

Q-Field from a 4D Brane: Cosmological Constant Cancellation and Minkowski Attractor

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Abstract

A 4D-brane realization of q -theory has been proposed a few years ago. The present paper studies the corresponding late-time cosmology and establishes the dynamic cancellation of an initial cosmological constant and an attractor behavior toward Minkowski spacetime.

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1. INTRODUCTION

The cosmological constant problem is perhaps the most important problem of modern physics [1]. A condensed-matter-inspired approach has been proposed and goes under the name of q -theory [2, 3, 4]. A particular realization of q -theory takes its cue from the physics of a freely suspended two-dimensional material film [5] and uses a generalization to four-dimensional “films” or 4D branes [6].

This 4D-brane realization of q -theory has a dimensionless chemical potential μ (details will be given shortly) and may be of relevance to a recent suggestion to replace the big bang by a quantum phase transition [7].

The goal of the present paper is to discuss the corresponding late-time cosmology, where the asymptotic vacuum energy density may or may not vanish. In particular, we would like to establish a possible attractor behavior toward Minkowski spacetime with a vanishing vacuum energy density. Throughout, we use natural units with $\hbar = 1$ and $c = 1$.

2. ACTION AND FIELD EQUATIONS

The following 4D-brane action has been proposed in [6]:

$$S = - \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G_N} + \epsilon \left(\frac{n}{\sqrt{-g}} \right) + \mathcal{L}^M[\psi] \right] + \mu \int d^4x n, \quad (1)$$

with a Lorentzian signature $(-, +, +, +)$ of the metric $g_{\alpha\beta}$, so that its determinant $g \equiv \det(g_{\alpha\beta})$ is negative. In (1), the term $\mathcal{L}^M(\psi)$ is the matter Lagrange density for a generic matter field ψ and the potential $\epsilon(x)$ is an essentially arbitrary function of x (the stability of the equilibrium state gives some conditions on the potential [2]). The crucial new ingredient of the above action is n as the 4D analog of the particle density of a 2D membrane. This 4D density n may refer to “spacetime atoms,” with the corresponding nonzero chemical potential μ (further discussion will be given in Section 4).

At this moment, it may be helpful to explain the terminology “spacetime atoms.” The physics of the freely suspended two-dimensional material film has been reviewed in the first two paragraphs of Section 3 in [6], with mention of the original reference [5]. The film there is made out of atoms, whose basic structure is known to be described by the Schrödinger equation. The Hamiltonian of this two-dimensional material

film has been generalized to a four-dimensional action and, for this reason, we can perhaps speak of a system describing “spacetime atoms,” whose structure is, of course, completely unknown at the present moment. What matters, for the following, is the structure of the action (1) with an unknown (conserved) quantity n having mass dimension 4.

The resulting n -field equation from (1) reads

$$\frac{d\epsilon}{dq} = \mu, \quad (2a)$$

$$q \equiv \frac{n}{\sqrt{-g}}. \quad (2b)$$

Here, we have identified a scalar q -field with mass dimension 4, so that μ has mass dimension 0, and this q -field realization may be relevant to the big-bang discussion of [7].

The resulting gravitational equation from (1) is the standard Einstein equation,

$$\frac{1}{8\pi G_N} \left(R_{\alpha\beta} - \frac{1}{2} R g_{\alpha\beta} \right) = \rho_V(q) g_{\alpha\beta} + T_{\alpha\beta}^M, \quad (3)$$

with the following gravitating vacuum energy density:

$$\rho_V(q) = \epsilon(q) - q \frac{d\epsilon(q)}{dq} = \epsilon(q) - \mu q, \quad (4)$$

where (2a) has been used to get the final expression for $\rho_V(q)$. Note that the constant μ here traces back to the 4D-brane action (1). This situation is different from the one for the q -field in the 4-form realization, where μ appears as an integration constant of the solution [2, 3].

In terms of the gravitating vacuum energy density (4), we can rewrite the n -field equation (2a) as

$$\frac{d\rho_V(q)}{dq} = 0. \quad (5)$$

Remark that for a homogeneous q -field (i.e., $q = q(t)$ in a suitable coordinate system), we can multiply (5) by dq/dt to get

$$\frac{d\rho_V(t)}{dt} = 0, \quad (6)$$

which carries over to cosmology, as will be discussed in the next section.

3. COSMOLOGY: MINKOWSKI ATTRACTOR

3.1. Setup

For cosmology, we take the standard spatially flat Robertson-Walker (RW) metric with cosmic scale factor $a(t)$ and Hubble

parameter $H \equiv [da/dt]/a$. We also add a homogeneous perfect fluid for the matter component with a constant equation-of-state parameter $w_M \equiv P_M/\rho_M$.

Next, introduce dimensionless variables: the cosmic time coordinate τ , the Hubble parameter $h \equiv a^{-1} da/d\tau$, the matter energy density r_M , and the vacuum energy density r_V . Then, the dimensionless ordinary differential equations (ODEs) are as follows:

$$\dot{r}_M + 3h(1 + w_M)r_M = +\Gamma, \quad (7a)$$

$$\dot{r}_V = -\Gamma, \quad (7b)$$

$$3h^2 = 8\pi(r_M + r_V), \quad (7c)$$

$$2\dot{h} + 3h^2 = -8\pi(w_M r_M - r_V), \quad (7d)$$

where the overdot stands for differentiation with respect to the dimensionless cosmic time coordinate τ and the source term Γ models vacuum-matter energy exchange, as discussed by [8] in general terms.

3.2. General ρ_V : Analytic Solution

An explicit calculation of Γ entering the ODEs (7) was presented in [9] and gave a Zeldovich-Starobinsky-type [10] source term:

$$\Gamma_{\text{particle-production}} = \frac{1}{36}\gamma|h|\mathcal{R}^2 = \gamma|h|(\dot{h} + 2h^2)^2, \quad (8)$$

with \mathcal{R} being the Ricci curvature scalar in terms of dimensionless variables. Observe that equations (7a) and (7b) are time-reversal noninvariant for the source term as given by (8). This time-reversal noninvariance corresponds to a dissipative effect, actually a quantum-dissipative effect as particle creation is a true quantum phenomenon.

The ODEs (7), for the source term (8) and $w_M = 1/3$, are *exactly the same* as those in [9], which were derived with the 4-form realization of the q -field.¹ Hence, there is the *same* attractor behavior toward Minkowski spacetime; see, in [9], the numerical results of Figure 3 and the analytic solution of Appendix B.

It may be instructive to recall the main steps for getting this exact solution. For the special case of relativistic matter, $w_M = 1/3$, we can add the two Friedmann equations (7c) and (7d), in order to eliminate r_M . The resulting equation relates the combination $\dot{h} + 2h^2$ to r_V . The special choice (8) for Γ involves the very same combination $\dot{h} + 2h^2$ and (7b) reduces to a single ODE for $r_V(\tau)$:

$$\frac{1}{|h|} \frac{dr_V}{d\tau} = -\gamma[(16\pi/3)r_V]^2. \quad (9)$$

Changing the τ coordinate to

$$\chi = \ln |a(\tau)|, \quad (10)$$

we obtain the following ODE:

$$\frac{dr_V}{d\chi} = -(16\pi/3)^2\gamma[r_V]^2. \quad (11)$$

The solution (denoted by a bar) is

$$\bar{r}_V(\chi) = \frac{1}{(16\pi/3)^2\gamma(\chi - \chi_0)}, \quad (12)$$

with an integration constant χ_0 . The corresponding solution $\bar{h}(\chi)$ involves the exponential integral function “Ei(χ)” and is given in Appendix B of [9]. As $a(\tau)$ goes to infinity with $\tau \rightarrow \infty$, so does χ and the vacuum energy density r_V from the exact solution (12) is seen to drop to zero.

3.3. Specific $\rho_V(q)$: ODEs

There is, however, an important caveat for the general discussion in Section 3.2: the dimensionless vacuum energy density r_V corresponding to the dimensional quantity ρ_V from (4) should be able to reach the value zero.

Denote the dimensionless version of the cosmological constant by λ and the dimensionless version of the q -variable from (2b) by ζ (the chemical potential μ is already dimensionless). Then, we can split the dimensionless version of the energy density ϵ from the action (1) into a constant part λ and a nonconstant part $\tilde{\epsilon}$,

$$\epsilon[\zeta] \equiv \lambda + \tilde{\epsilon}[\zeta], \quad (13)$$

and require for the existence of a Minkowski attractor:

$$\exists \bar{\zeta} \in \mathbb{R}^+ : r_V[\bar{\zeta}] = \lambda + \tilde{\epsilon}[\bar{\zeta}] - \mu\bar{\zeta} = 0. \quad (14)$$

The issue, now, is whether or not there exists such a $\bar{\zeta}$ and, if there exists such a $\bar{\zeta}$, whether or not it can be reached in the late-time cosmology.

For a specific realization of the vacuum energy density r_V as a function of ζ and with the Hubble parameter $h \equiv \dot{a}/a$, the ODEs (7) take the following form:

$$\dot{r}_M + 3h(1 + w_M)r_M = +\Gamma, \quad (15a)$$

$$\dot{\zeta} \frac{dr_V[\zeta]}{d\zeta} = -\Gamma, \quad (15b)$$

$$3h^2 = 8\pi(r_M + r_V[\zeta]), \quad (15c)$$

$$2\dot{h} + 3h^2 = -8\pi(w_M r_M - r_V[\zeta]), \quad (15d)$$

$$r_V[\zeta] = \lambda + \tilde{\epsilon}[\zeta] - \mu\zeta, \quad (15e)$$

where the dependence of r_V on the vacuum variable ζ has been made explicit.

As a simple example, take

$$\tilde{\epsilon}[\zeta] = \zeta^2, \quad (16a)$$

which turns condition (14) into a quadratic for $\bar{\zeta}$. For a given positive value of μ , there is now a suitable value $\bar{\zeta}$ for *any* value of the dimensionless cosmological constant in the following range:

$$\lambda \leq \mu^2/4, \quad (16b)$$

where one possible value of $\bar{\zeta}$ is given by

$$\bar{\zeta} = \frac{1}{2} \left(\mu + \sqrt{\mu^2 - 4\lambda} \right). \quad (16c)$$

¹The 4-form theory considered in [2, 3, 4, 9] is purely four-dimensional, so that the values of q are continuous and not quantized as happens for other types of 4-form theories [11].

$$\mu = 4, \lambda = 1$$

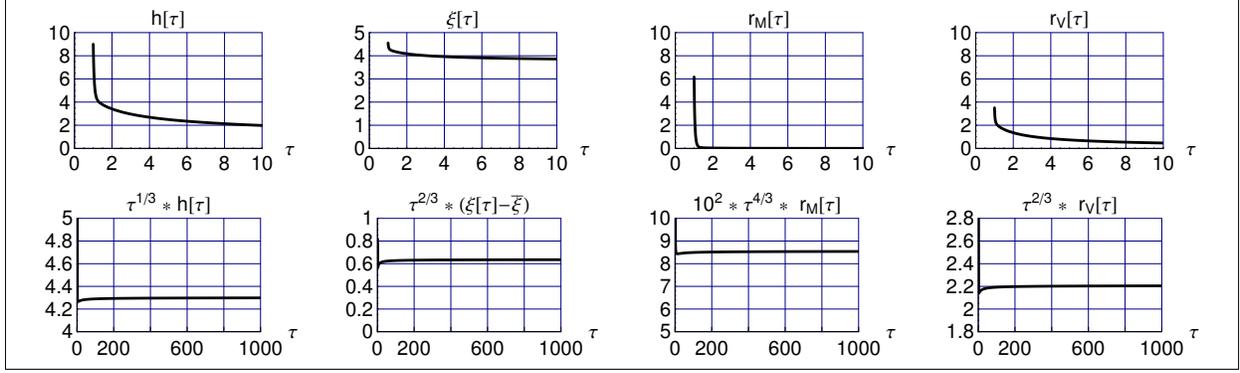


FIGURE 1: Numerical solution of the ODEs (15) with source term (17) and energy density (16a), for parameters $w_M = 1/3$, $\mu = 4$, $\lambda = 1$, and $\tilde{\gamma} = 10^{-3}$. The boundary conditions at $\tau = \tau_{\text{bcs}} = 1$ are $\{h, \xi, r_M\} = \{9, 4.55, 6.16616\}$, where the r_M value has been obtained from the first Friedman equation (15c). The top row shows the three basic variables: the Hubble parameter $h = \dot{a}/a$, the dimensionless vacuum variable ξ , and the matter energy density r_M . The bottom row shows their asymptotic behavior: $h(\tau) \sim \tau^{-1/3}$, $\xi(\tau) - \bar{\xi} \sim \tau^{-2/3}$, $r_M(\tau) \sim \tau^{-4/3}$, and $r_V(\tau) \sim \tau^{-2/3}$, where $\bar{\xi}$ is given by (16c). Both rows show the corresponding vacuum energy density r_V from (15e) and (16a).

$$\mu = 4, \lambda = -1$$

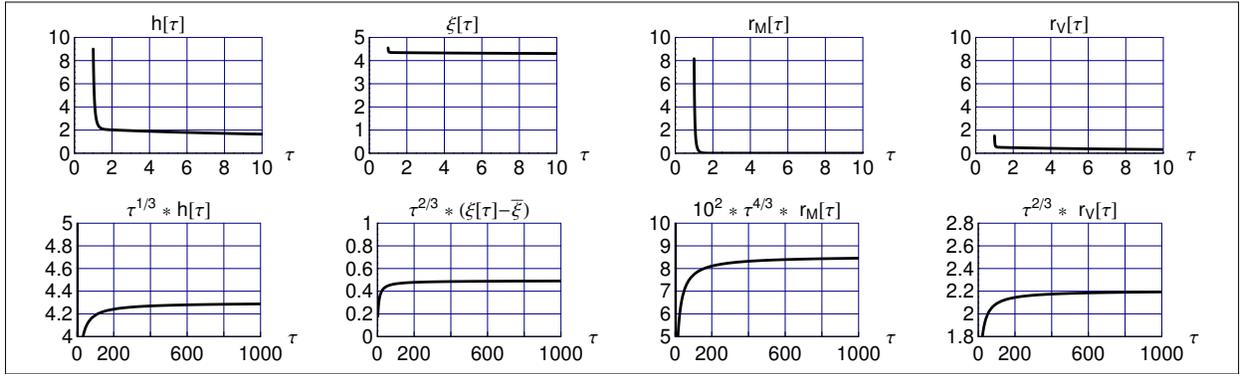


FIGURE 2: Numerical solution of the ODEs (15) with source term (17) and energy density (16a), for parameters $w_M = 1/3$, $\mu = 4$, $\lambda = -1$, and $\tilde{\gamma} = 10^{-3}$. The boundary conditions at $\tau = \tau_{\text{bcs}} = 1$ are $\{h, \xi, r_M\} = \{9, 4.55, 8.16616\}$, where the r_M value has been obtained from the first Friedman equation (15c). The variables shown are explained in the caption of Figure 1.

For $0 < \lambda < \mu^2/4$, the other possible value of $\bar{\xi}$ has a minus sign in front of the square root on the right-hand side of (16c).

A further point is the choice of Γ so that the numerics works in the simplest way. A suitable *ad hoc* choice is

$$\Gamma_{\text{ad-hoc}} = \tilde{\gamma}|h|h^4. \quad (17)$$

This basically has the structure of expression (8), but the squared Ricci factor has been simplified to the fourth power of the Hubble parameter h . Observe, again, that the ODE (15b) with source term (17) is time-reversal noninvariant.

3.4. Specific $\rho_V(q)$: Numerical Results

Numerical results from the ODEs (15), with (16) and (17), are presented in Figures 1 and 2 for $\mu = 4$ and $\lambda = \pm 1$ (similar results have been obtained for $\mu = 4$ and $\lambda = 0$). The asymptotic behavior in both figures is essentially the same,

$$h(\tau) \sim (\tilde{\gamma}\tau)^{-1/3}, \quad r_V(\tau) \sim (\tilde{\gamma}\tau)^{-2/3}. \quad (18)$$

This asymptotic behavior also follows directly from the ODEs (7b) and (7c), for the source term Γ from (17) and assuming that r_M is negligible compared to r_V . The asymptotic values of $\xi(\tau)$ in Figures 1 and 2 are numerically close to the analytic results from (16c).

The asymptotic behavior (18) illustrates the Minkowski attractor behavior ($r_V \rightarrow 0$). Indeed, we find numerically the same attractor behavior at the following four corners of the rectangle Δ of initial conditions:

$$\{h(\tau_{\text{bcs}}), \xi(\tau_{\text{bcs}})\} = \{9 \pm 1/10, 455/100 \pm 1/100\}, \quad (19)$$

equally for $\lambda = 1$ and $\lambda = -1$, at $\mu = 4$. We have also established numerically the same attractor behavior for several random points over the rectangle Δ .

These numerical results suggest that, for $\mu = 4$ and $|\lambda| \leq 1$, the attractor domain \mathcal{D} in the plane \mathbb{R}^2 of initial conditions is

$$\mu = 4, \lambda = 4$$

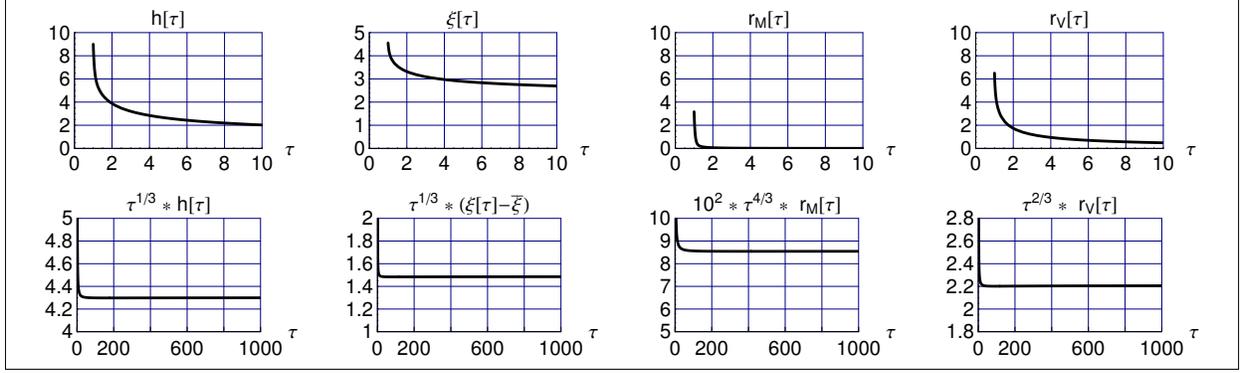


FIGURE 3: Numerical solution of the ODEs (15) with source term (17) and energy density (16a), for parameters $w_M = 1/3$, $\mu = 4$, $\lambda = 4$, and $\tilde{\gamma} = 10^{-3}$. The boundary conditions at $\tau = \tau_{\text{bcs}} = 1$ are $\{h, \xi, r_M\} = \{9, 4.55, 3.16616\}$, where the r_M value has been obtained from the first Friedman equation (15c). The variables shown are explained in the caption of Figure 1, but the asymptotic behavior of the dimensionless vacuum variable is now different: $\xi(\tau) - \bar{\xi} \sim \tau^{-1/3}$.

finite and includes the above-mentioned rectangle,

$$\begin{aligned} \mathcal{D}^{(\mu=4, |\lambda| \leq 1)} \\ \supseteq \Delta = \left\{ \left\{ h(\tau_{\text{bcs}}), \xi(\tau_{\text{bcs}}) \right\} \mid \right. \\ \left. 8.9 \leq h(\tau_{\text{bcs}}) \leq 9.1 \wedge 4.54 \leq \xi(\tau_{\text{bcs}}) \leq 4.56 \right\}. \end{aligned} \quad (20)$$

The actual attractor domain \mathcal{D} can be expected to be larger than the rectangle indicated.

In addition, we have similar numerical results for $\mu = 4$ and $\lambda = 4$ (shown in Figure 3 and Table 1), which will be discussed further in Section 3.5.

3.5. Discussion

Following up on the attractor-domain discussion of the last subsection with numerical results, we emphasize that the analytic solution [9] is clear about having a finite attractor domain. Let us give the details (expanding on the statement from the last paragraph of Appendix B in that reference): $\xi(\tau_{\text{bcs}})$ must be such as to make $r_V(\tau_{\text{bcs}})$ nonnegative and $h(\tau_{\text{bcs}})$ must also be nonnegative with a further condition that traces back to (15c). These conditions can be summarized as follows:

$$h(\tau_{\text{bcs}}) \geq \sqrt{\frac{8\pi}{3}} r_V(\tau_{\text{bcs}}) \geq 0. \quad (21)$$

How the actual attractor domain looks in the $\{h(\tau_{\text{bcs}}), \xi(\tau_{\text{bcs}})\}$ plane depends on the details of the *Ansatz* for $\tilde{e}[\xi]$ and the numerical values of μ and λ , possibly obeying a condition similar to (16b). Incidentally, there is no such condition on λ , for given μ , if the \tilde{e} *Ansatz* is changed to, for example, $\tilde{e}[\xi] = (\xi^2)^{1/4} + (\xi^2)^{-1/4}$, which allows for the cancellation of *any* value of $\lambda \in \mathbb{R}$ for arbitrary μ .

There is, however, a puzzle. Namely, the numerical results for $\mu = 4$ and $\lambda = \pm 1$ in Figures 1 and 2 show that Minkowski spacetime is approached, but how can that be as $\mu = 4$ differs from the fine-tuned value μ_0 for the Minkowski vacuum of [2]? Incidentally, for the $\tilde{e}(\xi)$ *Ansatz* in (16a), the equilibrium value of the chemical potential is given by $\mu_0 = 2\sqrt{\lambda}$, provided that λ is nonnegative (all the more surprising that our numerical

τ	$h(\tau)$	$\xi(\tau)$	$r_M(\tau)$	$r_V(\tau)$
10^2	0.926	2.32	1.85×10^{-4}	0.102
10^3	0.430	2.15	8.55×10^{-6}	0.0221
10^4	0.200	2.07	3.97×10^{-7}	0.00476
10^5	0.0927	2.03	1.84×10^{-8}	0.00102
10^6	0.0430	2.01	8.56×10^{-10}	0.000221

TABLE 1: Function values from the numerical solution of Figure 3, showing only 3 significant digits.

solution can approach Minkowski spacetime also for negative λ !).

The answer is simply that the Minkowski vacuum of [2] holds for *static* fields, whereas our numerical solution is *non-static*, $\xi = \xi(\tau)$ with $\dot{\xi} \neq 0$.

The numerical results for $\mu = 4$ and $\lambda = \pm 1$ do not appear to have a rigorous “limit” as $\tau \rightarrow \infty$ and the fields are essentially time-dependent. This is reminiscent of Dolgov’s model [12, 13] (see also Figure 2 in [4] and Appendix A in [14]), even though the time dependence of $h = h(\tau)$ and $\xi = \xi(\tau)$ in our 4D-brane model *diminishes* with time, whereas Dolgov’s time dependence stays constant with time (specifically, a linear time dependence of the massless vector field). Still, the final state with $q = \bar{q}$ is, in general, not the static-equilibrium state with $q = q_0$ and the question remains whether or not such a situation is acceptable (especially as concerns the gravitational dynamics of a solar-system-type subsystem; see Section I of [14] for further discussion and references).

For the special case $\mu = 4$ and $\lambda = 4$, there may be a strict limit, as we then reach the genuine Minkowski vacuum of [2] at $\mu_0 = 4$, with $\xi_0 = 2$ from (16c). The $\xi(\tau)$ values in Table 1 appear to approach the equilibrium value $\xi_0 = 2$. The corresponding dimensional vacuum variable $q(t)$ then approaches the equilibrium value q_0 , around which the vacuum energy density has a quadratic behavior, $\rho_V(q) \sim c_2(q - q_0)^2$ with a positive constant c_2 of mass dimension -4 (here, $c_2 \propto E_{\text{Planck}}^{-4}$).

4. OUTLOOK

The present paper has studied the cosmological behavior of a q -field in the 4D-brane realization [6]. The physical interpretation of this q -field scalar relates to a four-dimensional number density n of “spacetime atoms,” with a corresponding chemical potential μ . This number density n is a *new* variable, as the action (1) makes clear. It leads to an additional conservation equation (5) of the corresponding vacuum energy density $\rho_V(q)$.

A similar model has been presented recently [15], where there is also a four-dimensional number density n with a corresponding chemical potential μ . But, in that case, there is no need for a new variable as n is simply proportional to $\sqrt{-g}$, in terms of the *already available* metric determinant $g \equiv \det(g_{\alpha\beta})$. [This identification implies the restriction of the allowed coordinate transformations to those with unit Jacobian.] In that case, there is no additional conservation equation for ρ_V . Yet, in a cosmological context, the Friedmann equations do contain the equation $d\rho_V/dt = 0$; see the last paragraph of Section VI B in [15]. Moreover, there appears an attractor behavior toward Minkowski spacetime provided that there is vacuum-matter energy exchange.

Both manifestations of n apparently involve some fine-scale underlying structure of spacetime, called “atoms of spacetime” for the 4D-brane realization [6] and a “spacetime crystal” for the extended-unimodular-gravity approach [15]. The outstanding question is what the actual substructure of spacetime really is. A partial answer can perhaps be obtained from nonperturbative superstring theory as formulated by the IIB matrix model [16, 17, 18].

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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