An A_4 Model for keV-Scale Sterile Neutrino Dark Matter

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Abstract

We develop an $A_4 \times Z_4 \times Z_2$ symmetry model of neutrino masses and mixings within the minimal extended seesaw mechanism where three right-handed neutrinos ν_{R1}, ν_{R2}, and ν_{R3} and a keV-scale singlet sterile neutrino S are added to the Standard Model. This model breaks μ-τ symmetry of the neutrino mass matrix and successfully explains leptonic mixing with nonzero θ_{13}. We study the phenomenological results of the keV-scale sterile neutrino as a dark matter candidate by calculating the relic abundance and its decay rate to active neutrinos. Significant results are also observed for solar and atmospheric mixing angles within the 3σ bounds, sum of the active neutrino masses ($\sum m_i < 0.12$ eV), and effective neutrino mass from neutrinoless double beta decay, $m_{\text{eff}} \sim$ (0.001174–0.004367) eV for NH and $m_{\text{eff}} \sim$ (0.04766–0.05088) eV for IH.

Keywords: sterile neutrino, dark matter, A_4 model, beyond standard model
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1. INTRODUCTION

The observation of Higg's boson at the Large Hadron Collider (LHC) has successfully validated the Standard Model (SM) of particle physics. However, discoveries of solar and atmospheric mixing angles in SNO [1, 2], SK [3], etc. indicated the presence of nonzero neutrino mass and their oscillation between different flavors. Some other evidences such as the existence of dark matter (DM), matter-antimatter asymmetry, provide the need to go beyond the theory of SM (BSM). Among many hot problems in particle physics, the origin of flavor structure, Dirac CP-violating phase, absolute neutrino masses, neutrino mass hierarchy, and Dirac or Majorana nature of neutrino are extremely studied in the current scenario.

The extension of SM through discrete symmetry is one of the most interesting approaches in realizing the flavor structure of neutrino mass [4]. In particular, A_4 symmetry along with additional Z_n groups can provide a way to understand the underlying the symmetry of neutrino mass matrix, nonzero θ_{13}, Dirac CP-violating phase, etc. by considering Higgs-type scalar flavons [5, 6, 7, 8]. However, the addition of a number of flavon fields and additional symmetry groups are the main drawbacks of such approaches in neutrino model building.

Cosmological and astrophysical measurements have assured the presence of a mysterious, nonbaryonic, nonluminous matter, called dark matter which accounts for 26.8% of the total energy density of the Universe [9, 10]. In spite of the strong evidences for the presence of DM, the fundamental nature of dark matter, i.e., its origin, its constituents, and interactions, is still unknown. According to the latest Planck data [11], the relative abundance of DM in the Universe is observed as

$$\Omega_{\text{DM}} h^2 = 0.1187 \pm 0.0017. \tag{1}$$

The requirements of a DM particle [12] rule out all the SM particles. One of the most interesting BSM particles which behave as a warm dark matter (WDM) is the sterile neutrino. Particularly, sterile neutrinos with masses in the keV range having very small mixing with the active neutrinos of the order of $\sin^2 2\theta \sim \mathcal{O}(10^{-10})$ act as a dark matter [13, 14, 15, 16]. Dodelson-Widrow mechanism [17] provides the correct abundance of keV-scale sterile neutrinos as dark matter. The resulting relic abundance is proportional to the active-sterile mixing angle and mass of the sterile neutrino as

$$\Omega_{\text{DM}} h^2 \simeq 0.3 \left(\frac{\sin^2 2\theta}{10^{-10}}\right)\left(\frac{m_s}{100\,\text{keV}}\right)^2. \tag{2}$$

Sterile neutrinos can radiatively decay into an active neutrino and a monoenergetic photon through $\nu_s \to \nu_a + \gamma$. The decay width is given by [18]

$$\Gamma = 1.38 \times 10^{-32} \left(\frac{\sin^2 2\theta}{10^{-10}}\right)\left(\frac{m_s}{\text{keV}}\right)^5 \text{s}^{-1}, \tag{3}$$

where $\sin^2 2\theta = 4\sum_{i=e,\mu,\tau} |V_{i4}|^2$ is the effective active-sterile mixing angle and m_s is the mass of the sterile neutrino. This rate is very small, and the sterile neutrino lifetime is sufficiently large and exceeds the age of the Universe, so that the keV-scale sterile neutrino is a good DM candidate. There are constraints on the mass and mixing of the keV-scale sterile neutrino provided by X-ray, Lyman-α forests, Tremaine-Gunn bounds from phase-space analysis, etc. Combining these constraints, we can infer that the mass of the sterile neutrino should be $m_s \geq 4$ keV, and its mixing with SM neutrinos should be $\sin^2 2\theta \leq 10^{-6}$ [19].

Many authors have also discussed keV sterile neutrino dark matter based on different models [20, 21, 22]. Particularly, in [22], the authors considered different sets of five flavons extended with another two flavons for the perturbation terms separately for normal heirarchy (NH) and inverted heirarchy (IH). The present work is different and more efficient in the fact that we use less number of flavon fields and we try to address both NH and IH from a single neutrino mass model. Our model is based on $A_4 \times Z_4 \times Z_2$ by extending the SM with three A_4 singlet right-handed neutrinos ν_{R1}, ν_{R2}, and ν_{R3} and a singlet chiral field S in the 3+1 Minimal Extended Seesaw (MES) scheme [23]. The present letter is organized as follows. Section 2 gives a detailed description of the model followed by the numerical analysis in Section 3. We conclude in Section 4 with a brief summary.

Fields	l	e_R, μ_R, τ_R	H	H'	H''	ψ	ϕ	χ	ζ	ν_{R1}	ν_{R2}	ν_{R3}	S	η
$SU(2)_L$	2	1	2	2	2	1	1	1	1	1	1	1	1	1
A_4	3	$1, 1'', 1'$	1	$1'$	$1''$	3	3	1	$1''$	1	1	1	$1'$	3
Z_4	1	1	1	$-i$	i	1	$-i$	1	-1	i	-1	1	$-i$	$-i$
Z_2	1	1	1	-1	1	1	1	1	-1	1	-1	1	-1	1

TABLE 1: Particle content of the model and their group charges.

2. THE MODEL

In this model, we extend the SM with A_4 symmetry supplemented by Z_4 and Z_2. The SM lepton doublet l transforms as a triplet under A_4 while the right-handed charged leptons transform as singlets. Two A_4 singlet Higgs H' and H'' are used along with the SM Higgs H. Three flavon fields ϕ, ψ, and η which transform as triplet under A_4 are considered in which η is responsible for the breaking of μ-τ symmetry in the neutrino mass matrix. Two more A_4 singlet flavons χ and ζ provide the masses for heavy Majorana neutrino mass matrix M_R and sterile neutrino mass matrix M_S, respectively. The full particle content and their group charges are given in Table 1.

The invariant Lagrangian density for leptonic interactions is given by

$$\begin{aligned} -\mathcal{L} \sim & \frac{y_e}{\Lambda}\left(\bar{l}H\psi\right)_1 e_R + \frac{y_\mu}{\Lambda}\left(\bar{l}H\psi\right)_{1'} \mu_R + \frac{y_\tau}{\Lambda}\left(\bar{l}H\psi\right)_{1''} \tau_R \\ & + \frac{1}{\Lambda}\left(y_1\bar{l}\tilde{H}\phi + y_3\bar{l}\tilde{H}\eta\right)_1 \nu_{R1} \\ & + \frac{1}{\Lambda}\left(y_1\bar{l}\tilde{H}'\phi + y_3\bar{l}\tilde{H}'\eta\right)_1 \nu_{R2} \\ & + \frac{1}{\Lambda}\left(y_4\bar{l}\tilde{H}\psi + y_2\bar{l}\tilde{H}''\phi + y_3\bar{l}\tilde{H}''\eta\right)_1 \nu_{R3} \\ & + \frac{1}{2}\lambda_1\chi\overline{\nu}^c_{R1}\nu_{R1} + \frac{1}{2}\lambda_2\chi\overline{\nu}^c_{R2}\nu_{R2} + \frac{1}{2}\lambda_3\chi\overline{\nu}^c_{R3}\nu_{R3} \\ & + \frac{1}{2}\lambda_s\zeta\overline{S}^c\nu_{R1}. \end{aligned} \tag{4}$$

After electroweak symmetry breaking, the flavon fields obtain their vacuum expectation values (v.e.v.) along with alignments given as

$$\begin{aligned} \langle\psi\rangle = (v, 0, 0); \quad \langle\eta\rangle = (0, v, 0); \\ \langle\phi\rangle = (v, v, v); \quad \langle\chi\rangle = v; \quad \langle\zeta\rangle = u. \end{aligned} \tag{5}$$

In order to study sterile neutrinos in a keV scale, we consider an approximate scale of the flavon v.e.v and the cutoff scale Λ as follows:

$$v \sim 10^{14}\,\text{GeV}, \quad u \sim 10\,\text{TeV}, \quad \Lambda \sim 10^{15}\,\text{GeV}.$$

Using the multiplication rule of A_4 [4], the charged lepton mass matrix becomes diagonal:

$$M_L = \frac{\langle H\rangle v}{\Lambda}\text{diag}\left(y_e, y_\mu, y_\tau\right). \tag{6}$$

The Dirac, Majorana, and sterile neutrino mass matrices become

$$M_D = \begin{pmatrix} a & a & c+h \\ a & a & h \\ a+t & a+t & h+t \end{pmatrix}, \quad M_R = \begin{pmatrix} d & 0 & 0 \\ 0 & e & 0 \\ 0 & 0 & f \end{pmatrix}, \tag{7}$$

$$M_s = \begin{pmatrix} g & 0 & 0 \end{pmatrix}, \tag{8}$$

where

$$a = \frac{\langle H\rangle v}{\Lambda}y_1, \quad h = \frac{\langle H\rangle v}{\Lambda}y_2, \quad c = \frac{\langle H\rangle v}{\Lambda}y_4, \quad t = \frac{\langle H\rangle v}{\Lambda}y_3,$$
$$d = \lambda_1 v, \quad e = \lambda_2 v, \quad f = \lambda_3 v, \quad g = \lambda_s u.$$

In the MES mechanism, the (3×3) active neutrino mass matrix m_ν and the sterile neutrino mass m_s are given by

$$\begin{aligned} m_\nu \simeq\ & M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} M_S \left(M_R^{-1}\right)^T M_D^T \\ & - M_D M_R^{-1} M_D^T; \end{aligned} \tag{9}$$

$$m_s \simeq -M_S M_R^{-1} M_S^T. \tag{10}$$

Using equations (7)–(10), the active and sterile neutrino mass matrices become

$$m_\nu = -\begin{pmatrix} \frac{a^2}{e} + \frac{(c+h)^2}{f} & \frac{a^2}{e} + \frac{h(c+h)}{f} & \frac{a(a+t)}{e} + \frac{(c+h)(h+t)}{f} \\ \frac{a^2}{e} + \frac{h(c+h)}{f} & \frac{a^2}{e} + \frac{h^2}{f} & \frac{a(a+t)}{e} + \frac{h(h+t)}{f} \\ \frac{a(a+t)}{e} + \frac{(c+h)(h+t)}{f} & \frac{a(a+t)}{e} + \frac{h(h+t)}{f} & \frac{(a+t)^2}{e} + \frac{(h+t)^2}{f} \end{pmatrix}, \tag{11}$$

$$m_s = -\frac{g^2}{d}. \tag{12}$$

The full (4×4) active-sterile mass matrix is diagonalised by a unitary (4×4) mixing matrix given by [24]

$$V \simeq \begin{pmatrix} \left(1 - \frac{1}{2}RR^\dagger\right)U & R \\ -R^\dagger U & 1 - \frac{1}{2}R^\dagger R \end{pmatrix}, \tag{13}$$

where R represents the strength of active-sterile mixing given by

$$R = M_D M_R^{-1} M_S^T \left(M_S M_R^{-1} M_S^T\right)^{-1} = \begin{pmatrix} \frac{a}{g} \\ \frac{a}{g} \\ \frac{a+t}{g} \end{pmatrix}. \tag{14}$$

3. NUMERICAL ANALYSIS AND RESULTS

The active neutrino mass m_ν is diagonalized by a (3×3) unitary matrix U as

$$m_\nu = U m_\nu^{\text{diag}} U^T. \tag{15}$$

where $m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$.

For NH, $m_1 = 0$, $m_2 = \sqrt{\Delta m^2_{21}}$, and $m_3 = \sqrt{\Delta m^2_{21} + \Delta m^2_{31}}$ and for IH, $m_1 = \sqrt{\Delta m^2_{31}}$, $m_2 = \sqrt{\Delta m^2_{21} + \Delta m^2_{31}}$, and $m_3 = 0$, where $\Delta m^2_{ij} = |m_j^2 - m_i^2|$.

In PDG convention [25], U is parameterized using three mixing angles θ_{12}, θ_{13}, and θ_{23}, one Dirac phase δ_{CP}, and two Majorana phases α and β.

Parameters	NH (GeV)	IH (GeV)
$\lvert a \rvert$	0.46–0.60	1.09–1.44
$\lvert c \rvert$	2.53–4.72	5.43–6.61
$\lvert t \rvert$	0.80–1.59	0.10–3.14
$\lvert h \rvert$	3.20–3.94	1.11–2.71

TABLE 2: Allowed ranges of model parameters in both NH and IH.

For numerical analysis, we consider nondegenerate values of heavy Majorana masses $d = 10^{14}$ GeV, $e = 10^{11}$ GeV, and $f = 5 \times 10^{11}$ GeV, and the two Majorana phases α and β are varied in the range $(0, 2\pi)$. We solve the model parameters a, c, t, and h from equation (15) using the 3σ values of the mixing angles, mass-squared differences, and Dirac phase δ_{CP} from the latest Nufit data [26] for NH as well as IH. The detailed parameter space for both mass hierarchies is shown in Table 2. The parameters are further constrained by the Plank upper bound on the sum of active neutrino masses $\sum m_i < 0.12$ eV [27] and 3σ bounds of the neutrino mixing angles θ_{12}, θ_{13}, and θ_{23}. The remaining parameter g is solved by constraining the sterile neutrino mass in the range $m_s = (1\text{–}18.5)$ keV.

3.1. Neutrino Phenomenology

It is observed in our analysis that the active-sterile mixing is very small, as shown in Figure 3. As a result, we can ignore the effects of sterile neutrino on the active neutrino mixing angles. The three neutrino mixing angles can be determined from the elements of the unitary diagonalizing matrix U as

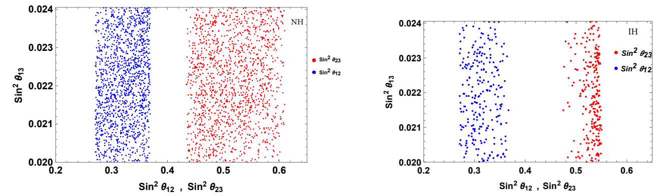
$$\sin^2\theta_{13} = |U_{13}|^2, \quad \sin^2\theta_{12} = \frac{|U_{12}|^2}{1-|U_{13}|^2}, \quad \sin^2\theta_{23} = \frac{|U_{23}|^2}{1-|U_{13}|^2}. \tag{16}$$

The variation plots of the mixing angles $\sin^2\theta_{23}$, $\sin^2\theta_{12}$ with $\sin^2\theta_{13}$ for both NH and IH are shown in Figure 1. As we can see from the plot, more data points of the mixing angles are obtained in case of NH compared to IH within the experimental bounds. For octant degeneracy of θ_{23}, we can see that data points are concentrated more in the region of $\sin^2\theta_{23} > 0.5$. This implies that our model predicts the higher octant of θ_{23}. Figure 2 shows the variation of active neutrino masses m_i with sum $\sum m_i$. It is found that the splitting between m_2 and m_3 is large in NH compared to the splitting between m_1 and m_2 in IH. The upper bound on $\sum m_i$ is also larger in case of IH than NH.

We also study the perturbativity as well as nonunitarity effects of active-sterile mixing. It is observed that the Yukawa couplings are in the acceptable range of (10^{-4}–10^{-1}). As a result, the perturbativity condition for the Yukawa couplings is not affected by the active-sterile mixing. The non-unitarity effect is measured by the parameter $RR^\dagger$ and it is also found to be very small, i.e.,

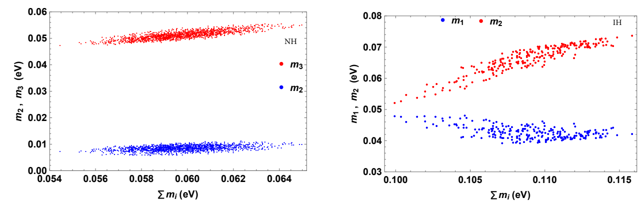
$$|RR^\dagger| \le 10^{-9} \quad \text{for NH}, \qquad |RR^\dagger| \le 10^{-8} \quad \text{for IH}. \tag{17}$$

Moreover, the study of effective neutrino mass m_{eff} from neutrinoless double beta decay($0\nu\beta\beta$) is also one of the most important parameters in neutrino physics. Combined bounds from



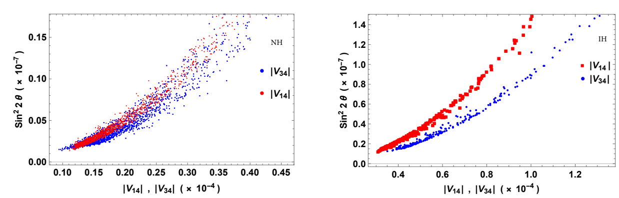
(a) (b)

FIGURE 1: Variation among mixing angles. Blue points represent the plot of $\sin^2\theta_{13}$ with $\sin^2\theta_{12}$ while the red points represent the variation of $\sin^2\theta_{13}$ with $\sin^2\theta_{23}$.



(a) (b)

FIGURE 2: Variations of active neutrino masses m_i with the sum of active neutrino masses $\sum m_i$ for NH as well as IH. Here, $m_1 = 0$ for NH while $m_3 = 0$ for IH.



(a) (b)

FIGURE 3: Variations of effective mixing angle $\sin^2 2\theta$ with elements of active-sterile mixing strength R.

KamLAND-Zen and GERDA experiments provide an upper limit on m_{eff} in the range 0.071–0.161 eV [28]. We calculate the effects of keV sterile neutrino in $0\nu\beta\beta$ as

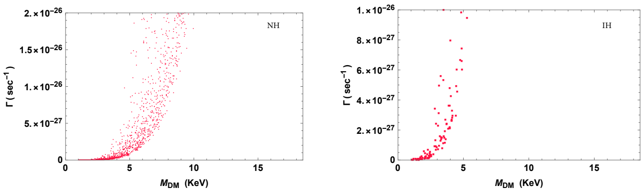
$$m_{\text{eff}} = \left| \sum_{j=1}^{4} \left| V_{ej} \right|^2 m_j \right|. \tag{18}$$

The results of our analysis for both NH and IH are

$$m_{\text{eff}} = (0.001174\text{–}0.004367)\,\text{eV} \quad \text{for NH}, \qquad m_{\text{eff}} = (0.047662\text{–}0.050885)\,\text{eV} \quad \text{for IH}. \tag{19}$$

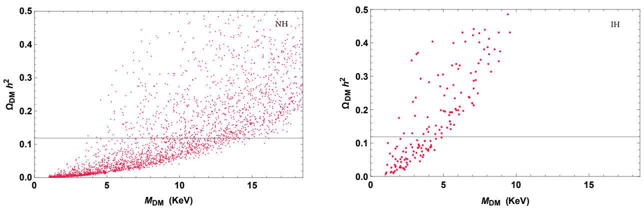
3.2. Dark matter

In order to study the keV sterile neutrino as dark matter, we need to calculate the decay width (Γ) of the sterile neutrino decay to SM neutrinos and its relic abundance ($\Omega_{\text{DM}} h^2$) in the early Universe using equations (3) and (2), respectively. Figures 4 and 5, respectively, show the variation of Γ and $\Omega_{\text{DM}} h^2$ with dark matter mass m_{DM}. In our analysis, we have considered the upper limit of decay width to be $\mathcal{O}(10^{-27})\,\text{sec}^{-1}$ so that the sterile neutrino behaves as a dark matter. From Figure 4, it is found that the allowed mass range for sterile neutrino is very narrow $m_s \sim (1\text{–}5)$ keV for IH whereas we observe a wider range of allowed sterile neutrino mass $m_s \sim (1\text{–}8)$ keV



(a) (b)

FIGURE 4: Variations of decay rate with dark matter mass in NH and IH. Here, $M_{\rm DM} = m_s$.



(a) (b)

FIGURE 5: Variations of relic abundance parameter $\Omega_{\rm DM}h^2$ with dark matter mass in NH and IH. Here, $M_{\rm DM} = m_s$.

in case of NH. Similarly, based on the plots of relic abundance in Figure 5, we find that in the case of NH, a wider mass range is observed (4–15) keV to be consistent with the current best-fit value given in equation (1), whereas a stricter range for m_s in the range (1–5) keV is observed in case of IH. This result for IH is also consistent with the bound from analysis of decay width given above.

4. CONCLUSION

We built an A_4 symmetry model supplemented by Z_4 and Z_2 to study neutrino masses and mixings in both NH and IH cases, as well as keV-scale sterile neutrino DM in MES framework considering five scalar flavons. The predicted neutrino masses and mixings are compatible with recent neutrino oscillation data and the cosmological upper bound $\sum m_i < 0.12$ eV. The decay width of the keV sterile neutrino as well as its relic abundance are calculated and it is found to satisfy the requirements of a sterile neutrino to be DM candidate for a narrow range of mass.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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