

Polarizability of the nucleon

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Abstract

The status of the experimental and theoretical investigations on the polarizabilities of the nucleon is presented. This includes a confirmation of the validity of the previously introduced *recommended* values of the polarizabilities [1, 2]. It is shown that the most reliable approach to a prediction of the polarizabilities is obtained from the nonsubtracted dispersion theory, where the appropriate degrees of freedom taken from other precise experimental data are taken into account. The present values of the *recommended* polarizabilities are $\alpha_p = 12.0 \pm 0.5$, $\beta_p = 1.9 \mp 0.5$, $\alpha_n = 12.6 \pm 1.2$, and $\beta_n = 2.6 \mp 1.2$ in units of 10^{-4}fm^3 and $\gamma_\pi^{(p)} = -36.4 \pm 1.5$, $\gamma_\pi^{(n)} = +58.6 \pm 4.0$, $\gamma_0^{(p)} = -0.58 \pm 0.20$, and $\gamma_0^{(n)} = +0.38 \pm 0.22$ in units of 10^{-4}fm^4 .

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given by [1]

$$\mathbf{d} = 4\pi \alpha \mathbf{E}, \quad (1)$$

$$\mathbf{m} = 4\pi \beta \mathbf{H}, \quad (2)$$

1. INTRODUCTION

The polarizabilities belong to the fundamental structure constants of the nucleon, in addition to the mass, the electric charge, the spin, and the magnetic moment. The proposal to measure the polarizabilities dates back to the 1950s. Two experimental options were considered: (i) Compton scattering by the proton and (ii) the scattering of slow neutrons in the Coulomb field of heavy nuclei. The idea was that the nucleon with its “pion cloud”, i.e., pions being part of the constituent-quark structure, obtains an electric dipole moment under the action of an electric field vector which is proportional to the electric polarizability. After the discovery of the photoexcitation of the Δ resonance, it became obvious that the nucleon also should have a strong paramagnetic polarizability, because of a virtual spin-flip transition of one of the constituent quarks due to the magnetic field vector provided by a real photon in a Compton scattering experiment. However, experiments showed that this expected strong paramagnetism is not observed. Apparently a strong diamagnetism exists which compensates the expected strong paramagnetism. Though this explanation is straightforward, how it may be understood in terms of the structure of the nucleon remained unknown [1]. A solution of this problem was found later when it was shown that the diamagnetism is a property of the structure of the constituent quarks [3, 4, 5, 6, 7]. In retrospect, this is not a surprise, because constituent quarks generate their mass mainly through the interaction with the QCD vacuum via the exchange of a σ meson. This mechanism is predicted by the linear σ model on the quark level (QL σ M) [2] which also predicts the mass of the σ meson to be $m_\sigma=666$ MeV. The σ meson has the capability of interacting with two photons being in parallel planes of linear polarization. We will show in the following that the σ meson as part of the constituent-quark structure, therefore, provides the largest part of the electric polarizability and the total diamagnetic polarizability.

2. DEFINITION OF ELECTROMAGNETIC POLARIZABILITIES

A nucleon in an electric field \mathbf{E} and a magnetic field \mathbf{H} obtains an electric dipole moment \mathbf{d} and magnetic dipole moment \mathbf{m}

in a unit system where the electric charge e is given by $e^2/4\pi = \alpha_{em} = 1/137.04$. The proportionality constants α and β are denoted as the electric and magnetic polarizabilities, respectively. These polarizabilities may be understood as a measure of the response of the nucleon structure to the fields provided by a real or virtual photon, and it is evident that we need a second photon to measure the polarizabilities. This may be expressed through the relations

$$\delta W = -\frac{1}{2} 4\pi \alpha \mathbf{E}^2 - \frac{1}{2} 4\pi \beta \mathbf{H}^2, \quad (3)$$

where δW is the energy change in the electromagnetic field due to the presence of the nucleon in the field. The definition implies that the polarizabilities are measured in units of a volume, i.e., in units of fm^3 ($1 \text{ fm}=10^{-15} \text{ m}$).

3. MODES OF TWO-PHOTON REACTIONS AND EXPERIMENTAL METHODS

Static electric fields of sufficient strength are provided by the Coulomb field of heavy nuclei. Therefore, the electric polarizability of the neutron can be measured by scattering slow neutrons in the electric field \mathbf{E} of a Pb nucleus. The neutron has no electric charge. Therefore, two simultaneously interacting electric field vectors (two virtual photons) are required to produce a deflection of the neutron. Then, the electric polarizability can be obtained from the differential cross section measured at a small deflection angle. A further possibility is provided by Compton scattering of real photons by the nucleon, where during the scattering process two electric and two magnetic field vectors simultaneously interact with the nucleon.

In the following, we discuss the experimental options we have to measure the polarizabilities of the nucleon. As outlined above, two photons are needed which simultaneously interact with the electrically charged parts of the nucleon. These photons may be in parallel or perpendicular planes of linear polarization and in these two modes measure the polarizabilities α , β , or spin polarizabilities γ , respectively. The spin polarizability is nonzero only for particles having a spin.

In total, the experimental options discussed above provide us with 6 combinations of two electric and magnetic field vectors. These are described in the following.

- For photons in parallel planes of linear polarization, we have

$$\begin{aligned} \text{(case 1)} \quad \alpha &: \mathbf{E} \uparrow \uparrow \mathbf{E}', \\ \text{(case 2)} \quad \beta &: \mathbf{H} \rightarrow \rightarrow \mathbf{H}', \\ \text{(case 3)} \quad -\beta &: \mathbf{H} \rightarrow \leftarrow \mathbf{H}'. \end{aligned} \quad (4)$$

- For photons in perpendicular planes of linear polarization, we have

$$\begin{aligned} \text{(case 4)} \quad \gamma_E &: \mathbf{E} \uparrow \rightarrow \mathbf{E}', \\ \text{(case 5)} \quad \gamma_H &: \mathbf{H} \rightarrow \downarrow \mathbf{H}', \\ \text{(case 6)} \quad -\gamma_H &: \mathbf{H} \rightarrow \uparrow \mathbf{H}'. \end{aligned} \quad (5)$$

Case (1) corresponds to the measurement of the electric polarizability α via two parallel electric field vectors \mathbf{E} and \mathbf{E}' . These parallel electric field vectors may be provided as longitudinal photons either by the Coulomb field of a heavy nucleus or by Compton scattering in the forward direction or by reflecting the photon by 180° . Real photons simultaneously provide transverse electric \mathbf{E} and magnetic \mathbf{H} field vectors. This means that in a Compton scattering experiment linear combinations of electric and magnetic polarizabilities and linear combinations of electric and magnetic spin polarizabilities are measured. The combination of case (1) and case (2) measures $\alpha + \beta$ and is observed in forward-direction Compton scattering. The combination of case (1) and case (3) measures $\alpha - \beta$ and is observed in backward-direction Compton scattering. The combination of case (4) and case (5) measures $\gamma_0 \equiv \gamma_E + \gamma_H$ and is observed in forward-direction Compton scattering. The combination of case (4) and case (6) measures $\gamma_\pi \equiv \gamma_E - \gamma_H$ and is observed in backward-direction Compton scattering. Compton scattering experiments exactly in the forward direction and exactly in the backward direction are not possible from a technical point of view. Therefore, the respective quantities have to be extracted from Compton scattering experiments carried out at intermediate angles.

4. EXPERIMENTAL RESULTS

The experimental polarizabilities of the proton (p) and the neutron (n) may be summarized as follows:

$$\begin{aligned} \alpha_p &= 12.0 \pm 0.5, & \beta_p &= 1.9 \mp 0.5, \\ \alpha_n &= 12.6 \pm 1.2, & \beta_n &= 2.6 \mp 1.2, \end{aligned} \quad (6)$$

in units of 10^{-4} fm^3 .

The experimental spin polarizabilities of the proton (p) and neutron (n) are

$$\gamma_\pi^{(p)} = -36.4 \pm 1.5, \quad \gamma_\pi^{(n)} = 58.6 \pm 4.0, \quad (7)$$

in units of 10^{-4} fm^4 .

The experimental polarizabilities of the proton have been obtained as an average from a larger number of Compton scattering experiments [1]. In addition, a recent reanalysis of these data leading to $\alpha_p = 12.03 \pm 0.72$ has been taken into account [8]. The experimental electric polarizability of the neutron is the average of an experiment on electromagnetic scattering of

a neutron in the Coulomb field of a Pb nucleus and a Compton scattering experiment on a quasifree neutron, i.e., a neutron separated from a deuteron during the scattering process. The two results are [1] $\alpha_n = 12.6 \pm 2.5$ from electromagnetic scattering of a slow neutron in the electric field of a Pb nucleus, and $\alpha_n = 12.5 \pm 2.3$ from quasifree Compton scattering by a neutron initially bound in the deuteron. In addition, the result obtained from the experimental electric polarizability of the proton α_p and the predicted ratio α_n/α_p leading to $\alpha_n = 12.7 \pm 0.9$ has been taken into account [9]. The average given above is obtained from these three numbers.

Furthermore, there have been experiments at the University of Lund (Sweden), where the electric polarizability of the neutron is determined through Compton scattering by the deuteron. The results obtained in this way are model dependent.

5. CALCULATION OF POLARIZABILITIES

Recently, great progress has been made in disentangling the total photoabsorption cross section into parts separated by the spin, the isospin, and the parity of the intermediate state [10, 11], using the meson photoproduction amplitudes of Drechsel et al. [12]. The spin of the intermediate state may be $s = 1/2$ or $s = 3/2$ depending on the spin directions of the photon and the nucleon in the initial state. The parity change during the transition from the ground state to the intermediate state is $\Delta P = \text{yes}$ for the multipoles $E1, M2, \dots$ and $\Delta P = \text{no}$ for the multipoles $M1, E2, \dots$. Calculating the respective partial cross sections from photomeson data, the following sum rules can be evaluated:

$$\alpha + \beta = \frac{1}{2\pi^2} \int_{\omega_0}^{\infty} \frac{\sigma_{\text{tot}}(\omega)}{\omega^2} d\omega, \quad (8)$$

$$\begin{aligned} \alpha - \beta &= \frac{1}{2\pi^2} \int_{\omega_0}^{\infty} \sqrt{1 + \frac{2\omega}{m}} [\sigma(\omega, E1, M2, \dots) \\ &\quad - \sigma(\omega, M1, E2, \dots)] \frac{d\omega}{\omega^2} + (\alpha - \beta)^t, \end{aligned} \quad (9)$$

$$\gamma_0 = -\frac{1}{4\pi^2} \int_{\omega_0}^{\infty} \frac{\sigma_{3/2}(\omega) - \sigma_{1/2}(\omega)}{\omega^3} d\omega, \quad (10)$$

$$\begin{aligned} \gamma_\pi &= \frac{1}{4\pi^2} \int_{\omega_0}^{\infty} \sqrt{1 + \frac{2\omega}{m}} \left(1 + \frac{\omega}{m}\right) \\ &\quad \cdot \sum_n P_n [\sigma_{3/2}^n(\omega) - \sigma_{1/2}^n(\omega)] \frac{d\omega}{\omega^3} + \gamma_\pi^t, \end{aligned} \quad (11)$$

$$\begin{aligned} P_n &= -1 \text{ for } E1, M2, \dots \text{ multipoles,} \\ P_n &= +1 \text{ for } M1, E2, \dots \text{ multipoles,} \end{aligned} \quad (12)$$

$$\begin{aligned} (\alpha - \beta)^t &= \frac{1}{2\pi} \left[\frac{g_{\sigma NN} M(\sigma \rightarrow \gamma\gamma)}{m_\sigma^2} \right. \\ &\quad + \frac{g_{f_0 NN} M(f_0 \rightarrow \gamma\gamma)}{m_{f_0}^2} \\ &\quad \left. + \frac{g_{a_0 NN} M(a_0 \rightarrow \gamma\gamma)}{m_{a_0}^2} \tau_3 \right], \end{aligned} \quad (13)$$

$$\gamma_{\pi}^t = \frac{1}{2\pi m} \left[\frac{g_{\pi NN} M(\pi^0 \rightarrow \gamma\gamma)}{m_{\pi^0}^2} \tau_3 + \frac{g_{\eta NN} M(f_0 \rightarrow \gamma\gamma)}{m_{\eta}^2} + \frac{g_{\eta' NN} M(\eta' \rightarrow \gamma\gamma)}{m_{\eta'}^2} \right], \quad (14)$$

where ω is the photon energy in the lab frame. The sum rules for $\alpha + \beta$ and γ_0 depend on nucleon-structure degrees of freedom only, whereas the sum rules for $\alpha - \beta$ and γ_{π} have to be supplemented by the quantities $(\alpha - \beta)^t$ and γ_{π}^t , respectively. These are t -channel contributions which may be interpreted as contributions of scalar and pseudoscalar mesons being parts of the constituent-quark structure. The sum rule for $\alpha + \beta$ depends on the total photoabsorption cross section and, therefore, does not require a disentangling with respect to quantum numbers. The sum rule for $\alpha - \beta$ requires a disentangling with respect to the parity change of the transition. The sum rule for γ_0 requires a disentangling with respect to the spin of the intermediate state. The sum rule for γ_{π} requires a disentangling with respect to spin and parity change.

The t -channel contributions depend on those scalar and pseudoscalar mesons which (i) are part of the structure of the constituent quarks and (ii) are capable of coupling to two photons. These are the mesons $\sigma(600)$, $f_0(980)$, and $a_0(980)$ in case of $(\alpha - \beta)^t$ and the mesons π^0 , η , and η' in case of γ_{π}^t . The contributions are dominated by the σ and the π^0 mesons whereas the other mesons only lead to small corrections.

6. RESULTS OF CALCULATION

The results of the calculation are summarized in the following eight equations [10, 11]:

$$\alpha_p = +4.5 \text{ (nucleon)} + 7.6 \text{ (const. quark)} = +12.1, \quad (15)$$

$$\beta_p = +9.4 \text{ (nucleon)} - 7.6 \text{ (const. quark)} = +1.8, \quad (16)$$

$$\alpha_n = +5.1 \text{ (nucleon)} + 7.6 \text{ (const. quark)} = +12.7, \quad (17)$$

$$\beta_n = +10.1 \text{ (nucleon)} - 7.6 \text{ (const. quark)} = +2.5, \quad (18)$$

in units of 10^{-4}fm^3 ,

$$\gamma_0^{(p)} = -0.58 \pm 0.20 \text{ (nucleon)}, \quad (19)$$

$$\gamma_0^{(n)} = +0.38 \pm 0.22 \text{ (nucleon)}, \quad (20)$$

$$\gamma_{\pi}^{(p)} = +8.5 \text{ (nucleon)} - 45.1 \text{ (const. quark)} = -36.6, \quad (21)$$

$$\gamma_{\pi}^{(n)} = +10.0 \text{ (nucleon)} + 48.3 \text{ (const. quark)} = +58.3, \quad (22)$$

in units of 10^{-4}fm^4 .

The electric polarizabilities α_p and α_n are dominated by a smaller component due to the pion cloud (nucleon) and a larger component due to the σ meson as part of the constituent-quark structure (const. quark). The magnetic polarizabilities β_p and β_n have a large paramagnetic part due to the spin structure of the nucleon (nucleon) and an only slightly smaller diamagnetic part due to the σ meson as part of the constituent-quark structure (const. quark). The contributions of the σ meson may be supplemented by small corrections due to $f_0(980)$ and $a_0(980)$ mesons [6, 7, 10, 11]. These contributions are disregarded here because of their smallness and uncertainties [9].

The spin polarizabilities $\gamma_0^{(p)}$ and $\gamma_0^{(n)}$ are dominated by destructively interfering components from the pion cloud and the spin structure of the nucleon. The different signs obtained for the proton and the neutron are due to this destructive interference [11]. The spin polarizabilities $\gamma_{\pi}^{(p)}$ and $\gamma_{\pi}^{(n)}$ have a minor component due to the structure of the nucleon (nucleon) and a major component due to the pseudoscalar mesons π^0 , η , and η' as structure components of the constituent quarks (const. quark).

Differing from other theoretical approaches, the presently applied dispersion theory is based on fundamental relations only. The precision of the results of the present calculation only depends on the precision of the photomeson data used as an input. A consideration shows that the errors of the results given in Equations (15)-(22) are of the same order of magnitude as or somewhat smaller than those of the corresponding experimental results.

7. DISCUSSION

In a first approach, the electric polarizabilities of proton and neutron have been related to the dipole moment of the transition

$$p \rightarrow n + \pi^+ \quad \text{and} \quad n \rightarrow p + \pi^-. \quad (23)$$

Since the $n + \pi^+$ dipole moment is smaller than the $p + \pi^-$ dipole moment, we expect that the related contributions to the electric polarizabilities in Equations (15) and (17) are smaller for the proton than those for the neutron. This is in agreement with the observation where $\alpha_p(\text{nucleon}) = +4.5$ (Eq. 15) and $\alpha_n(\text{nucleon}) = +5.1$ (Eq. 17) are given. The difference between the two numbers, namely, $\alpha_n(\text{nucleon}) - \alpha_p(\text{nucleon}) = 0.6$, precisely corresponds to the difference between the electric polarizabilities of neutron and proton, as seen in Equations (15) and (17). The reason for this agreement is that the constituent-quark parts of the two polarizabilities are the same.

The quantity 7.6 (const. quark) entering into Equations (15) to (18) corresponds to the σ meson as part of the constituent-quark structure. This quantity has a positive sign when being part of the electric polarizabilities or a negative sign when representing the diamagnetic polarizabilities. The investigation of these quantities has been carried out previously in a number of publications [13, 14, 15, 16, 17]. All the relevant information may be found in these publications.

The meaning of the spinpolarizabilities in relation to the structure of the nucleon is less straightforward than that of the polarizabilities.

In addition to dispersion theory, chiral perturbation theory plays a prominent role in the current investigations of nucleon Compton scattering and polarizabilities. Therefore, it is advisable to carry out a comparison of the two approaches. This is done in Table 1.

One essential difference between the two versions is the missing t -channel contribution in the BChPT version. The t -channel provides the total diamagnetism and the largest part of the electric polarizability. Another essential difference is contained in the $N\pi$ component of the electric polarizability. The procedure used in the BChPT method corresponds to the Born approximation of the DR method, leading to an error in the BChPT method of more than a factor of 2.

	$\alpha_p(\text{BChPT})$	$\alpha_p(\text{DR})$	$\beta_p(\text{BChPT})$	$\beta_p(\text{DR})$
$N\pi$	+6.9	+3.09	-1.8	+0.48
$\Delta\pi$	+4.4	+1.4	-1.4	+0.4
Δ -pole	-0.1	-0.01	+7.1	+8.56
t-channel	-	+7.6	-	-7.6
Total	+11.2	+12.1	+3.9	+1.8

TABLE 1: Predicted electric α_p and magnetic β_p polarizabilities for the proton, where BChPT [18] denotes covariant chiral perturbation theory and DR dispersion theory

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