Capture of Dark Matter Particles by a Galaxy in the Case of a Bimodal Distribution of Their Velocities

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Abstract

We have analyzed the rate of capture of dark matter (DM) particles by the galaxy in the case of the existence of two different types of DM or a bimodal velocity distribution function for DM. It is shown that, in addition to the scenario considered in our previous work which is based on the assumption of a unimodal distribution, more complex scenarios are possible in which the transition to the state of intense capture and/or exit from it can occur in two stages. A detailed description is given of the change in the curve describing the rate of capture of dark matter particles as a function of the rate of increase in the baryon mass of the galaxy for various values of the rate of decrease of the DM density.

1. INTRODUCTION

From the analysis of galaxy rotation curves, it is known that dark matter (DM) is responsible for 85% of the mass of a typical galaxy. It forms a dark halo that surrounds the galaxy and extends far beyond the limits of its visible part, consisting of baryonic matter (BM). Evidence for the existence of dark matter is not limited to rotation curves. Reviews [1, 2, 3] describe other astronomical observations attributed to the influence of this still enigmatic entity. There are other points of view, in which the observed effects are explained by the modification of the laws of physics or with the help of alternative theories of gravity. There is also a hypothesis that dark matter consists of a large number of black holes; as a rule, typically these are primordial black holes with masses comparable to or even smaller than the solar mass.

In this article, we proceed from the assumption that DM consists of some still unknown particles that are not affected by strong and electromagnetic interactions. They interact between themselves and with baryonic matter gravitationally. A weak interaction is possible. However, the so-called mirror DM incapable of the weak interaction is also acceptable. In principle, it is also possible that DM particles can interact with each other using forces unknown to us, which do not act or act very weakly on BM. Candidates for particle dark matter are described in the paper [3]. The possibility that DM is a mixture of particles of different types is also acceptable.

These particles fill our Universe. Sometimes, they fly from intergalactic space into galaxies, and the gravitational field of the latter affects the speed of dark matter particles, deflecting them or changing their speed. Consider DM particles flying in and out of a dark halo. Their velocities inside the halo are greater than those in the intergalactic space due to falling into a gravitational well. Therefore, the density of DM particles passing through the halo is less than that outside the halo. If we con-

sider the galaxy as a source of a static gravitational field, then the incoming particle has enough energy to leave the galaxy.

Galaxies were formed after recombination as a result of the growth of density fluctuations. So, a natural question arises as to where this dark matter came from. The answer clearly refers to the time when the gravitational field of a galaxy is not constant. For example, it can be associated with the stage of galaxy formation. Galaxies formed after recombination as a result of the growth of small initial density fluctuations. This idea is in good agreement with the small fluctuations observed in the temperature of the cosmic microwave background [4, 5]. An overdensity leads to the infall of the surrounding matter. In this case, BM is basically captured, increasing the mass of the overdensity, while the DM can fly through it without being captured.

As long as perturbations are small, they can be studied with linear or higher-order perturbation theory. But for the formation of galaxies, such an approach is not suitable for two reasons. First of all, the overdensities of galaxies, $\rho_{gal}/\rho_m \gtrsim 10^5$, are much larger than 1 so that the perturbation theory cannot be expected to converge. Furthermore, we assume DM particles to be collisionless. In this case, they should be described by Vlasov's equation in phase space. While this leads to a similar Jeans scale as the fluid approach, just replacing the sound speed with the velocity dispersion [6, 7], nonlinear aspects are very different as soon a shell crossing becomes relevant which leads to singularities in the fluid approach.

Therefore, the nonlinear regime of the cosmological structure formation is usually treated either via N-body simulations, which have, at present, achieved an impressive amount of detail [8, 9, 10, 11] or by simple analytical models, usually spherical collapse or secondary infall models [12, 13, 14, 15, 16].

In article [17], we have proposed to use a new approach different from those mentioned above. Unlike the N-body simulations, we consider a continuous density distribution of particles without their concentration in massive objects, which allows one to use analytical treatment of the motion of DM particles and guarantees the absence of divergent acceleration. We do not consider the complex processes of accretion and thermal-

ization of BM, concentrating on the investigation of the possibility of capturing DM particles. The influence of baryonic matter is reduced to a single parameter, namely, the rate of increase of the overdensity due to BM accretion. As a result, we show the possibility of a catastrophic increase of the DM particle capture rate associated with the strong nonlinearity of this process.

The main mechanism of the increase in the DM mass is the capture of DM particles flying into the halo from intergalactic space. However, without BM accretion by the galaxy, they would fly through the galaxy without being captured. If the baryonic mass of the galaxy increases due to the accretion of BM or the merger of galaxies (see the details of galaxy formation in [7]), then some of the passing DM particles cannot escape due to a lack of kinetic energy. In this case, the depth of the potential well has time to increase during the time the particle passes through the galaxy. Correspondingly, the height of the potential barrier, caused by the gravitational interaction of the particle with the galaxy, also increases.

This is determined by the total mass of the galaxy M, which can increase due to the accretion of BM and the capture of DM. Accordingly, the rate of its increase $\dot{M}$ is equal to the sum of the rates of increase in DM, designated as $\dot{M}_{DM}$, and the rate of increase in BM, designated as $\dot{M}_b$:

$$\dot{M} = \dot{M}_{DM} + \dot{M}_b. \tag{1}$$

We consider $\dot{M}_b$ as an external quantity that can change with time, and we try to calculate $\dot{M}$ as a function of $\dot{M}_b$. We assume that the mass of the galaxy M is known at the moment of time under consideration (assuming that it does not change significantly during the passage of the DM particle, which cannot be captured due to its rather high initial velocity v_0 far away from the galaxy). We also know the halo radius R, which we consider to be constant, and the parameters of dark matter particles in intergalactic space.

They include the particle mass m, particle density N, and the particle velocity distribution. The last two quantities are determined by the distribution function $f(v_0)$. Let the number of particles with velocities in the range from v_0 to $v_0 + dv_0$ in a unit volume in extragalactic space be equal to $dN = f(v_0)dv_0$. So, the total density of DM particles is $N = \int_0^\infty f(v_0)dv_0$.

We consider DM particles to be nonrelativistic, and we use the formulas of classical mechanics. Indeed, astronomical observations show that the characteristic velocities of dark matter are much less than the speed of light. This type of dark matter is called cold dark matter, in contrast to hot dark matter with particles moving at relativistic speed. Estimates of the minimum speed of a DM particle in intergalactic space, which allows it to fly through the Milky Way without capture, were made in [17] and give a value not exceeding $100\,\mathrm{km/s}$, which is significantly less than the speed of light. Faster particles fly through the Galaxy and do not contribute to the increase in the DM mass in it. Actually, nothing prevents us from taking into account the effects of Special Relativity and considering the case of hot DM, but this apparently will not lead to a qualitative change in the main result of our model, specifically the catastrophic increase of the DM particle capture rate. We assume that the rest mass of the DM particle is large enough not to take into account its wave properties but otherwise arbitrary.

We take into account the Hubble expansion to account for changes in the DM density in intergalactic space. N decreases rapidly with time because of both Hubble expansion and the capture of a part of the DM particles by galaxies. The first of these effects is stronger and ensures a dependence according to the law $N \propto (1+z)^3$. The second makes the decrease in N even faster. Other parameters of the galaxy like its mass, M, or of the velocity distribution of DM, $f(v_0)/N$, change much more slowly with time.

The model described in the article [17] assumes that the halo is spherically symmetrical and homogeneous; i.e., has no inhomogeneities, on which the DM particles could perform a gravitational maneuver. Therefore, particles moving radially can either be captured at $v_0 < v_{pc}$ or fly through the galaxy at $v_0 > v_{pc}$ (the subscript pc indicates passage through the center). The estimate detailed in [17] gives

$$v_{pc} \simeq a\sqrt{M}, \quad a = \left(\frac{2GR}{M}\right)^{1/4}. \tag{2}$$

A nonlinear equation

$$\dot{M}_{DM} \simeq \pi R^2 \int_0^{v_{pc}} \left(1 - \frac{v_0^4}{v_{pc}^4}\right) m f(v_0) v_0 dv_0 \tag{3}$$

was also obtained in the article [17]. From the analysis of equations (1)–(3), we obtained a qualitative picture of the process of capture of DM particles and of the accumulation of DM mass in the galaxy. In this article, we will continue the analysis, considering, additionally, some more exotic variants.

These options arise due to the fact that equation (3) includes an unknown distribution function $f(v_0)$. We have no observational data to establish its type or convincing theories to predict it. We do not know if DM particles are in a state of thermal equilibrium. It is only known that dark matter has characteristic velocities much smaller than the speed of light and belongs to the so-called cold or warm dark matter. The latter has properties intermediate between those of hot dark matter and cold dark matter, but the differences are mainly related to the details of structure formation.

Nevertheless, one can make a number of reasonable assumptions about the form of $f(v_0)$. We suppose that the function $f(v_0) \geq 0$ is continuous, vanishes at $v_0 = 0$, reaches a maximum at some value $v_0 = v_{\max}$, and quickly decreases to zero at the high-speed end, most likely exponentially in v_0^2. At small v_0, we can put

$$f(v_0) \approx a_1 v_0^2 \tag{4}$$

due to the three independent Cartesian velocity components. The coefficient $a_1 \propto N$.

This is enough to analyze the solution of the system (1)–(3) and derive a number of qualitative conclusions. They are based primarily on the shape of the curve $\dot{M}(\dot{M}_b)$. We immediately note that this form is constantly changing due to the time dependence of the parameters included in equations (2) and (3), primarily N.

In this case, as we have found in [17] a qualitative change in the shape of the curve occurs. When N is less than some critical value N_1, the curve increases monotonically. In this case, the capture of dark matter occurs weakly and only at $\dot{M}_b > 0$. At $N > N_1$, the curve acquires a characteristic s-shape. In this case, it consists of three branches. Two of them have a positive derivative and correspond to stable solutions. They are connected by a section with a negative derivative corresponding to

unstable solutions. The behavior of the solution in this case is well studied in catastrophe theory. It is well described in many books including [18, 19]. This shape is called the fold catastrophe.

The solution $\dot{M}_b = \dot{M} = 0$ is stable, as is any point on the stable branches. Therefore, in the absence of accretion, there is no capture of DM particles on the lower branch. But for $\dot{M}_b > 0$, we have $\dot{M}_{DM} > 0$. So, capture in this case is a consequence of synergy, i.e., combined action of BM accretion and passage of DM particles. If the system is on the lower stable branch and an external influence, i.e., the value of $\dot{M}_b$ increases so much that it reaches the right boundary of the lower branch (we will refer to it as point B) and continues to increase, then the state of the system jumps to the upper stable branch and remains on it even after decreasing $\dot{M}_b$.

The upper branch describes the galaxy in a state of intense capture of DM particles. In this case, the ratio between the growth rates of the dark and baryon masses can be very large. This explains the high abundance of dark matter in galaxies. In our opinion, the galaxies captured almost all of their DM in the state of intense capture. In [17], the mass of dark matter that could be captured by the Galaxy was estimated if the process of intense capture began at the redshift parameter z_0. It turned out that the proposed mechanism can ensure the capture of all dark matter in the galaxy at $z_0 = 15 \ldots 25$. The process of intense capture could have begun even before the appearance of the first stars and reionization.

The upper stable branch has a left boundary, i.e., the minimum value $\dot{M}_b$. We will refer to it as point C. As $\dot{M}_b$ decreases, the system located on the upper branch can reach point C and, as $\dot{M}_b$ decreases further, it jumps to the lower branch. In this case, the galaxy leaves the stage of intense capture. Note that this is possible only if $\dot{M}_b > 0$ in point C. As N decreases, the curve changes so that point C shifts to the right, crossing at some moment the y-axis. Only after this crossing is it possible to end the stage of intensive capture with a decrease in $\dot{M}_b$.

Something like a phase transition occurs at $N = N_1$ with a change in the regime of particle capture. But by this time, the intensive capture mode will most likely have had time to end.

We can illustrate all of the above with the example of the Maxwell-Boltzmann distribution

$$ f(v) = \frac{4Nv^2}{\sqrt{\pi}v_{\max}^3} \exp\left[-\left(\frac{v}{v_{\max}}\right)^2\right], \quad v_{\max} = \sqrt{\frac{2k\Theta}{m}} \quad (5) $$

with temperature Θ and maximum at $v = v_{\max}$. Figure 1 shows the dependence of quantities proportional to $\dot{M}$ and $\dot{M}_b$:

$$ x = \gamma\dot{M}_b, \quad y = \gamma\dot{M}, \quad \gamma = \sqrt{\frac{2GR}{M}}v_{\max}^{-2}. \quad (6) $$

All parameters included in equations (1)–(5) are reduced to a single parameter

$$ \mu = \frac{2R^2mN}{v_{\max}}\sqrt{\frac{2\pi GR}{M}}, \quad (7) $$

which decreases due to the decrease of N. In Figure 1, one can see both the transition from the s-shaped curve to a monotone one and the intersection of point C with the y-axis. We see that after the jump to the upper stable branch, the rate of increase in the mass of the galaxy due to the capture of DM particles increases greatly at the same rate of increase in the baryonic
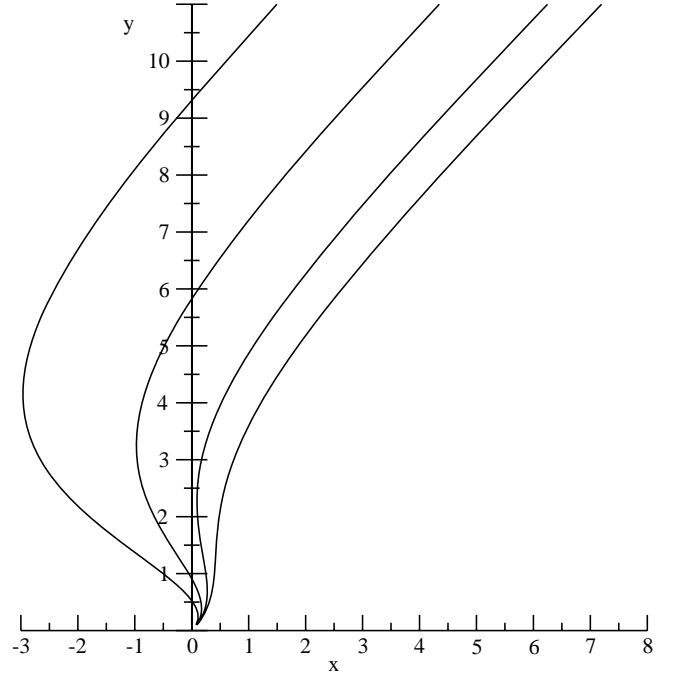
FIGURE 1: Dependence of x and y defined in equation (6) proportional to the rates of increase in the total and baryonic masses of the galaxy for the case of the Maxwell distribution of DM particle velocities. The curves from left to right correspond to the different values of the parameter μ from equation (7) equal to 10, 7, 5, and 4.

mass. For the curves depicted in Figure 1, it increases 11 times at $\mu = 5$, 55 times at $\mu = 7$, and 85 times at $\mu = 10$. This shows how significantly the catastrophic transition to the intensive capture regime affects the ratio of the growth rates of dark and baryonic matter.

Again, we are using the Maxwell-Boltzmann distribution for illustration purposes only. It has all the properties that, according to our assumption, are inherent in the function $f(v)$, including the asymptotic behavior of equation (4) at low speeds and fast exponential decay at large values of v. However, we do not claim that it describes the actual velocity distribution of dark matter particles or that dark matter particles are in a state of thermal equilibrium.

We can describe the process of accumulation of dark matter inside the halo of the galaxy. As a result of the growth of small density fluctuations, regions of increased density emerge, in which galaxies can form. The surrounding matter, both baryonic and dark, begins to fall into them. In the absence of accretion of baryonic matter, DM particles fly through protogalaxies and are not captured.

But the situation changes with an increase in the mass of the baryonic component of the galaxy. A fraction of the DM particles passing into the galaxy at sufficiently low speed is being captured. The mass of dark matter inside the galaxy begins to grow, but the rate of its increase is smaller than the rate of increase in the mass of BM.

If the rate of increase in the mass of the BM exceeds a certain threshold value, corresponding to point B, a jump on the upper branch occurs. The galaxy goes into the regime of intense capture of DM particles. The system remains on the up-

per branch until either the particle density drops below N_1 or point C crosses the y-axis, and the baryon mass growth rate becomes less than the abscissa of point C. The galaxy comes out of the regime of intense capture of DM particles.

2. EXOTIC EXAMPLES OF DM CAPTURE

Let us try to add something to this picture and show that sometimes the process of transition to the stage of intense capture or exit from it occurs in a more complicated way. This possibility arises in some cases when the distribution function $f(v)$ is bimodal. Can such a function be realized? As already mentioned, we do not know the form of $f(v)$. It is possible that some processes in the early Universe could lead to a such distribution of DM particle velocities.

But there is also another possibility. There may be two (or more) different types of dark matter with different particles and different velocity distributions; then, we can include both in equation (3). Both types of particles have the properties that we attributed to DM particles. They gravitationally attract BM and dark matter of any kind and do not participate in strong and electromagnetic interactions. The capture of DM particles of any kind increases the mass of dark matter in the galaxy and its total mass but does not affect the rate of increase in the mass of the baryonic component.

In this case, it is easier to consider the velocity distribution not of the number of particles but of their mass density $mf(v)$. Since only the product of $f(v)$ and the particle mass m enters into the formulas, the transition is simple. It is obtained by summing the distribution for each particle type. If the properties of different DM particles are very different, then the velocities corresponding to the modes of the two distributions are also likely to be different. By summing two distributions with different modes, we can get a total bimodal distribution.

If DM particles of both types do not born and do not decay, then the ratio between their total number in the Universe (but not in the halo of an individual galaxy) is preserved and the average DM mass density is proportional to their average concentration N. So, it is worth analyzing the behavior of the curves describing the dependence of $\dot{M}$ on $\dot{M}_b$ for the case of an arbitrary distribution $f(v)$ that satisfies the formulated conditions.

Note that all points on the curve that are of most interest to us, like points B and C, have vertical tangents. The same applies to the moment of the phase transition at $N = N_1$, when a point with a vertical tangent appears on the curve. At these points, we have the condition $d\dot{M}/d\dot{M}_b = \infty$ or $d\dot{M}_b/d\dot{M} = 0$. From equation (1), we then get $d\dot{M}_{DM}/d\dot{M} = d\dot{M}_{DM}/dv_{pc} \cdot dv_{pc}/d\dot{M} = 1$. From equations (3) and (2), we then obtain

$$H\left(v_{pc}\right) = \frac{1}{2\pi R^2 mN}\left(\frac{M}{2GR}\right)^{1/2}, \qquad (8)$$

where we have introduced the function H defined by

$$H\left(v_{pc}\right) \equiv N^{-1} v_{pc}^{-6} \int_0^{v_{pc}} v_0^5 f\left(v_0\right) dv_0. \qquad (9)$$

The factor N^{-1} is introduced such that this function does not directly depend on N. It compensates for the fact that $f \propto N$. As a result, the function H changes with time, but slowly, not as fast as the particle density N.
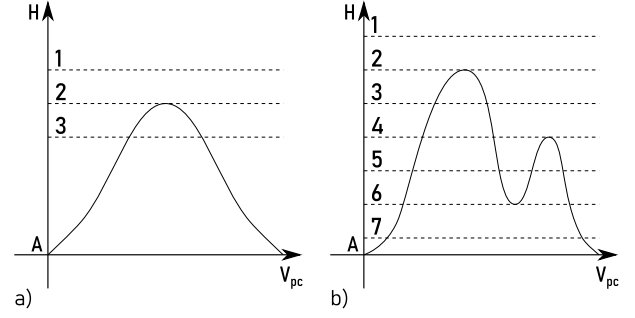
FIGURE 2: Unimodal (a) and bimodal (b) functions H associated with the velocity distribution of dark matter particles.

Let us consider its properties. The function $H(v_{pc}) \geq 0$ and tends to zero as $v_{pc} \to 0$ because of asymptotic behavior (4). It also tends to zero as $v_{pc} \to \infty$ because the integral in equation (9) tends to a finite value, but the factor v_{pc}^{-6} ensures that it falls according to the law $H(v_{pc}) \xrightarrow[v_{pc} \to \infty]{} O(v_{pc}^{-6})$. This means that it has a maximum at a certain $v_{pc} = v_1$.

Can it have several local maxima with minima in between? At each extremum, the condition

$$\frac{dH\left(v_{pc}\right)}{dv_{pc}} = N^{-1} v_{pc}^{-7}\left(v_{pc}^6 f\left(v_{pc}\right) - 6\int_0^{v_{pc}} v_0^5 f\left(v_0\right) dv_0\right) = 0 \qquad (10)$$

must be satisfied. After integration by parts, it reduces to

$$\int_0^{v_{pc}} v_0^6 f'\left(v_0\right) dv_0 = 0, \qquad (11)$$

where the prime means the derivative with respect to the argument. If the function $f(v_0)$ is bimodal and has two maxima at the points $v_{\max 1}$ and $v_{\max 2} > v_{\max 1}$ and a local minimum at $v_{\min 1}$ between them, then f' is positive on the segments $(0, v_{\max 1})$, $(v_{\min 1}, v_{\max 2})$ and negative on the segments $(v_{\max 1}, v_{\min 1})$, $(v_{\max 2}, \infty)$. Condition (11) can be met at some point between $v_{\max 1}$ and $v_{\min 1}$ only if

$$\int_0^{v_{\max 1}} v_0^6 f'\left(v_0\right) dv_0 < \int_{v_{\max 1}}^{v_{\min 1}} v_0^6 |f'\left(v_0\right)| dv_0. \qquad (12)$$

If this condition is not satisfied, then there are no extrema of the function H also on the segment $(v_{\min 1}, v_{\max 2})$. A simple analysis shows that H can be bimodal only if f is bimodal or multimodal, and the maxima of f are at a sufficiently large distance from each other. But this variant is possible in principle. However, the bimodality of f does not guarantee that H is bimodal.

Consider the function H at some moment. It can be unimodal as in Figure 2(a) or bimodal. Figure 2(b) shows an example of such a function. Of course, any of the two maxima can be the global maximum at $v_{pc} = v_1$. In Figure 2, it is the left one that has a smaller value of v_{pc} and hence $\dot{M}$. But, it is easy to construct curves for the case when the right maximum is higher.

According to equation (8), the points with vertical tangents on the dependence of $\dot{M}$ on $\dot{M}_b$ correspond to the intersections of the curve $H(v_{pc})$ with the horizontal line of height given by the right-hand side of equation (8) which decreases with in-
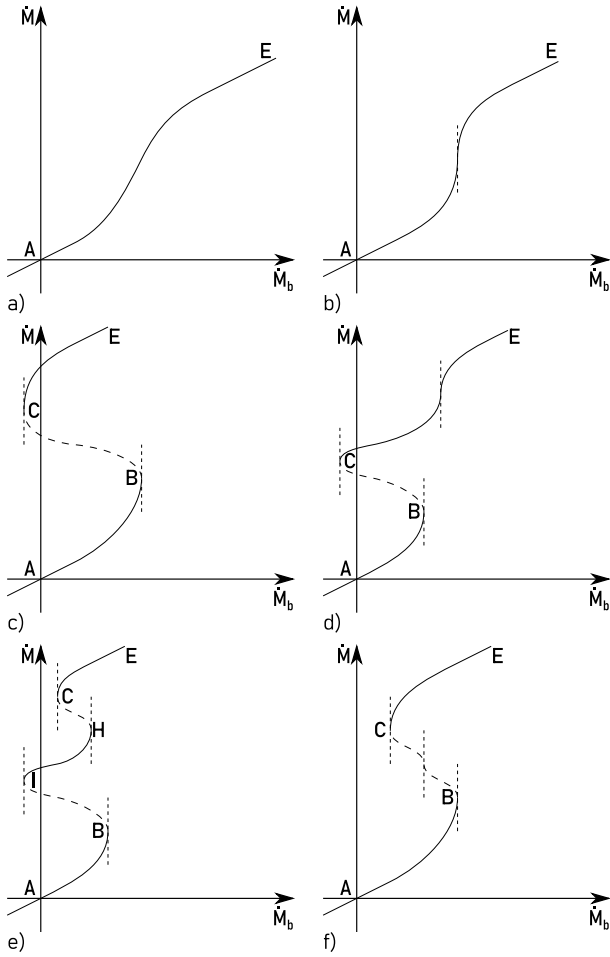
FIGURE 3: Characteristic curves of dependence $\dot{M}$ on $\dot{M}_b$ at different values of particle density N. (a) $N < N_1$, line 1 in Figure 2. (b) $N = N_1$, line 2 in Figure 2. (c) S-shaped curve for lines 3 and 7. (d) The same as (c) but with a vertical tangent point outside BC segment for line 4. (e) Double s-shaped curve for line 5. (f) The same as (c) but with a vertical tangent point inside the BC segment for line 6.

creasing N. We denote

$$N_1 = \frac{1}{2\pi R^2 m H(v_1)} \left(\frac{M}{2GR} \right)^{1/2}. \tag{13}$$

At $N < N_1$, we have no solution, and the points with vertical tangents do not exist. In Figure 2, this case corresponds to the horizontal line 1 passing above the curve. The characteristic form of dependence $\dot{M}$ on $\dot{M}_b$ for this case is shown in Figure 3(a).

At $N = N_1$, line 2 touches the curve at its maximum. This value corresponds to the transition to an s-shaped curve from the monotonic one. The characteristic form of the curve for this case is shown in Figure 3(b). At $N > N_1$, we have several possibilities. For line 3 below the maximum (but above the second mode for the bimodal H), we find two solutions of equation (8). The solution with smaller v_{pc} corresponds to point B, one with a larger v_{pc} to point C. For small v_{pc}, we can use the approximation (4) and obtain the estimate for the coordinates of point

B. The characteristic form of the curve for this case is shown in Figure 3(c).

The following lines are only of interest in the case of the bimodal distribution shown in Figure 2(b). Line 4 touches the top of the lower mode. The characteristic curve for this case is depicted in Figure 3(d). Line 5 has four points of intersection with $H(v_{pc})$. The characteristic curve (its shape) depicted in Figure 3(e) resembles two letters s one above the other. Line 6 passing through the local minimum provides the characteristic curve depicted in Figure 3(f). The characteristic curve for line 7 below the local minimum in Figure 2(b) is qualitatively the same as in Figure 3(c).

Recall that lines 1–7 correspond to an increase in the particle density N, which corresponds to earlier stages of the evolution of the galaxy. In the process of this evolution, the curve $\dot{M}$ on $\dot{M}_b$ changes its form, successively acquiring the forms shown in Figures 3(c), 3(f), 3(e), 3(d), 3(c), 3(b), and 3(a).

Of all these stages, the most exotic is by far the one whose graph is shown in Figure 3(e). However, this conditional image corresponds to 4 possible variants of the mutual arrangement of the abscissas of points with vertical tangents. All of them are shown in Figure 4. The curve has three stable branches AB, IH, and CE and two unstable ones BI and HC.

Let us consider the different possibilities one by one. The curve in Figure 4(a) corresponds to the case when the transition to and exit from the stage of intense capture of DM particles occur in two jumps BD + HJ and CK + IF. In Figure 4(b), both transitions occur in one jump. The states on the branch IH are stable, but unattainable for the system. In Figure 4(c), the upward jump BD occurs in one stage and downward in two; Figure 4(d) depicts the opposite case.

Let us discuss the question of which direction the dependence curve shifts during galaxy formation, when the value of N rapidly decays. Consider an arbitrary point on this curve that has some fixed ordinate, i.e., constant value $\dot{M}$. From equation (2), we find the dependence of v_{pc}. The value of a does not depend on N. It slowly decreases due to the increase in the mass of the galaxy M, but this can be neglected in comparison with the effect of the rapidly falling particle density N. The value of $\dot{M}_{DM} \propto N$ also decreases rapidly. The abscissa of the point $\dot{M}_b$ shifts to the right with decreasing N according to equation (1). Therefore, in the process of evolution, the curve $\dot{M}$ versus $\dot{M}_b$ shifts to the right. Figure 1 illustrates this.

This also applies to the abscissas of points B, C and H, I, if the last two are present. But their ordinate also changes. Figure 2 shows that the value of v_{pc} and hence also $\dot{M}$ increases for points B and H and decreases for points C and I as N decreases.

Is the number of stages required to jump to the upper stable branch and to descend to the lower stable branch predetermined by the initial conditions? A jump to another branch requires the simultaneous fulfillment of two conditions: the presence of a point with a vertical tangent at some value $\dot{M}_{b0}$ on the curve $\dot{M}(\dot{M}_b)$ and a change in $\dot{M}_b$ so that its value coincides with or passes through the value of $\dot{M}_{b0}$. Both the shape of the curve $\dot{M}(\dot{M}_b)$ and the external parameter $\dot{M}_b$ change with time. If we know exactly how they change, we can determine the number of jumps during the evolution of the galaxy.

However, the rate of increase in the mass of the baryonic component $\dot{M}_{b0}$ depends on the rate of accretion and merger of galaxies, and the shape of the curve is also determined by the parameters of the galaxy that change depending on accretion, for example, its total mass and size. Therefore, the number of
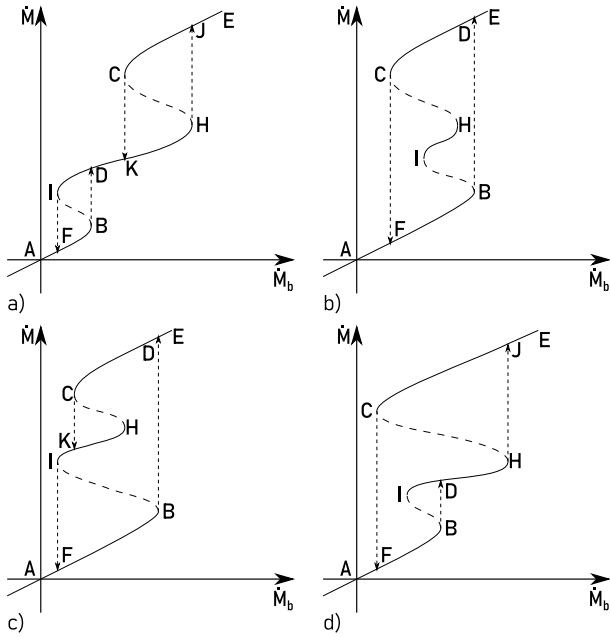
FIGURE 4: Different options of the curve in Figure 3(e) provide a transition to the upper and lower branches with a different number of jumps.

jumps during the formation of a galaxy depends on the values determined by the environment of the given galaxy, especially $\dot{M}_b$, and is not determined only by the initial conditions. One can expect one jump on a branch with an intense capture and a second jump when returning to a branch with a moderate capture in the case of a unimodal distribution without strong changes in the $\dot{M}_b$ value. But more than two jumps are also possible in the case of a bimodal distribution of DM particles velocities. The transition to any of the above-mentioned branches can occur in two stages with a jump to an intermediate stable branch.

3. CONCLUSIONS

We have analyzed the rate of capture of DM particles by the galaxy for two different types of DM particles or a bimodal velocity distribution function of DM particles. It is shown that, in addition to the scenario considered in our previous work which was based on the assumption of unimodality of the velocity distribution, more complex scenarios are possible in which the transition to the state of intense capture and/or exit from it occurs in two stages.

A detailed description is given of the change in the curve describing the rate of capture of dark matter particles as a function of the rate of increase in the baryon mass of the galaxy for various values of the density of dark matter which decreases with time, N. As N decreases, the curve shifts to the right, increasing the value of $\dot{M}_b$ at a practically constant $\dot{M}$.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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