FRW Cosmology with a Varying Cubic Deceleration Parameter

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Abstract

In this work, a new law of a varying deceleration parameter of a third degree has been proposed. The solutions of the modified field equations have been derived under the newly proposed law of the deceleration parameter. The model exhibits the big bang singularity at a cosmic time ($t = 0$) and shows a big rip at $t = n$, and then, it reenters the phase of initial singularity at $t = 2n$ and ends its cyclic behavior at $t = 3n$. The evolution of the physical and dynamical parameters of the Universe has been studied, and the graphical representation has also been shown. Further, $Om(z)$ diagnostic parameter and the energy conditions have also been studied together with their graphical representations.

Keywords: Friedmann-Robertson-Walker model, dark energy, singularity, big rip, quintessence, phantom energy
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1. INTRODUCTION

From several observations of Riess et al. (1998, 1999), Perlmutter et al. (1999), Clocchiatti et al. (1999), CMB data of Page (2003), WMAP of Spergel et al. (2003, 2007), Knop et al. (2003) [1, 2, 3, 4, 5, 6, 7, 8], and Planck collaborations of Ade et al. (2015) [9], it is shown that our Universe is undergoing an accelerated expansion. However, the cause behind the accelerating expansion of the Universe is not yet completely formulated. From existing literature like Copeland et al. (2006) [10], Friedman et al. (2008) [11], and Bamba et al. (2012) [12], we can say that dark energy plays a big role in the accelerating expansion of the Universe. But dark energy is still a mystery, although substantial lead has been formulated theoretically and observationally. Many cosmologists/researchers generally use two different methods to explain the phenomenon of dark energy. One method is to adopt some exotic matter sources such as quintessence (Martin 2008) [13], Chaplygin gas (Bento et al. 2002) [14], polytropic gas (Karami et al. 2009) [15], phantom models (Nojiri et al. 2003, 2009 and Bilic et al. 2008), tachyons (Padmanabhan et al. 2002) [16, 17, 18, 19], and the cosmological constant Λ (Alcaniz, 2006) [20]. The second one is to modify the field equation of Einstein's general theory of relativity. The cosmological constant, which was introduced by Einstein in the field equation of general relativity, is also considered a candidate for dark energy (DE). However, the cosmological constant term Λ has suffered the so-called cosmological constant problem (Weinberg 1989 [21] and Martin 2012 [22]), and fine-tuning and cosmic coincidence problem (Peebles and Ratra 2003 [23]). Secondly, modifying the Einstein field equation, many modified theories of gravity have been obtained. Some notable examples are $f(R)$ (Sotiriou and Faraoni 2010 [24] and De Felice and Tsujikawa 2010 [25]), $f(R,G)$ (De Laurentis 2015 [26], Santos da Costa et al. 2018 [27], Odintsov et al. 2019 [28], and Singh 2021 [29]), $f(R,T)$ (Harko et al. 2011 [30]), and $f(Q,T)$ (Xu et al. 2019 [31]) gravity theories.

Presently modified theories of gravity have become an interesting area of modern cosmology. Many researchers have developed different models of modified theory of gravities to accommodate the ever-growing accelerating expansion of the Universe. In this setting, $f(R)$ gravity, where R is the Ricci scalar is one of the most simplest modifications. It was first introduced by Buchdahl (1970) [32] and later used to find nonsingular isotropic de Sitter type cosmological solutions (Starobinsky 1980) [33]. Many cosmologists/researchers have studied different features of $f(R)$ gravity in varied cosmological models (Nojiri and Odintsov 2007 [34], Carroll et al. 2004 [35], and Reddy et al. 2014 [36]). A more general modified cosmological model than $f(R)$ theory is the $f(R,T)$ gravity theory proposed by Harko et al. (2011) [30]. It is established by coupling the geometry and matter in the gravitational action. Besides this, $f(R)$ gravity has been expanded to $f(R,G)$ gravity, where G is the Gauss-Bonnet scalar. It has been shown that $f(R,G)$ gravity obviously leads to an effective cosmological constant Λ, quintessence, or phantom cosmic acceleration (Elizalde et al. 2010) [37]. Xu et al. (2019) [31] have recently proposed $f(Q,T)$ gravity, which is an extension of symmetric teleparallel gravity. In this model, the gravitational action L is represented by arbitrary function f of the nonmetricity Q and trace of the energy-matter momentum tensor T. Simram and Sahoo (2020) [38] investigated energy conditions in $f(Q,T)$ gravity and observed that weak, null, and dominant energy conditions are all satisfied while the strong energy conditions (sec) are violated as per the present accelerating expansion of the Universe.

Many authors have investigated the cosmological models with a varying deceleration parameter. The deceleration parameter explains the accelerations and deceleration behavior of the Universe. More precisely, it acts as a geometric parameter. In the same direction of reasoning, in this work, within the context of current acceleration of the universe on its sign, we propose a deceleration parameter of third degree as an application to assess the early temporal behavior of the cosmos in terms of graphical representation, based on existing literature. Berman (1983) [39] and Berman and Gomide (1988) [40] had introduced a new law of time-dependent Hubble parameter in the framework of Robertson-Walker space-time which yield constant deceleration parameter. Singh et al. (2009) [41] investigated bulk viscous cosmological models of Universe in Lyra's Manifold by considering a time-varying deceleration parameter and coefficient of viscosity which is constant in FRW space-time, where exact solutions of Sen's Equations in Lyra Geometry have been derived and show that the Universe starts with a big bang singularity initially. Singh et al. (2010) [42] studied a new class of bulk viscous cosmological models in a scale covariant theory of gravitation in which they have studied the false vacuum model, the stiff fluid model, and radiating model with a time-varying deceleration parameter and found that the Universe has an initial singularity. It also concludes that the Universe begins with a big bang and is expanding. Singh (2015) [43] investigated Friedmann-Robertson-Walker (FRW) Universe in the presence of viscous fluid based on Lyra's manifold by considering a linearly varying deceleration parameter and coefficient of bulk viscosity to be a constant. Exact solutions have also been obtained where cosmological models have been derived. It has been found that the derived model of the Universe starts with an initial singularity and has a finite lifetime, which ends with a big rip. Akarsu and Dereli (2012) [44] studied cosmological models considering the deceleration parameter to be linearly varying, where the Universe shows quintom-like behavior and ends with a big rip. Lohakare et al. (2021) [45] also investigated the cosmological model by assuming the deceleration parameter to be time-varying in the context of $f(R, G)$ gravity, which shows quintessence behavior at late times. Bakry and Shafeek (2019) [46] also investigated the model of the Universe with a time-dependent deceleration parameter of the second degree and conclude that our Universe passes through a big rip, and then retreats back as it was initially in the instant of the big bang. Tiwari et al. (2021) [47] investigated a time-dependent deceleration parameter in the framework of $f(R, T)$ gravity and conclude that our Universe has a cyclic expansion history. That is, the Universe starts with deceleration expansion and later shifts to accelerated expansion and further to super-exponential accelerating expansion period. Bishi et al. (2022) [48] also investigated the behavior of the FRLW cosmological model in the context of $f(R, T)$ gravity with a quadratic deceleration parameter. Tiwari and Sofuoglu (2020) [49] investigated a quadratic varying deceleration parameter in the context of $f(R, T)$ gravity and established an interesting result that the Universe begins with a big bang and finally ends with a big rip. It also concludes that the Universe is filled with a quintessence like fluid in the early Universe and with a phantom-like fluid at late time.

Motivated by these studies and investigations of the above literature, we present a new law with a varying deceleration parameter of the third degree. A brief introduction about FRW cosmological model with a time-dependent deceleration parameter is given in Section 1. Modified field equation and solution of the model by introducing the deceleration parameter of third degree are discussed in Sections 2 and 3, respectively. Physical behavior and the graphical representations of the cosmological parameters of the proposed model are presented in Section 4. We discussed the energy conditions and $Om(z)$ diagnostic parameter in Sections 5 and 6, respectively, and the conclusion is given in Section 7.

2. THE FIELD EQUATIONS

The Einstein field equations are given by

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, and $T_{\mu\nu}$ is the energy-momentum tensor.

Consider the homogeneous and isotropic space-time Friedmann-Robertson-Walker (FRW) universe whose metric is given by

$$ds^2 = dt^2 - a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2 \right) \right], \tag{2}$$

where $a(t)$ is the scale factor and the spatial curvature index $k = -1$, $k = 0$, and $k = 1$ represents spatially open, flat, and closed Universe, respectively. We consider a co-moving fluid, where the FRW metric allows perfect fluid only for the energy-momentum tensor, which can be put in the form as follows:

$$T_{\mu\nu} = (p + \rho) u_\mu u_\nu - p g_{\mu\nu}, \tag{3}$$

where ρ is the energy density, p is the pressure, $g_{\mu\nu}$ is the fundamental Einstein tensor, and u_μ is the four-velocity vector satisfying $u_\mu u^\mu = 1$. For the metric (2), equation (1) takes the form

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}, \tag{4}$$

$$\frac{\ddot{a}}{a} = \frac{-4\pi G}{3} (\rho + 3p). \tag{5}$$

Equation (4) gives the mathematical expression of energy density (ρ) as

$$\rho = \frac{3\dot{a}^2 + 3k}{8\pi G a^2}. \tag{6}$$

And equation (5) gives the mathematical expression of pressure (p) as

$$p = -\frac{1}{8\pi G} \left(\frac{2\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right). \tag{7}$$

The equation of the state of the model is given by

$$p = \omega \rho, \tag{8}$$

where ω is the equation of state parameter and $-1 \leq \omega \leq 1$.

3. TIME-DEPENDENT DECELERATION PARAMETER OF THE THIRD DEGREE

Berman (1983) [39] and Berman and Gomide (1988) [40] put forward a new law of time-varying Hubble parameter in the framework of Robertson-Walker space-time in general relativity that gives deceleration parameter which is constant $q = m - 1$, where a is the scale factor and $m \geq 0$ is a constant. In physical cosmology, the deceleration parameter is the key to exemplify the expansion period of the Universe such as the acceleration and deceleration behavior of the universe. More precisely, it acts as a geometric parameter. At the back of the discovery of acceleration and expanding Universe, several authors are investigating the rate of Universe expansion and derived models by using a time-varying deceleration parameter. Bakry and Shafeek (2019) [46] also proposed a quadratic generalized varying deceleration parameter $q = (8n^2 - 1) - 12nt + 3t^2$, where n is a constant greater than zero. Based on existing literature, we proposed a new form of time-dependent deceleration parameter which is of third degree as follows:

$$q = (6n^3 - 1) - 22n^2 t + 18nt^2 - 4t^3, \tag{9}$$

where n is a positive constant. At $t = 0$, $q = 6n^3 - 1$ which is always positive for all $n \geq 1$, indicating that the expansion of the Universe starts with the deceleration phase. For $0 < n < 1$, we have the value of the deceleration parameter to be negative which may be interpreted as inflation epoch just after the big bang. The equation for the deceleration parameter is given by

$$q = \frac{-a\ddot{a}}{\dot{a}^2} = \frac{dH^{-1}}{dt} - 1. \tag{10}$$

From equations (9) and (10), we get the Hubble parameter as

$$H = \frac{\dot{a}}{a} = \frac{1}{t(n-t)(2n-t)(3n-t)}. \tag{11}$$

From equation (11), we can see that Hubble parameter H is always positive for $(n-t) < 0$, $(2n-t) < 0$, and $(3n-t) > 0$; i.e., Hubble parameter H is positive in the range $0 \leq t \leq 3n$. Also, we can observe that the Hubble parameter diverges at $t = 0$, $t = n$, $t = 2n$, and $t = 3n$. Integrating equation (11), we get

$$a = \left(\frac{t}{3n-t}\right)^{\frac{1}{6n^3}} \left(\frac{2n-t}{n-t}\right)^{\frac{1}{2n^3}} \tag{12}$$

which gives the expression of the scale factor a as a function of cosmic time. It is observed that the scale factor $a = 0$ for $t = 0$ and $t = 2n$, and it diverges for $t = n$ and $t = 3n$, which shows that the Universe begins with the big bang at $t = 0$ and ends with a big crunch at $t = 3n$. Putting the value of a, $\dot{a}$, and $\ddot{a}$ in the equation of ρ and p, we get

$$\begin{aligned} \rho = \frac{3}{8\pi G} \Bigg[& \frac{1}{t^2(n-t)^2(2n-t)^2(3n-t)^2} \\ & + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}} \left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}} \Bigg], \end{aligned} \tag{13}$$

$$\begin{aligned} p = -\frac{1}{8\pi G} \Bigg[& \frac{3 - 12n^3 + 44n^2t + 4t^2 - 36nt^2 + 4t^3}{t^2(n-t)^2(2n-t)^2(3n-t)^2} \\ & + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}} \left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}} \Bigg]. \end{aligned} \tag{14}$$

And the equation of state parameter ω is given by

$$\begin{aligned} \omega = & -\frac{(3 - 12n^3 + 44n^2t + 4t^2 - 36nt^2 + 4t^3)\, t^{\frac{1}{3n^3}} (2n-t)^{\frac{1}{n^3}}}{3\left[t^{\frac{1}{3n^3}}(2n-t)^{\frac{1}{n^3}} + kt^2(n-t)^{\frac{2n^3+1}{n^3}}(2n-t)^2(3n-t)^{\frac{6n^3+1}{3n^3}}\right]} \\ & - \frac{kt^2(n-t)^{\frac{2n^3+1}{n^3}}(2n-t)^2(3n-t)^{\frac{6n^3+1}{3n^3}}}{3\left[t^{\frac{1}{3n^3}}(2n-t)^{\frac{1}{n^3}} + kt^2(n-t)^{\frac{2n^3+1}{n^3}}(2n-t)^2(3n-t)^{\frac{6n^3+1}{3n^3}}\right]}. \end{aligned} \tag{15}$$

The commonly known examples of cosmological fluids with constant ω are dust ($\omega = 0$), radiation ($\omega = \frac{1}{3}$), and vacuum energy ($\omega = -1$) which is also mathematically equivalent to the cosmological constant Λ. When $\omega < -\frac{1}{3}$, it is considered in the context of dark energy as it gave rise to accelerated expansion of the Universe. Also $-1 < \omega < -\frac{1}{3}$ indicates the quintessence model and $\omega < -1$ represents the phantom phase of the model. In our model, ω approaches -1 as $t \to \infty$; this shows the accelerating expansion of the Universe at the late epoch. The findings of Supernova Legacy Survey (Mark Sullivan, 2004) [50] support the dark energy model that can change into the epoch ($\omega < -1$) (Eisentein et al. 2005) [51]; this observation is in agreement with our derived model. Since the model passes into the dark energy epoch, then it comes to the phantom region $\omega < -1$, and the existence of a big rip in future epochs is not ruled out by observational data and our model.

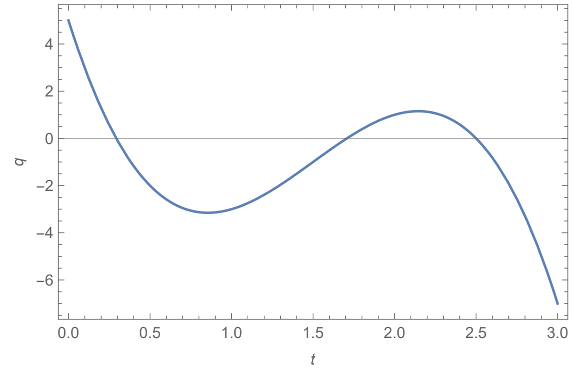

FIGURE 1: Plot of q versus t for $n = 1$.

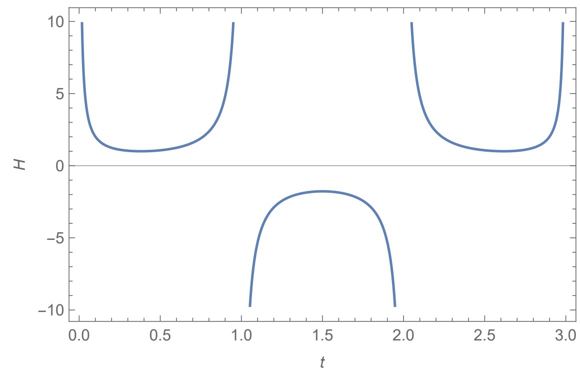

FIGURE 2: Plot of H versus t for $n = 1$.

4. PHYSICAL BEHAVIOR OF THE COSMOLOGICAL PARAMETERS

For simplicity, let us consider $n = 1$ for our discussion. Figure 1 shows the variation of the deceleration parameter q w.r.t cosmic time t, which shows that the Universe starts with decelerating expansion ($q > 0$) and moves to an expansion phase with a constant rate ($q = 0$) and later transitioned to the acceleration expansion phase ($-1 \leq q < 0$) and to the exponential acceleration phase (de Sitter expansion) ($q = -1$) and further to the super-exponential expansion phase ($q < -1$), and it retreats back again to decelerating expansion in a periodic manner. Initially, the deceleration parameter $q = 5$, and then enters into the acceleration phase $q < 0$ at $t \in [0.29, 1.7]$, and it enters into the deceleration phase $q > 0$ at $t \in [1.7, 2.5]$ and ends at $t \in [2.5, 3]$. This confirms the oscillating behavior of our model (Broadhurst et al. 1990 [52] and Morikawa 1990 [53]). The model shows the accelerating expansion at two phases with the value of $q = -0.73$ at $t = 0.35$ and $t = 2.58$ which are in agreement with Cunha and Lima 2008 [54]. Compared with the observational results of Cunha (2009) [55], the present value of the deceleration parameter is -0.73 which gives the present age of the Universe as $t = 0.35$. The transitional phase of the Universe from deceleration to acceleration is observed. From the plot of Hubble parameter H in Figure 2, we can say the Universe has a singularity at $t = 0$, big rip at $t = 1, 2$, and a big crunch at $t = 3$. Figure 3 shows the graphical representation of the scale factor a w.r.t the cosmic time t. From the graph, we can say that our model Universe has an initial singularity at $t = 0$ as $a(t) = 0$. Then, $a(t)$ diverges at $t = 1$, which gives the big rip behavior of the universe (Caldwell et al. 2003) [56] and later retreats into a moment of singularity at $t = 2$ and again

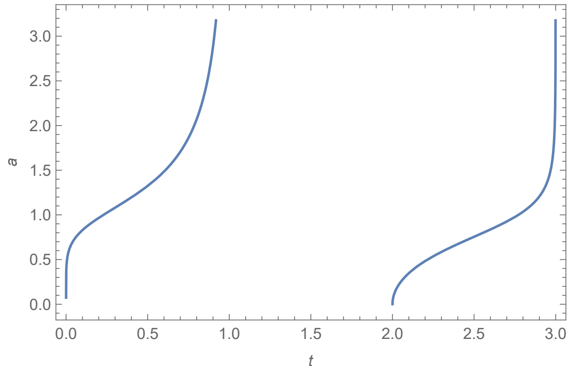

FIGURE 3: Plot of a versus t for $n = 1$.

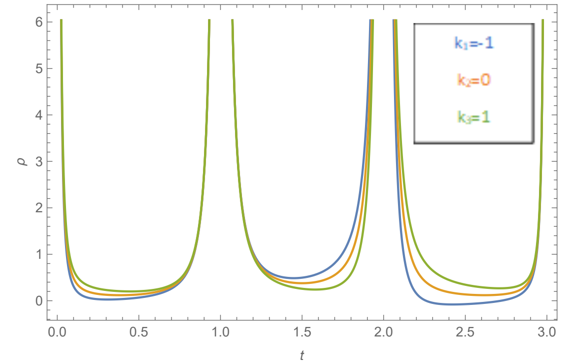

FIGURE 4: Plot of ρ versus t for $n = 1$ and $G = 1$.

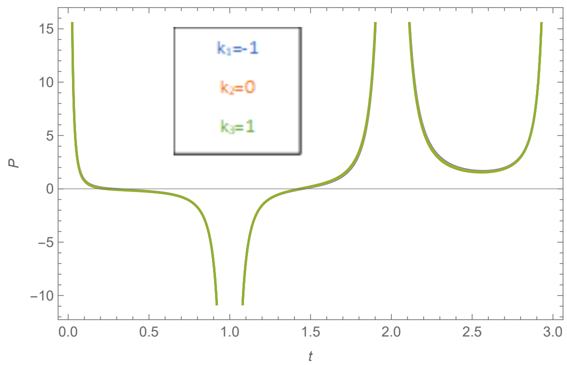

FIGURE 5: Plot of p versus t for $n = 1$ and $G = 1$.

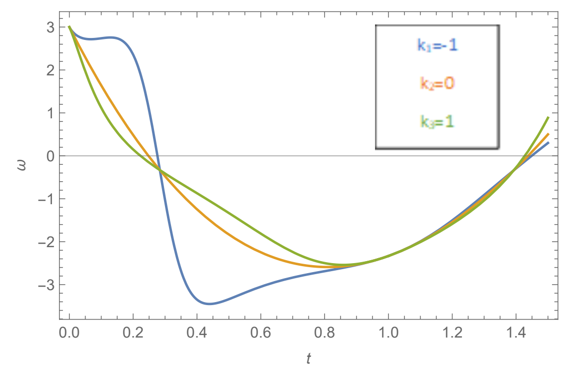

FIGURE 6: Plot of ω versus t for $n = 1$ and $G = 1$.

diverges at $t = 3$ again observing big rip behavior (Caldwell et al. 2003), which shows the periodic nature of the Universe predicting a periodic Universe. From Figure 4, we can say that the positive energy density condition for spatially open, flat, and closed Universe holds true. Further, we can also say that the Universe has an initial singularity at $t = 0$, big rip at $t = 1, 2$, and big crunch at $t = 3$ (Caldwell et al. 2003) [56]. Figure 5 highlights the graph of pressure w.r.t cosmic time t, which also shows that the Universe has an initial singularity at $t = 0$, big rip at $t = 1, 2$, and big crunch at $t = 3$ for spatially open, flat, and closed Universe. From the plot, we know that the positivity condition of pressure is violated for spatially open, flat, and closed Universe, which reckons for the accelerating expansion of the Universe. Figure 6 shows the evolution of the equation of the state parameter (EOS) ω with respect to cosmic time t for spatially open, flat, and closed Universe. It manifests simple different behaviour for spatially open, flat, and closed Universe but behaves in a similar manner, which starts from a positive value (stiff matter dominated $\omega = 1$ and $\omega = 1/3$) and tends to zero (dust filled model, $\omega = 0$) and further moves toward a negative value (vacuum energy model $\omega = -1$ and phantom energy $\omega < -1$). The solutions of this model with a varying deceleration parameter of cubic degree indicate that the Universe begins with inflation for few moments after the big bang and then it decelerates and accelerated until the moment of big rip, and then further accelerates to the moment of future crunch.

5. ENERGY CONDITIONS

Energy conditions are vital and important tools to know and investigate the geodesics of the Universe. It is primarily the boundary conditions to maintain the positivity condition of the energy density (Hawking and Ellis 1973 [57] and Poisson 2004 [58]). These energy conditions do not correspond to physical reality. The observable effects of dark energy which is manifested in the violation of strong energy conditions are the most recent examples of this reality. The four basic/fundamental energy conditions are as follows.

(i) Strong energy condition: $\Rightarrow \rho + 3p \geq 0$.

(ii) Null energy condition: $\Rightarrow \rho + p \geq 0$.

(iii) Weak energy condition: $\Rightarrow \rho \geq 0, \rho + p \geq 0$.

(iv) Dominant energy condition: $\Rightarrow \rho \geq 0, |p| \leq \rho$.

Substituting the values of equations (13) and (14) in the energy conditions, we get the following.

SEC:

$$\begin{aligned}\frac{3}{8\pi G}\Bigg[&-\frac{3-12n^3+44n^2t+4t^2-36nt^2+4t^3}{t^2(n-t)^2(2n-t)^2(3n-t)^2}\\&+k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\\&+\frac{1}{t^2(n-t)^2(2n-t)^2(3n-t)^2}\\&+k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\Bigg]\geq 0,\end{aligned} \tag{16}$$

WEC:

$$\frac{3}{8\pi G}\left[\frac{1}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right] \geq 0, \tag{17}$$

$$-\frac{1}{8\pi G}\left[\frac{3-12n^3+44n^2t+4t^2-36nt^2+4t^3}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right], \\ +\frac{3}{8\pi G}\left[\frac{1}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right] \geq 0, \tag{18}$$

NEC:

$$-\frac{1}{8\pi G}\left[\frac{3-12n^3+44n^2t+4t^2-36nt^2+4t^3}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right], \\ +\frac{3}{8\pi G}\left[\frac{1}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right] \geq 0, \tag{19}$$

DEC:

$$\frac{3}{8\pi G}\left[\frac{1}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right] \geq 0, \tag{20}$$

$$\left|-\frac{1}{8\pi G}\left[\frac{3-12n^3+44n^2t+4t^2-36nt^2+4t^3}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right]\right| \\ \leq \frac{3}{8\pi G}\left[\frac{1}{t^2(n-t)^2(2n-t)^2(3n-t)^2} + k\left(\frac{3n-t}{t}\right)^{\frac{1}{3n^3}}\left(\frac{n-t}{2n-t}\right)^{\frac{1}{n^3}}\right]. \tag{21}$$

The 3D graph of the strong energy condition is given in Figure 7. From the graph, we see that the strong energy condition does not hold true for our model Universe, which agrees with the present observation. Figures 8, 9, and 10 show the 3D plot of the weak energy condition, null energy condition, and dominant energy condition, respectively. We see from the graph that the null energy condition, weak energy condition, and dominant energy conditions holds true in our model Universe.

6. $Om(z)$ DIAGNOSTIC PARAMETER

The relation between the scale factor and redshift is given by

$$a(t) = \frac{1}{1+z}. \tag{22}$$

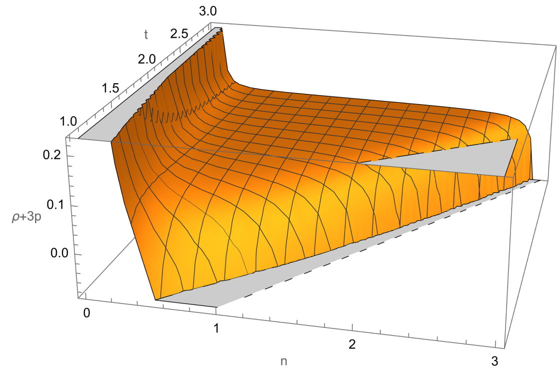

FIGURE 7: 3D Plot of SEC for $k = 1$, $n = 1$, and $G = 1$.

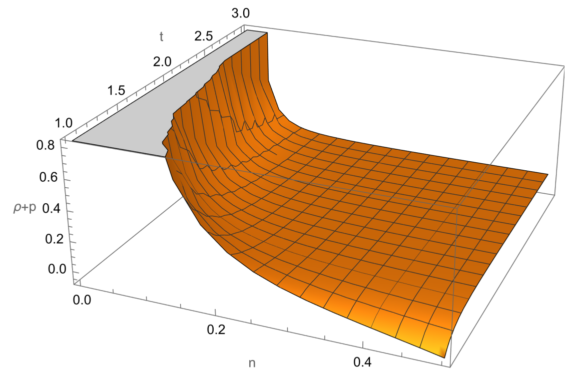

FIGURE 8: 3D Plot of NEC for $k = 1$, $n = 1$, and $G = 1$.

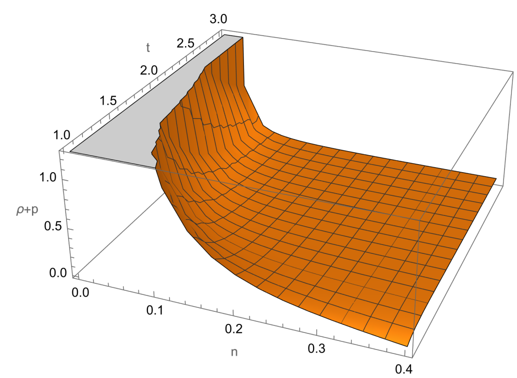

FIGURE 9: 3D Plot of WEC for $k = 1$, $n = 1$, and $G = 1$

This relation further relates cosmic time and redshift as

$$\frac{1}{(1+z)^6+1} = \frac{t^6}{2} - \frac{9}{4}t^2 + \frac{21}{8}t - \frac{t}{8(3-2t)} \\ \Rightarrow \frac{t^3}{2} \simeq \frac{1}{(1+z)^6+1} \\ \Rightarrow t \simeq \left[\frac{2}{(1+z)^6+1}\right]^{\frac{1}{3}} \tag{23}$$

$Om(z)$ diagnostic parameter is an important geometrical diagnostic approach used in the literature. Literature shows that the dark energy models are associated with positive Hubble parameters and negative deceleration parameters. H and q alone are not enough to differentiate between different dark energy models. To investigate such analysis, the $Om(z)$ diagnostic parameter is presented. When $Om(z)$ is constant w.r.t redshift, the dark energy model is in the form of Λ. The nature of

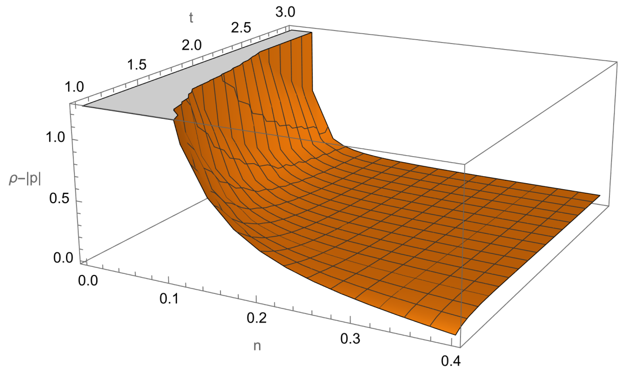

FIGURE 10: 3D Plot of DEC for $k = 1$, $n = 1$, and $G = 1$.

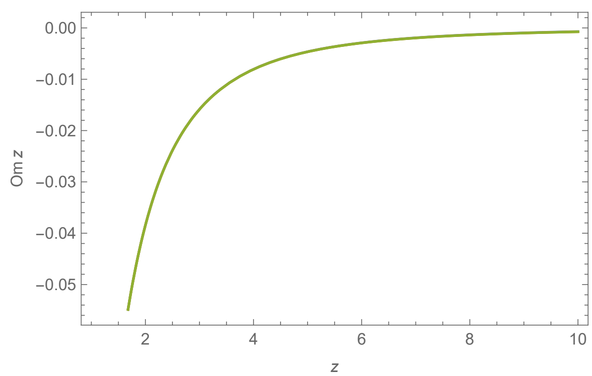

FIGURE 11: Plot of $Om(z)$ versus z for $n = 1$ and $H_0 = 67.77$.

the gradient of $Om(z)$ characterizes the dark energy models in this manner: when the gradient of the $Om(z)$ is positive, it indicates the phantom phase $\omega < -1$, and when the gradient of the $Om(z)$ is negative it indicates quintessence phase $\omega > -1$ (Sahni et al. 2008) [59].

The $Om(z)$ diagnostic parameter can be defined as

$$Om(z) = \frac{\left[\frac{H(z)}{H_0}\right]^2 - 1}{(1+z)^3 - 1}. \quad (24)$$

Here, H_0 denotes the present value of the Hubble parameter.

Figure 11 represents the plot of the $Om(z)$ diagnostic parameter versus redshift z for $n = 1$ and $H_0 = 67.77$. From Figure 11, we observe that the slope of $Om(z)$ is negative which indicates the phantom nature ($\omega < -1$) of our Universe.

7. CONCLUSION

In this work, we have investigated FRW cosmology with variable deceleration parameter of the third degree of the form $q = (6n^3 - 1) - 22n^2t + 18nt^2 - 4t^3$. At an initial moment, the Universe shows the inflation for $n < 1$; then, it decelerates for $n \geq 1$. In this model, we also observe the transition phase from deceleration to acceleration for the valid range $n \geq 1$ which is in agreement with observations. From the behavior of the cosmological parameters discussed above, we see that the parameters a, H, q, ρ, and p indicate that our model has a point-type singularity initially at $t = 0$ and similar singularities occur periodically at $t = n, 2n$ and observe future crunch at late time $t = 3n$. We also see that the parameters a, H, q, ρ, and p are infinite initially and preserve the periodic behavior against cosmic time. Thus, our model exhibits a periodic Universe. Also the positivity condition of energy density ρ is maintained and satisfied in our model. Strong energy condition is violated due to the negative nature of the pressure, accounting for the accelerated expansion of the Universe. But weak energy condition, null energy condition, and dominant energy condition are satisfied. The slope of $Om(z)$ diagnostic parameter with respect to redshift z is negative indicating the phantom phase of our model Universe. Thus, the model presented in this paper can be helpful in understanding the evolution of the Universe.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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