) \times U(1)_\chi$  so that  $(-1)^{Q_\chi + 2j}$  distinguishes dark matter from matter, previous simple models of dark matter are reappraised. Furthermore, the notion is put forward that naturally light seesaw dark matter exists in parallel with naturally light seesaw neutrinos. In the latter, the left-handed doublet neutrino  $\nu$  couples to a heavy singlet right-handed neutrino  $\nu^c$  through the SM Higgs doublet  $\Phi$ , and  $\nu^c$  acquires a large Majorana mass through the singlet scalar  $\zeta$  which also breaks  $U(1)_\chi$  and makes  $Z_\chi$  massive. As a result,  $\nu$  gets a small seesaw mass. In the former, the fermion singlet  $N \sim (1, 0)$  under  $SU(5) \times U(1)_\chi$  has an allowed large Majorana mass, whereas the singlet  $D_\chi \sim (1, -10)$  couples to  $N$  through  $\zeta$ , thereby generating a small Majorana mass for  $D_\chi$ . As an example,  $m_{\nu^c} \sim 10^7 \text{ GeV}$ ,  $m_\nu \sim 0.1 \text{ eV}$ ,  $m_N \sim 10^{14} \text{ GeV}$ ,  $m_{D_\chi} \sim \text{GeV}$  may be obtained. Note that the anchor scale  $\langle \zeta \rangle = u$  for seesaw neutrino mass is the intermediate scale for seesaw dark matter.

Below the temperature of order  $T \sim 1 \text{ MeV} (m_{Z_\chi}/m_Z)^{4/3}$ ,  $D_\chi$  is out of thermal equilibrium with the SM particles. However, there is a suppressed Yukawa interaction  $f_H HD_\chi D_\chi$  which allows it to be produced through Higgs decay before the Universe cools below  $m_H$ . It may thus be freeze-in FIMP dark matter and escape present experimental detection, directly or indirectly.

As for the grand unification of  $SU(5) \times U(1)_\chi$ , it is well-known that the SM particle content is inadequate for the gauge couplings to converge at a common mass scale. This is however easily solved by the addition of new particles at intermediate scales as explicitly shown in Ref. [1]. It is also shown that it is possible to have the unification scale greater than  $10^{16} \text{ GeV}$ , thus avoiding the constraint from proton decay. For each model variation considered in this paper, a full discussion of unification would require a similar set of new particles. However, the main purpose of this paper is to point out the rich physics possibilities of the  $U(1)_\chi$  extension regarding dark matter. Other possible new particles depend on the specific (but mostly arbitrary) scenario chosen for unification. The details of how any

previous proposed simple model may be fully developed in the  $SU(5) \times U(1)_\chi$  context are left for future investigations.

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