Applying New Physics to the Problems of the $\eta' \to \pi^0\gamma\gamma$ Decay

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Abstract

Rare decays of light mesons may be a discovery window for a new weakly coupled forces hidden at a low-energy QCD scale. BES-III Collaboration reported an observation of the rare decay $\eta' \to \pi^0\gamma\gamma$. The observed decay width disagrees with the preliminary theoretical estimates. We show that this tension may be attributed to New Physics, presumably Dark Photon. For completeness, we consider a possible influence of New Physics on a similar well-measured decay $\eta \to \pi^0\gamma\gamma$ and a recently measured one, $\eta' \to \eta\gamma\gamma$, showing that the impact of the hypothetical Dark Photon may be also present in these decays.

Keywords: Rare decays, $\eta' \to \pi^0\gamma\gamma$ decay, VMD, ChPT, $L\sigma M$, unitarity conditions, New Physics, Dark Photon.
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1. INTRODUCTION

Dark Photon is a hypothetical particle that should be a force carrier similar to the photon in electromagnetism and possibly connected to Dark Matter particles and can be weakly coupled with the visible charged particles by a kinetic mixing with the usual photon [1]. There are a number of anomalies which potentially could be caused by Dark Photon since it may be coupled with the usual photon, for instance, beryllium anomaly [2], Dark Matter effects and astrophysics [3, 4], muon $(g-2)$ [5, 6, 7], and possibly "proton radius puzzle" [8, 9]. Moreover, Dark Photon is a DM candidate itself [10]. There are Dark Photon searches carried out at JLab [11] and at CERN [12].

Nevertheless, in these anomalies, searches, and observations, the hypothetical Dark Photon has predominantly *leptonic* coupling. On the contrary, in [13], a model of the Dark Photon (or "$\mathcal{B}$ boson") was proposed, which has a coupling to quarks dominating over the coupling to leptons.

Moreover, this area is not yet covered by the Beyond Standard Model searches. Consequently, rare decays $\eta' \to \pi^0\gamma\gamma$, $\eta' \to \eta\gamma\gamma$, and $\eta \to \pi^0\gamma\gamma$ could serve as a probe of Beyond Standard Model Physics of such kind [13].

The proposed interaction Lagrangian has the following form:

$$\mathcal{L}_{int} = (\frac{1}{3}g_B + \epsilon \cdot Q_q \cdot e) \cdot \bar{q}\gamma^\mu q \cdot \mathcal{B}_\mu - \epsilon \cdot e \cdot \bar{l}\gamma^\mu l \cdot \mathcal{B}_\mu, \quad (1)$$

where ϵ is an adjustable parameter. $\mathcal{B}$ boson mass was estimated in the range 140 MeV–1 GeV. It should have the same quantum numbers as ω meson $I^G(J^{PC}) = 0^-(1^{--})$ to preserve the symmetries of low-energy QCD [13].

Dark Photon (or "$\mathcal{B}$ boson") could manifest itself as a resonance in rare decays of η, η', π, and ω mesons, including $\eta' \to \pi^0\gamma\gamma$ and similar ones, $\eta \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$.

Since hypothetical Dark Photon may mix with the regular photon, it may also be coupled with all the three lightest vector mesons. However, the mixing with ω meson should be dominant. Photon is a linear combination of isoscalar and an isovector. Both ω meson and hypothetical Dark Photon are isoscalars, and ρ meson is an isovector. $\phi(1020)$ is also an isoscalar; however it gives a negligibly small contribution $\sim 1\%$ to the width of these decays. Therefore, for purposes of our paper, we neglect possible mixings of Dark Photon with vector mesons other than ω.

The branching ratio of $\eta' \to \pi^0\gamma\gamma$ decay reported by BES-III Collaboration is as follows [14]:

$$BR(\eta' \to \gamma\gamma\pi^0)_{Incl.} = (3.20 \pm 0.07(stat) \pm 0.23(sys)) \times 10^{-3}, \quad (2)$$

where the subscript *"Incl."* indicates the branching ratio of the inclusive decay $\eta' \to \gamma\gamma\pi^0$.

Unlike QED, in QCD, low-energy processes cannot be described by the strong coupling constant since it is large at low energies making the perturbative expansion meaningless.

Vector Meson Dominance (VMD) model [15] and Chiral Perturbation Theory (ChPT) [16] are low-energy effective theories of QCD. Preliminary estimations were done using their combination or a combination of VMD and $L\sigma M$ (Linear Sigma Model) which takes into account the scalar meson effects explicitly [17, 18, 19]. For the case of the $\eta \to \pi^0\gamma\gamma$ decay, a reasonable agreement with the experiment was achieved, thus supporting the validity of such an approach.

These estimations show that the decay is dominated by the intermediate vector mesons ω and ρ subsequently decaying into $\pi^0\gamma$ (Figure 1), and the decay width is estimated to be $\Gamma_{\eta' \to \pi^0\gamma\gamma} = 1.29$ keV [19], which is two times larger than the observed result [14]. Contributions of both the chiral loops and linear σ-terms are suppressed with respect to VMD on the level $\sim 10^{-3}$.

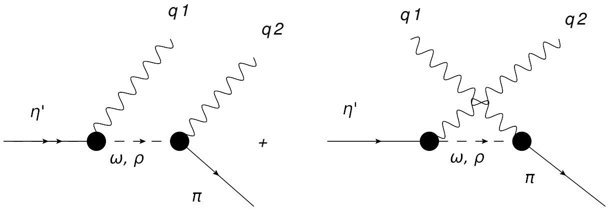

FIGURE 1: Leading order diagrams of $\eta' \to \pi^0\gamma\gamma$ decay.

In our calculation, we used the values of the coupling constants of vector mesons extracted directly from the known decays. Extracted in such a way, their uncertainty is determined only by the uncertainties of the branching ratios of the particles and their widths and masses. *However, as we will see, any choice of coupling constants is unable to explain the discrepancy between the theoretical prediction and the experimental result.*

We find that the theoretical branching ratio of the decay lies in the following range:

$$BR^{VMD+L\sigma M}(\eta' \to \pi^0\gamma\gamma)|_{Theoretical} = (7.88 \pm 1.26) \times 10^{-3}, \tag{3}$$

which is in a direct contradiction to the observed value, given in formula (2).

As we show, this discrepancy may be explained if we assume the existence of Dark Photon mixing with the ordinary ω meson without changing anything else in the model.

Besides, it is worth noting that recently an upper limit of the branching fraction of another decay $\eta' \to \eta\gamma\gamma$ was reported to be 1.33×10^{-4} at the 90% CL by BES-III Collaboration [20], which is again in a *direct contradiction* with the theoretical prediction [19]. So, there are discrepancies in other similar rare decays which could be possibly attributed to Dark Photon effects.

Consequently, for the completeness of our analysis, we consider a possible impact of $\mathcal{B}$ boson on the decay $\eta \to \pi^0\gamma\gamma$ in a similar manner to $\eta' \to \pi^0\gamma\gamma$ since there is sufficient data available [21, 22], and we show that its impact may be present in this decay also. However, if it is present, it should be smaller than that for the case of $\eta' \to \pi^0\gamma\gamma$.

For the case of $\eta' \to \eta\gamma\gamma$, the fit is not possible; yet, since the invariant mass spectrum is not provided by BES-III, only the overall branching ratio and its upper limit are given.

Nevertheless, we show that this tension can also be relaxed if we assume that the existence of Dark Photon effectively changes the coupling constant of ω meson.

We postpone the joint fit of the parameters of $\mathcal{B}$ boson from these three decays simultaneously till more data on $\eta' \to \eta\gamma\gamma$ becomes available, in particular $\gamma\gamma$ spectrum.

2. THEORETICAL PREDICTIONS

In the general case, the VMD amplitude of the decay corresponding to the diagram shown in Figure 1 is given by

$$A = (\frac{c_\omega}{D_\omega(t)} + \frac{c_\rho}{D_\rho(t)} + \frac{c_\phi}{D_\phi(t)})B(q_2) + \tag{4}$$

$$+(\frac{c_\omega}{D_\omega(u)} + \frac{c_\rho}{D_\rho(u)} + \frac{c_\phi}{D_\phi(u)})B(q_1), \tag{5}$$

where $t = (P_{\eta'} - q_2)^2$, $u = (P_{\eta'} - q_1)^2$, $q_{1,2}$ is the 4-momenta of outgoing photons, $P_{\eta'}$ is the 4-momentum of η' meson, and $D_{\omega,\rho,\phi}(t,u) = m^2_{\omega,\rho,\phi} - (t,u) - im_{\omega,\rho,\phi}\Gamma_{\omega,\rho,\phi}(t,u)$ is the propagator of vector meson (Breit−Wigner function). $B(q_{1,2})$ are kinematic coefficients representing the spin structure of the particles [19].

The VMD coupling constants are determined by g (the vector-pseudoscalar-pseudoscalar coupling constant of VMD), φ_P ($\eta' - \eta$ mixing angle), and φ_V ($\omega - \phi$ mixing angle, which is zero if OZI rule is applied). All three quantities appear without any reference to New Physics. In the limit of an exact OZI $\varphi_V = 0$, the coupling constants of ω and ρ mesons are the same $c_\omega = c_\rho$ and determined by the pseudoscalar mixing angle φ_P, and for ϕ meson, $c_\phi = 0$.

$c^{OZI}_\omega = c^{OZI}_\rho = (\frac{Ge}{\sqrt{2}g})^2 \cdot \frac{1}{3} \cdot Sin[\varphi_P]$, $c^{OZI}_\phi = 0$, where $G = \frac{3g^2}{4\pi^2 f_\pi}$, $g \approx 4.2$, and f_π is the pion decay constant.

Nevertheless, the $\eta - \eta'$ mixing angle is not uniquely defined. We derived the mixing angle in our previous work [23], to be $\varphi_P = 37.4^\circ \pm 0.4^\circ$ from the analysis of charge exchange reactions $\pi^- p$ and $K^- p$.

In [24], the previous results on the determination of the mixing angle data from different processes including strong decays of tensor and higher-spin mesons, electromagnetic decays of vector and pseudoscalar mesons, J/ψ decays into a vector and a pseudoscalar meson, and other transitions are summarized. They provide several values extracted in different ways: $\varphi_P = 44.2^\circ \pm 1.4^\circ; 43.2^\circ \pm 2.8^\circ; 40.7^\circ \pm 3.7^\circ; 42.7^\circ \pm 5.4^\circ; 41.0^\circ \pm 3.5^\circ; 41.2^\circ \pm 3.7^\circ; 50^\circ \pm 26^\circ; 36.5^\circ \pm 1.4^\circ; 42.4^\circ \pm 2.0^\circ; 40.2^\circ \pm 2.8^\circ$

Consequently, the coupling constants derived from the mixing angles can vary up to ∼ 30% which can lead to a variation in the predicted decay width up to ∼ 50%.

Therefore, we derive coupling constants directly from the known decays since their uncertainties are related only to the masses of particles, their widths, and branching ratios thus being smaller. However, as we show further, *for any choice of the coupling constants, there is a discrepancy between the theoretical prediction and the experimental results.*

In our approach, the constants of electromagnetic decays are $c_\omega = G_{\eta' \to \omega\gamma} \cdot G_{\omega \to \pi^0\gamma}$, $c_\rho = G_{\eta' \to \rho\gamma} \cdot G_{\rho \to \pi^0\gamma}$, and $c_\phi = G_{\phi \to \pi^0\gamma} \cdot G_{\phi \to \eta'\gamma}$. They are determined from the known decay widths $\eta' \to \omega\gamma$, $\eta' \to \rho\gamma$, $\omega \to \pi^0\gamma$, $\rho \to \pi^0\gamma$, $\phi \to \pi^0\gamma$, and $\phi \to \eta'\gamma$.

$$\Gamma(\omega \to \pi^0\gamma) = \frac{1}{3} \cdot G^2_{\omega \to \pi^0\gamma} \cdot \frac{(m^2_\omega - m^2_{\pi^0})^3}{32\pi \cdot m^3_\omega}, \tag{6}$$

$$\Gamma(\eta' \to \omega\gamma) = G^2_{\eta' \to \omega\gamma} \cdot \frac{(m^2_{\eta'} - m^2_\omega)^3}{32\pi \cdot m^3_{\eta'}}. \tag{7}$$

So, ω coupling constant equals $c^{\eta' \to \pi^0\gamma\gamma}_\omega = G_{\omega \to \pi^0\gamma} \cdot G_{\eta' \to \omega\gamma} = 0.08872(587)\ GeV^{-2}$, where the total uncertainty is determined by the propagation of errors.

Analogously, for $c^{\eta' \to \pi^0\gamma\gamma}_\rho$ coupling constant,

$$\Gamma(\rho \to \pi^0\gamma) = \frac{1}{3} \cdot G^2_{\rho \to \pi^0\gamma} \cdot \frac{(m^2_\rho - m^2_{\pi^0})^3}{32\pi \cdot m^3_\rho}, \tag{8}$$

$$\Gamma(\eta' \to \rho\gamma) = G^2_{\eta' \to \rho\gamma} \cdot \frac{(m^2_{\eta'} - m^2_\rho)^3}{32\pi \cdot m^3_{\eta'}}. \tag{9}$$

The corresponding decay constant is $c^{\eta' \to \pi^0\gamma\gamma}_\rho = G_{\rho \to \pi^0\gamma} \cdot G_{\eta' \to \rho\gamma} = 0.08871(892)\ GeV^{-2}$.

Finally, for ϕ meson,

$$\Gamma(\phi \to \pi^0\gamma) = \frac{1}{3} \cdot G^2_{\phi \to \pi^0\gamma} \cdot \frac{(m^2_\phi - m^2_{\pi^0})^3}{32\pi \cdot m^3_\phi}, \tag{10}$$

$$\Gamma(\phi \to \eta'\gamma) = \frac{1}{3} \cdot G^2_{\phi \to \eta'\gamma} \cdot \frac{(m^2_\phi - m^2_{\eta'})^3}{32\pi \cdot m^3_\phi}. \tag{11}$$

So, $c^{\eta' \to \pi^0\gamma\gamma}_\phi = G_{\phi \to \pi^0\gamma} \cdot G_{\phi \to \eta'\gamma} = 0.00879(36)\ GeV^{-2}$. Such a small value in comparison with $c^{\eta' \to \pi^0\gamma\gamma}_\omega$ and $c^{\eta' \to \pi^0\gamma\gamma}_\rho$ is due to OZI. The coupling constants $c^{\eta' \to \pi^0\gamma\gamma}_\omega$ and $c^{\eta' \to \pi^0\gamma\gamma}_\rho$ in our approach are approximately the same, and the difference between them is of order ∼ 10%; c_ϕ is small in comparison with c_ρ and c_ω but nonzero.

For similar decays $\eta' \to \eta\gamma\gamma$ and $\eta \to \pi^0\gamma\gamma$, the coupling constants extracted in a similar way from the known decays are

$$\begin{cases} c_\omega^{\eta\to\pi^0\gamma\gamma} = 0.09435(641)\ GeV^{-2} \\ c_\rho^{\eta\to\pi^0\gamma\gamma} = 0.10718(1138)\ GeV^{-2} \\ c_\phi^{\eta\to\pi^0\gamma\gamma} = 0.00852(27)\ GeV^{-2}, \end{cases} \tag{12}$$

$$\begin{cases} c_\omega^{\eta'\to\eta\gamma\gamma} = 0.01707(168)\ GeV^{-2} \\ c_\rho^{\eta'\to\eta\gamma\gamma} = 0.19116(01388)\ GeV^{-2} \\ c_\phi^{\eta'\to\eta\gamma\gamma} = -0.04657(140)\ GeV^{-2}. \end{cases} \tag{13}$$

The ω meson is quite narrow and its peaks are clearly seen on a Dalitz plot, but ρ meson is much wider, so we have to include the corrections due to the dependence of the width of ρ meson on energy which is dictated by the unitarity conditions [25].

We use a new parametrization of the ρ meson width [26], instead of the one used previously [27], since, as it was shown in [26], it gives equally good or better fits to the CMD2, SND, and KLOE Collaborations data:

$$\Gamma_\rho(s) = \Gamma_\rho \cdot \frac{m_\rho}{\sqrt{s}} \cdot \Big(\frac{s - 4\cdot m_{\pi^+}^2}{m_\rho^2 - 4\cdot m_{\pi^+}^2}\Big)^{\frac{3}{2}} \cdot \theta(s - 4\cdot m_{\pi^+}^2). \tag{14}$$

The ρ meson width effects were also considered in [28].

The VMD contribution is split in the following way: $\Gamma_{total}^{VMD} = \Gamma_\omega^{VMD} + \Gamma_\rho^{VMD} + \underbrace{\Gamma_{\omega-\rho}^{VMD}}_{Interference\ \omega-\rho}$. Their relative contributions are the following: $\frac{\Gamma_\omega^{VMD}}{\Gamma_{total}^{VMD}} \approx 75\%$, $\frac{\Gamma_\rho^{VMD}}{\Gamma_{total}^{VMD}} \approx 5\%$, and $\frac{\Gamma_{\omega-\rho}^{VMD}}{\Gamma_{total}^{VMD}} \approx 20\%$. Such a sharp difference between the contributions of ρ and ω is due to the fact that $\Gamma_\rho \gg \Gamma_\omega$. Nevertheless, the interference term is crucial in the area of a Dalitz plot outside the range of ω meson.

We also include the contributions of the kaon loops and $a_0(980)$ resonance in our calculation with the mixing angle determined in our previous work [23]. The corresponding spectrum $\frac{d\Gamma_{\eta'\to\pi^0\gamma\gamma}^{VMD+L\sigma M}}{dm_{\gamma\gamma}^2}$ is shown in Figure 2.

The VMD contribution to the decay width is

$$BR^{VMD}(\eta' \to \pi^0\gamma\gamma)_{Theory} = (8.23 \pm 1.16) \times 10^{-3}. \tag{15}$$

The total decay width taking into account the coherent sum of VMD, kaon loops, and $a_0(980)$ resonance is

$$BR^{VMD+L\sigma M}(\eta' \to \pi^0\gamma\gamma)_{Theory} = (7.88 \pm 1.26) \times 10^{-3}, \tag{16}$$

which has a tension with the experimental result, given in formula (2).

After the appearance of our manuscript, the new analysis of this decay appeared [29]. The authors of this work carried out a detailed and comprehensive fit of $\eta' \to \pi^0\gamma\gamma$, $\eta' \to \eta\gamma\gamma$, and $\eta \to \pi^0\gamma\gamma$ decays. In their approach, they simultaneously adjust three parameters: φ_P, φ_V, and $|g|$. In general, [29] confirms our conclusion about the discrepancy between the theoretical prediction and the experimental data and our predictions are close; the difference is due to different VMD coupling constants, mixing angles, and parametrizations of ρ meson width used.

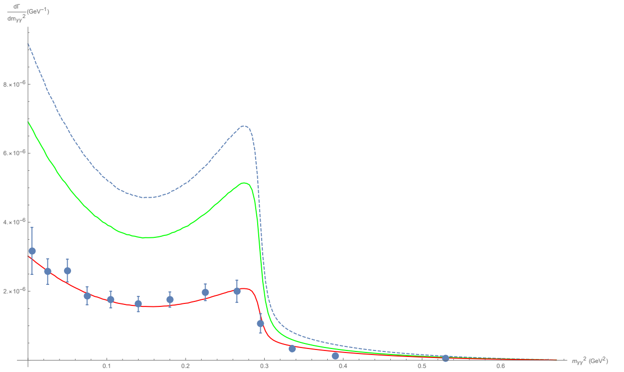

FIGURE 2: $\frac{d\Gamma_{\eta'\to\pi^0\gamma\gamma}^{VMD+L\sigma M}}{dm_{\gamma\gamma}^2}$ compared to BES-III experimental points. The upper blue dashed and the green lines correspond to the maximum and minimum values of the VMD coupling constants, and the lower red line includes the possible contribution of "$\mathcal{B}$ boson".

Nevertheless, in their approach, they have *three fitting parameters* and simultaneously fit *three* coupling constants.

In our approach, on the contrary, *we do not have free parameters.* To extract the values of the VMD coupling constants, we used the known masses, decay widths, and branching ratios of the particles. Moreover, in [29], the origin of the discrepancy was not determined.

In the next section, we discuss other possible contributions which could be the reason for these discrepancies.

In addition to the discrepancy in the decay we considered, $\eta' \to \pi^0\gamma\gamma$, recently BES-III [20] provided the results for a similar decay $\eta' \to \gamma\gamma\eta$ which has a clear tension with the theoretical prediction [19].

Therefore, in addition to $\eta' \to \pi^0\gamma\gamma$, it is worth considering the possible impact of hypothetical Dark Photon on similar decays $\eta \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$.

3. OTHER POSSIBLE CONTRIBUTIONS

As we have seen, $\phi(1020)$ gives $\sim 1\%$ contribution to the overall decay width of $\eta' \to \pi^0\gamma\gamma$. Also, as it was first shown in [19], scalar meson effects of $a_0(980)$, $f_0(980)$, and $\sigma(600)$ which are included in $L\sigma M$ contribution give a negligibly small contribution to all three decays.

Other possible intermediate states which could give the contribution to this decay can be found by constructing the invariant amplitudes. There are such options: $0^{-+} \to \gamma^{--} + 1^{--}$, $0^{-+} \to \gamma^{--} + 1^{+-}$, and $0^{-+} \to \gamma^{--} + 2^{+-}$.

Additional intermediate vector states could be $\omega(1420)$, $\rho(1450)$, $\rho(1570)$, $\omega(1650)$, $\phi(1680)$, $\rho(1700)$, $\rho(1900)$, $\rho(2150)$, and $\phi(2170)$. Possible axial intermediate states are $h_1(1150)$, $b_1(1235)$, and $h_1(1380)$.

However, all these additional intermediate states are even heavier than $\phi(1020)$ and the aforementioned scalar mesons; thus, they are even further from the boundaries of the Dalitz plot. Therefore, neglecting them seems a safe assumption, and it is very unlikely that they could explain such a large discrepancy.

Another opportunity that could provide an explanation of this discrepancy is Dark Photon (or "$\mathcal{B}$ boson"). On an experimental $\pi^0\gamma$ invariant mass spectrum, the clear sharp peak of a new particle is not seen [14].

Nevertheless, since "$\mathcal{B}$ boson" should have the same quantum numbers as ω meson, it can have mixing with ω meson and thus give a significant contribution to this decay.

If a possible contribution of "$\mathcal{B}$ boson" is taken into account, formula (4) should be modified in such a way:

$$A \to \left(\frac{c_\omega}{D_\omega(t)} + \frac{c_\rho}{D_\rho(t)} + \frac{c_\phi}{D_\phi(t)} + \frac{c_{\mathcal{B}}}{D_{\mathcal{B}}(t)}\right)B(q_2) + \tag{17}$$

$$+\left(\frac{c_\omega}{D_\omega(u)} + \frac{c_\rho}{D_\rho(u)} + \frac{c_\phi}{D_\phi(u)} + \frac{c_{\mathcal{B}}}{D_{\mathcal{B}}(u)}\right)B(q_1). \tag{18}$$

We consider the simplest option to show the viability of such a scenario. If we assume that the new $\mathcal{B}$ boson is hidden within the range of ω meson (so $m_{\mathcal{B}} = m_\omega$), then, its peak is not seen on the $\pi^0\gamma$ invariant mass spectrum. Nevertheless, it could give a significant contribution to the overall $\pi^0\gamma\gamma$ decay width. For instance, if the hypothetical $\mathcal{B}$ has a width such that $\Gamma_{\mathcal{B}} = \Gamma_\omega$ and the coupling constant $c_{\mathcal{B}}$ has an opposite sign than c_ω, then, the effective ω coupling constant would be lower than c_ω: $c_\omega^{Effective} = c_\omega - |c_{\mathcal{B}}| < c_\omega$.

As it was indicated in [29], the BES-III result may be explained if we decrease the *overall* normalization and so *simultaneously* decrease c_ω, c_ρ, and c_ϕ constants by decreasing $|g|$ and $Sin[\phi_P]$.

On the contrary, we introduce the effective ω meson coupling constant which may be caused by the possible mixing with "$\mathcal{B}$ boson" *without touching anything else* (ρ and ϕ mesons, kaon loops, and $a_0(980)$).

Since this decay is dominated by the ω meson ($\sim$ 80%), its decay width is very sensitive to ω coupling constant. For our numerical estimations, we take $c_\omega^{Effective} = 0.48\ GeV^{-2}$ and receive quite a similar result to [29] shown in Figure 2 by a lower red line.

Consequently, assuming that the BES-III result on $\eta' \to \pi^0\gamma\gamma$ is correct, the scenario with the hypothetical Dark Photon (or $\mathcal{B}$ meson) is quite viable.

Additionally, a recent measurement of $\eta' \to \eta\gamma\gamma$ by BES-III Collaboration gives an upper limit on the branching ratio 1.33×10^{-4} at the 90% CL [20], which has also a tension with the theoretical prediction [19].

Consequently, the possible impact of hypothetical Dark Photon may be present in other similar rare decays $\eta' \to \eta\gamma\gamma$ and $\eta \to \pi^0\gamma\gamma$, and we would like our model of Dark Photon to be flexible enough to explain the results of all three decays $\eta' \to \pi^0\gamma\gamma$, $\eta' \to \eta\gamma\gamma$, and $\eta \to \pi^0\gamma\gamma$ simultaneously.

Analogously to a regular ω meson, Dark Photon should have couplings to η, η', and π^0, and in each of those decays, like for a regular ω meson, the corresponding coupling constants should be determined as $c_{\mathcal{B}}^{\eta'\to\pi^0\gamma\gamma} = G_{\mathcal{B}\eta'\gamma} \cdot G_{\mathcal{B}\pi^0\gamma}$, $c_{\mathcal{B}}^{\eta'\to\eta\gamma\gamma} = G_{\mathcal{B}\eta'\gamma} \cdot G_{\mathcal{B}\eta\gamma}$, and $c_{\mathcal{B}}^{\eta\to\pi^0\gamma\gamma} = G_{\mathcal{B}\eta\gamma} \cdot G_{\mathcal{B}\pi^0\gamma}$.

The measured branching ratio is less than the theoretical prediction for $\eta' \to \pi^0\gamma\gamma$ decay, bigger than that for $\eta \to \pi^0\gamma\gamma$, and smaller than that for $\eta' \to \eta\gamma\gamma$. This leads to the assumption that $G_{\mathcal{B}\eta'\gamma} > 0$, $G_{\mathcal{B}\eta\gamma} < 0$, and $G_{\mathcal{B}\pi^0\gamma} < 0$; therefore, the "effective ω couplings" will be modified:

$$\begin{cases} c_{\mathcal{B}}^{\eta'\to\pi^0\gamma\gamma} < 0 \Rightarrow c_{\omega,Effective}^{\eta'\to\pi^0\gamma\gamma} < c_\omega^{\eta'\to\pi^0\gamma\gamma} \\ c_{\mathcal{B}}^{\eta\to\pi^0\gamma\gamma} > 0 \Rightarrow c_{\omega,Effective}^{\eta\to\pi^0\gamma\gamma} > c_\omega^{\eta\to\pi^0\gamma\gamma} \\ c_{\mathcal{B}}^{\eta'\to\eta\gamma\gamma} < 0 \Rightarrow c_{\omega,Effective}^{\eta'\to\eta\gamma\gamma} < c_\omega^{\eta'\to\eta\gamma\gamma} \end{cases} \tag{19}$$

Therefore, for $\eta' \to \pi^0\gamma\gamma$ and $\eta' \to \eta\gamma\gamma$, "effective ω coupling" would be reduced, and for $\eta \to \pi^0\gamma\gamma$, it would be increased.

For completeness, we consider a well-studied decay $\eta \to \pi^0\gamma\gamma$ in a similar manner to $\eta' \to \pi^0\gamma\gamma$ and get the value which is *smaller* than the experimental result [21, 22]:

$$BR^{VMD+L\sigma M}(\eta \to \pi^0\gamma\gamma)|_{Theoretical} = (1.30 \pm 0.23) \times 10^{-4}, \tag{20}$$

$$BR^{VMD+L\sigma M}(\eta \to \pi^0\gamma\gamma)|_{Experimental} = (2.56 \pm 0.22) \times 10^{-4}. \tag{21}$$

Our result is *smaller* than the theoretical prediction given in [29] since we used the coupling constants extracted directly from the known decays (and they are *significantly smaller*), another parametrization for the ρ meson width and value of the mixing angle. And, as in the case of $\eta' \to \pi^0\gamma\gamma$, we do not have free-fit parameters.

As can be seen in Figure 2 of [29], the theoretical curve, although being on the edge of the error bars, lies noticeably lower than the majority of experimental points. Consequently, both our prediction and the prediction provided in [29] are *smaller* than the experimental value [21, 22].

Nevertheless, this discrepancy can be relaxed in a similar manner to $\eta' \to \pi^0\gamma\gamma$ if we assume that the coupling constants of $\mathcal{B}$ have the signs provided in formula (19), so by increasing the effective ω coupling. For our numerical estimations, we take $c_{\mathcal{B}}^{\eta\to\pi^0\gamma\gamma} = 0.5 \cdot c_\omega^{\eta\to\pi^0\gamma\gamma}$. For the decay $\eta \to \pi^0\gamma\gamma$, the effect of increasing the effective ω coupling on the $\gamma\gamma$ spectrum is shown in Figure 3.

The $\eta \to \pi^0\gamma\gamma$ decay is not dominated by ω meson like $\eta' \to \pi^0\gamma\gamma$, so the effects of $\mathcal{B}$ meson distort the shape of the $\gamma\gamma$ spectrum, not just shift it. Nevertheless, as it can be seen in Figure 3, the uncertainty bars are quite large, and the spectrum with the contribution of $\mathcal{B}$ boson approaches the allowed experimental values and the overall branching is increased approaching the experimental value.

Finally, for the case of $\eta' \to \eta\gamma\gamma$, the direct comparison between theory and experiment is not possible yet since the $\gamma\gamma$ spectrum is not provided for this decay now. Nevertheless, it is clear that the theoretical branching ratio of this decay may be decreased in a similar manner to $\eta' \to \pi^0\gamma\gamma$ if we assume the signs of the coupling constants of $\mathcal{B}$ boson to be given by formula (19), since in the case $\eta' \to \eta\gamma\gamma$ the effective ω coupling constant has to be reduced.

For example, taking into account the fact that $c_\omega^{\eta\to\pi^0\gamma\gamma} \approx c_\omega^{\eta'\to\pi^0\gamma\gamma}$,

$$\begin{cases} c_{\mathcal{B}}^{\eta'\to\pi^0\gamma\gamma} = G_{\mathcal{B}\eta'\gamma} \cdot G_{\mathcal{B}\pi^0\gamma} \approx -0.5 \cdot c_\omega^{\eta'\to\pi^0\gamma\gamma} \\ c_{\mathcal{B}}^{\eta\to\pi^0\gamma\gamma} = G_{\mathcal{B}\eta\gamma} \cdot G_{\mathcal{B}\pi^0\gamma} \approx 0.5 \cdot c_\omega^{\eta\to\pi^0\gamma\gamma} \\ c_{\mathcal{B}}^{\eta'\to\eta\gamma\gamma} = G_{\mathcal{B}\eta'\gamma} \cdot G_{\mathcal{B}\eta\gamma}, \end{cases} \Rightarrow \tag{22}$$

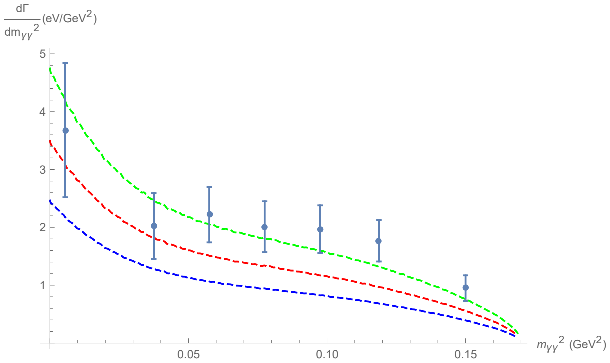

FIGURE 3: $\gamma\gamma$ spectrum of $\eta \to \pi^0\gamma\gamma$. Two lower lines correspond to the largest and smallest values of the coupling constants. The upper green dashed line corresponds to a possible contribution of hypothetical $\mathcal{B}$ boson. The data points are taken from [22].

$$\Rightarrow \frac{G_{\mathcal{B}\eta'\gamma}}{G_{\mathcal{B}\eta\gamma}} \approx -1,\; c_{\mathcal{B}}^{\eta'\to\eta\gamma\gamma} \approx -(G_{\mathcal{B}\eta'\gamma})^2. \qquad (23)$$

Taking $(G_{\mathcal{B}\eta\gamma})^2 \approx c_\omega^{\eta'\to\eta\gamma\gamma}$ and knowing the relative contributions of ω, ρ, and ϕ, we can make $c_\omega^{\eta'\to\eta\gamma\gamma,\ Effective} \approx 0$. Consequently, for $\eta' \to \eta\gamma\gamma$ decay, the theoretical decay width may be reduced by $\sim 40\%$ in a similar manner to the other two aforementioned decays, thus reducing the tension with the experimental result.

We postpone the detailed analysis of $\eta' \to \eta\gamma\gamma$ decay and the simultaneous fit of parameters of hypothetical $\mathcal{B}$ boson, $\{m_{\mathcal{B}}, \Gamma_{\mathcal{B}}, G_{\mathcal{B}\eta'\gamma}, G_{\mathcal{B}\pi^0\gamma}, G_{\mathcal{B}\eta\gamma}\}$, till more experimental data becomes available, in particular, $\gamma\gamma$ spectrum of $\eta' \to \eta\gamma\gamma$ decay.

4. CONCLUSIONS

For any choice of the coupling constants, there is a clear discrepancy between $\Gamma(\eta' \to \pi^0\gamma\gamma)^{VMD+L\sigma M}_{Theory}$ and the observed result by BES-III which can be attributed to New Physics, presumably Dark Photon (or "$\mathcal{B}$ boson").

As may be seen in Figure 2, the scenario with the $\mathcal{B}$ boson giving a contribution to $\eta' \to \pi^0\gamma\gamma$ is quite viable.

Considering in a similar manner the decay $\eta \to \pi^0\gamma\gamma$, we see that both our approach and the approach provided in [29] give the values which are below the experimental result [21, 22].

Unlike $\eta' \to \pi^0\gamma\gamma$, $\eta \to \pi^0\gamma\gamma$ is not dominated by ω boson, so the shape of $\gamma\gamma$ spectrum is distorted. Nevertheless, taking into account that the error bars are large, there is more space for the change of spectrum shape by taking into account $\mathcal{B}$ boson contribution (Figure 3).

Additionally, a recent measurement of $\eta' \to \eta\gamma\gamma$ [20] gives the branching ratio which is also significantly smaller than the theoretical prediction [19]. Clearly, by reducing the "effective ω coupling", we can reduce the tension between theory and experiment in a similar manner to $\eta' \to \pi^0\gamma\gamma$ in this case also. However, we postpone this analysis till more data on this decay becomes available.

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