Dark Matter Freeze-out and Freeze-in beyond Kinetic Equilibrium

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Abstract

In the standard approach to the determination of the dark matter (DM) thermal relic abundance, both from thermal freeze-out and freeze-in mechanisms, one takes into account only the zeroth moment of the Boltzmann equation, i.e., the equation for the evolution of the particle number density. In case of freeze-out, this comes from the assumption of local thermal equilibrium, while for freeze-in, this comes from neglecting DM annihilation processes. This proceedings report discusses how to go beyond this assumption, reports on the introduction of DRAKE—a numerical precision tool that can trace not only the dark matter relic density but also its velocity dispersion and full phase space distribution function—and finally presents a new mechanism of *decreasing* the dark matter relic density through injecting even more DM particles into the thermal bath.

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1. INTRODUCTION

The origin of dark matter (DM) still remains a mystery. A number of mechanisms for DM production in the Early Universe have been proposed, but which of them, if any, was indeed realized in Nature is yet to be discovered. Nevertheless, *any* mechanism that results in establishing the observed dark matter relic abundance had to involve a phase in which the DM particles had a nonequilibrium distribution.

An attractive assumption is that the interactions between the DM particles and Standard Model (SM) plasma are sufficiently frequent to enforce chemical equilibrium at early stages of its history. When the rate of annihilation drops below the rate of the Universe expansion, the dark sector departs from the chemical equilibrium with the SM plasma and the number of DM particles in the comoving volume ceases to change with time (*freezes-out*), hence establishing the relic population. Another well-studied scenario lives on the opposite end of the interaction strength spectrum: when the coupling of the DM to the SM particles is so weak that its initial population after reheating was minuscule and DM was gradually produced from SM bath but never reached equilibrium. The production rate drops with time due to decreasing the number of densities of SM states, and when it effectively stops, then the final relic DM population forms (*freezes-in*).

A general approach to determine the relic abundance in both cases relies on solving the Boltzmann equation (BE) that describes the evolution of the DM distribution function including all the relevant collision operators. To calculate the abundance, often only the 0-th moment of this equation, tracing the particle number density, is considered [6], which is what is used in widely used numerical packages. However, this formalism captures only the departure from *chemical* equilibrium but is not able to incorporate the distortion of the distribution whenever the *kinetic* equilibrium is not enforced.

In recent years, there has been a dedicated effort to improve the aforementioned formalism to be able to include effects of going beyond the kinetic equilibrium assumption. In this proceedings, I report some of the new findings on this topic, with an emphasis on new intriguing observation that adding more DM particles, e.g., from decays or annihilations of heavier states, can counter-intuitively lead to a decrease in its final relic abundance [1]. This observation stems from being able, for the first time in the literature, to solve the evolution of such a system at the level of DM phase space distribution.

2. KINETIC DECOUPLING IN FREEZE-OUT

In thermal freeze-out scenarios, kinetic equilibrium between DM and the SM plasma is maintained at least as long as the two components remain in chemical equilibrium. In fact, typically kinetic equilibrium persists for a long time after departure from the chemical one. This is a consequence of the fact that elastic scatterings are typically much more frequent than the number of changing processes, due to the relatively large abundance of the light SM states. However, this is not universally true and exceptions to this standard scenario exist in even simple models (see, e.g., [2] and references therein) and are expected to occur much more often in more involved scenarios containing processes actively disrupting local thermal equilibrium, e.g., decays of heavier states or self-heating [3] due to semiannihilations, cannibalization, or conversions (see, e.g., [4]).

Recently, a new public tool called DRAKE was released [2], which allows us to trace the evolution of the DM distribution function in generic single-component dark matter scenarios. In [2, 5], the classes of cases that lead to a significant change with respect to the more standard approach have been highlighted.

In essence, DRAKE solves the Boltzmann equation (fBE) for the DM phase space distribution function f_χ of the DM particle χ in an expanding Friedmann-Robertson-Walker universe [7]:

$$E\left(\partial_t - H\mathbf{p}\cdot\nabla_{\mathbf{p}}\right) f_\chi = C\left[f_\chi\right], \tag{1}$$

where H is the Hubble parameter and the collision term $C[f_\chi]$ contains all interactions between DM and SM particles f. Using coordinates $x(t,p) \equiv m_\chi/T$ and $q(t,p) \equiv p/T$, where p is the length of three-momentum, m_χ the DM mass, and T the SM plasma temperature, allows us to rewrite the fBE as follows [5]:

$$\begin{aligned}\partial_x f_\chi(x,q) = {} & \frac{m_\chi^3}{\tilde{H}x^4}\frac{g_{\bar{\chi}}}{2\pi^2}\int d\tilde{q}\tilde{q}^2\frac{1}{2}\int d\cos\theta v_{\text{Møl}}\sigma_{\bar{\chi}\chi\to\bar{f}f} \\ & \times\left[f_{\chi,\text{eq}}(q)f_{\chi,\text{eq}}(\tilde{q}) - f_\chi(q)f_\chi(\tilde{q})\right] \\ & + \frac{\gamma(x)}{2\tilde{H}x}\left[x_q\partial_q^2 + \left(q + \frac{2x_q}{q} + \frac{q}{x_q}\right)\partial_q + 3\right]f_\chi \\ & + \tilde{g}\frac{q}{x}\partial_q f_\chi,\end{aligned} \tag{2}$$

where $\sigma_{\bar{\chi}\chi\to\bar{f}f}$ is the annihilation cross section, $\gamma(x)$ is the momentum transfer rate, $x_q \equiv \sqrt{x^2+q^2}$, and θ is the angle between $\mathbf{q}$ and $\tilde{\mathbf{q}}$. This equation is discretized with the use of the method-of-lines and solved as a system of coupled ordinary differential equations.

3. DM TEMPERATURE IN FREEZE-IN

In thermal freeze-in scenarios, there is no equilibration, kinetic or chemical, within the dark sector nor with the SM plasma.[1] The DM distribution function can be highly nonthermal, depending on the exact production history. However, in most cases, this does not translate to significantly altered relic abundance, since typically the back reaction of DM annihilations is not relevant for its formation (for recent exceptions, see, e.g., [8]). Nevertheless, this is not universally true, and moreover, the shape of the distribution can be relevant for the structure formation (see, e.g., [9]).

A notable freeze-in production mechanism where the departure from kinetic equilibrium has a huge impact on the relic abundance is the so-called semiproduction scenario [10] (a.k.a. "pandemic DM" [11]). In this case, the main process that drives DM production is the inverse of semi-annihilation $\phi\chi^* \to \chi\chi$, where ϕ can be SM or dark sector states and may or may not be in equilibrium. The collision term for such a process has the form

$$\begin{aligned} C\left[f_\chi\left(p_i\right)\right] &= \frac{1}{g_\chi}\int d\Pi_j d\Pi_k d\Pi_l (2\pi)^4 \delta^{(4)}\left(p_i+p_j-p_k-p_l\right) \\ &\times |\mathcal{M}|^2_{\phi\chi^*\leftrightarrow\chi\chi}\Big\{f_\phi\left(p_k\right) f_{\chi^*}\left(p_l\right)\left[1+f_\chi\left(p_i\right)\right]\left[1+f_\chi\left(p_j\right)\right] \\ &\qquad - f_\chi\left(p_i\right) f_\chi\left(p_j\right)\left[1+f_\phi\left(p_k\right)\right]\left[1+f_{\chi^*}\left(p_l\right)\right]\Big\}. \end{aligned} \tag{3}$$

Here, the collision term cannot be rephrased with the use of the cross section. It is also quite more involved to solve numerically, especially for nonleading terms in the powers of f_χ.

In contrast to usual production from decay or pair-annihilation, here, the production rate is dependent on the DM population and in particular its momentum distribution. It has been found that taking into account the fact that the temperature of the dark sector can be different than the one of the SM plasma can lead to more than an order of magnitude difference in the relic abundance [10].

4. DISTORTION OF DM DISTRIBUTION THROUGH ADDITIONAL PROCESSES

The above brief summary of the nonequilibrium effects discussed in the literature stays within the context of a relatively simple dark sector, where there are no additional dynamics that drive the DM population to depart from the thermal distribution. Indeed, the impact of processes like decays or annihilations of heavier states into DM particles has not been yet considered beyond their number-changing effect. In the work [1], which this proceedings report stems from, and in results shown below, we have made the first step toward general implementation of such scenarios. We also studied in detail the impact of dark matter self-scatterings on its relic abundance. The latter is expected to be particularly important for scenarios in which on top of the thermal component, a portion of DM is produced nonthermally.

Particle physics models quite often contain multiple particles that can decay leading to the production of one or several DM states (see, e.g., [12]). If the lifetime of these particles is sufficiently long, the contribution from such decays does not simply annihilate away but can noticeably alter the evolution of the DM distribution. If the velocity-averaged cross section of DM annihilation is essentially momentum-independent, this injection will have an impact on the rate of annihilation solely by the increase of the DM density—the resulting relic density will be determined by the interplay of the prolongation of annihilation process that depletes the DM population and the continuous supply of new particles that increases its resulting relic abundance. However, if the annihilation is strongly velocity-dependent, the effect of the distribution on the annihilation rate is more complicated. If the injected component is rather energetic with respect to the thermal one, self-scattering processes will lead to the redistribution of DM particles into the region of the phase space with a larger momentum and hence they can noticeably affect the velocity-averaged cross section.

To study the self-thermalization of a nonthermal component in [1], we took a sterile-neutrino-like model of DM [13] that is coupled to a scalar singlet field S (the model has been extensively studied in the literature) with an additional $U'(1)$ gauge interaction. In [14], a similar model is considered in the context of the impact of nonthermal processes on the DM distribution function; however, not including the additional $U'(1)$ making the self-scattering processes absent. The Lagrangian of the model reads

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{\rm SM} + \frac{1}{2}\left(\partial_\mu S\right)^2 - V(S,H) + yS\bar{\chi}\chi + m_\chi\bar{\chi}\chi \\ &\quad + \bar{\chi}i\mathcal{D}_\mu\gamma^\mu\chi - \frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} - \frac{\epsilon}{2}F'_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_A^2 A'_\mu A'^\mu, \end{aligned} \tag{4}$$

where ϵ is the mixing parameter between the photon and the dark photon A'_μ, $\mathcal{D}_\mu = \partial_\mu - ie'A'_\mu$ and $V(S,H)$ is the Z_2 symmetric scalar potential.

The collision term for self-scattering has the following general expression (neglecting the $[1\pm f_\chi]$ factors, as appropriate for temperatures well below the DM mass):

$$\begin{aligned} C_{\rm self} &= \frac{1}{2g_\chi}\int d\Pi_{\tilde{p}} d\Pi_k d\Pi_{\tilde{k}} (2\pi)^4\delta^{(4)}(\tilde{p}+p-\tilde{k}-k) \\ &\times\Big\{\frac{1}{2}|\mathcal{M}|^2_{\chi\chi\leftrightarrow\chi\chi}\left[f_\chi(k)f_\chi(\tilde{k}) - f_\chi(p)f_\chi(\tilde{p})\right] \\ &\quad + |\mathcal{M}|^2_{\chi\bar{\chi}\leftrightarrow\chi\bar{\chi}}\left[f_\chi(k)f_{\bar{\chi}}(\tilde{k}) - f_\chi(p)f_{\bar{\chi}}(\tilde{p})\right]\Big\}. \end{aligned} \tag{5}$$

This model exhibits phenomenology interesting from the point of view of the importance of departure from kinetic equilibrium. In [1], we presented an example of a point in the parameter space of that model that predicts a particularly striking effect of the DM self-interaction processes. Here, instead, let me show a result for a different case, focusing on a region when the thermal and nonthermal components are of similar size leading to a strongly nonthermal shape of the final population.

[1] Here, we use the term *dark sector* to denote the part of the total matter and radiation in the Universe that contains the DM and fields coupled to it, but very weakly coupled to the SM.

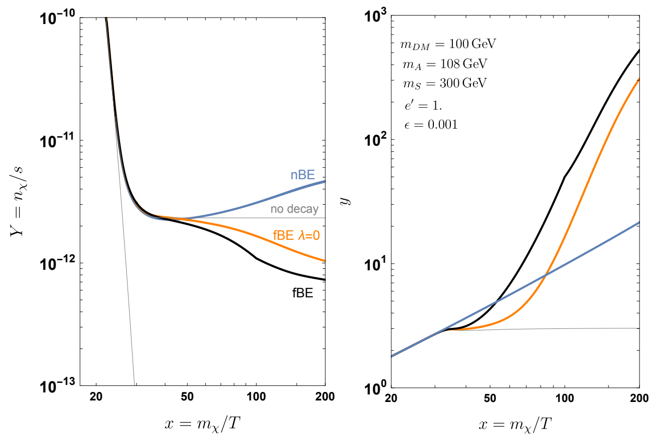

FIGURE 1: An example evolution of the particle yield Y (left panel) and the temperature parameter y (right panel) for the benchmark point specified in the main text. On both panels, the blue line gives the result with the standard treatment (nBE), orange fBE without self-scatterings, and black of the full calculation. For comparison, gray lines show the equilibrium yield and the one obtained in a model without the component injected through the decay.

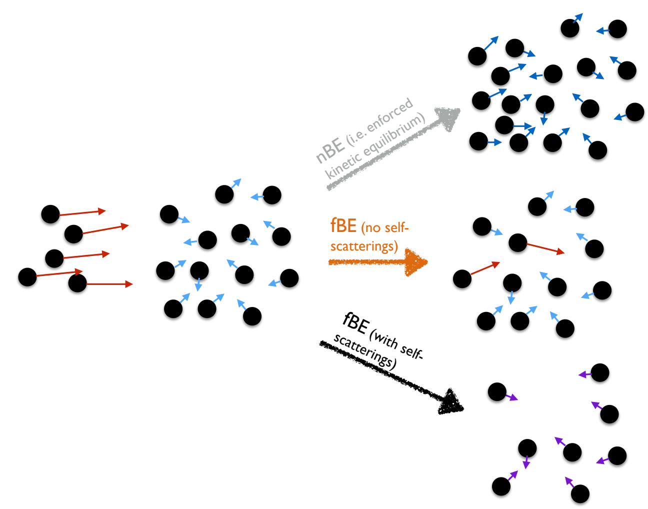

FIGURE 2: An illustration of possible outcomes of the scenario presented in the main text depending on the level of simplification of calculations. When the energetic DM component (on the left) is injected into the cold DM population, the assumption of kinetic equilibrium (top right) would enforce very efficient, unphysical, energy redistribution that would not lead to increased annihilations. Performing calculations at the phase space level, but neglecting self-interactions, allows us to capture the increased annihilation rate. However, full fBE with self-interactions may enhance this effect even more through the reshuffling of the momenta.

An example of the evolution of particle yield Y and the temperature parameter $y(x) \equiv m_\chi T_\chi s^{-2/3}$, with s being the entropy density, is shown in Figure 1 for a benchmark set of parameters $m_\chi = 100\,\mathrm{GeV}$, $m_A = 108\,\mathrm{GeV}$, $e' = 1$, $\epsilon = 10^{-3}$, and $y = 9 \cdot 10^{-10}$ and the coupling between S and the Higgs, $\lambda_{SH} = 0.08$. The presented example shares the same feature as the one given in [1], namely, that the injection of more DM particles leads to lower relic abundance and that this effect is made stronger through efficient self-interactions. Also, in both cases, the elastic scatterings between the DM and the SM particles are suppressed leading to an early kinetic decoupling, though with some qualitative differences.

While the behavior of the nBE curve is expected as the production of additional DM particles should increase its density, the fact that solving the actual DM momentum distribution function leads to a decrease in the DM density may seem indeed surprising. However, it can be understood from the velocity-dependent annihilation pattern of the model and the momentum distributions that correspond to the different approaches. Namely, since the annihilations of DM particles with higher momenta are preferred in this parameter point, due to the existence of an annihilation threshold at $\sqrt{s} = 2m_A$, the injected component annihilates much more effectively with itself and also with the particles of the thermal cold component. On top of that, the self-interactions, if their interaction rate is large enough, redistribute the additional kinetic energy between all the DM particles and as a consequence up-scatter even more particles to energies allowing us to surpass the annihilation threshold. A graphical illustration of this is given in Figure 2.

The evolution of $f_\chi(p)$ is shown in Figure 3. In the nBE approach (blue solid line), the shape of the distribution is taken to be unchanged even if the DM particles from S decay are in fact more energetic; hence, the rate of annihilation is only slightly affected by the presence of additional DM particles. In the fBE case with the self-scatterings switched off (orange curve), the decays create a clear bump in the distribution function, supplying particles with momenta sufficient to overcome the annihilation threshold, and therefore, the rate of annihilations is significantly boosted. However, there is not enough redistribution of energy between DM particles, and due to the occurrence of an early kinetic decoupling, the thermal component is even colder than the SM plasma.

In the presence of self-scattering, the fBE result (black curve) displays an even stronger annihilation. This is because the injected DM component heats up the DM gas via elastic collisions and thus more DM particles have energies to overcome the annihilation threshold. It is clearly visible that the up-scattering of low momenta DM particles leads to a much broader distribution.

Finally, in Figure 4, the importance of the effect of increased annihilation due to the injected component is shown. It is given in the plane of the size of this nonthermal component coming from the decay (governed by the coupling of the S particle to the SM, λ_{SH}) and the lifetime of S (governed by the coupling y). The contours highlight the ratio of the relic density obtained with fBE and nBE methods, i.e., $(\Omega h^2)_{\mathrm{fBE}}/(\Omega h^2)_{\mathrm{nBE}}$. The plot shows that for most of the chosen regions of the parameter space the resulting relic density is significantly suppressed. The plot range has been chosen such to emphasize the limiting cases when the nBE and fBE calculations lead to the same outcome. This is when either the decay happen very early (for low values of y), when the annihilation rate is large enough to still keep the DM population in equilibrium, or when decay happens very late (for the high end of y values) when the plasma is diluted

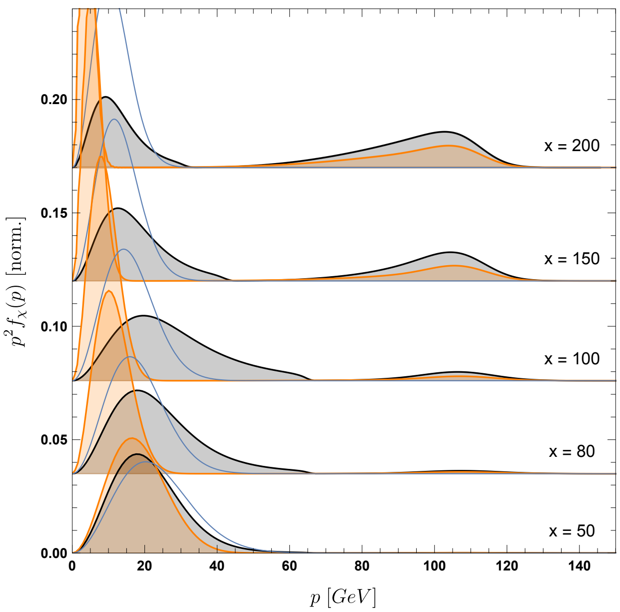

FIGURE 3: Time snapshots around the freeze-out of the evolution of the normalized momentum distribution for the benchmark model. Black (orange) lines show $f_\chi(p)$ with (without) self-scatterings, while for comparison blue line shows equilibrium distribution at the SM plasma temperature.

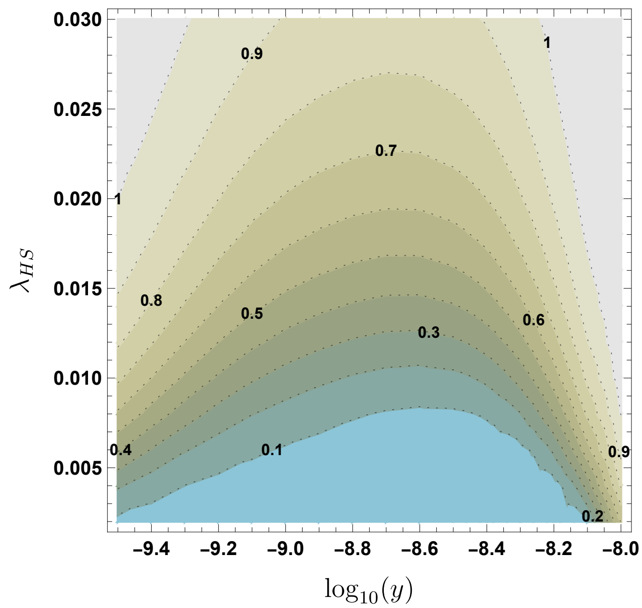

FIGURE 4: The impact of the calculation at the phase space density level in the plane of the size of the decaying component (governed by the coupling of the S particle to the SM, λ_{SH}) and the lifetime of S (governed by the coupling y). The contours show the ratio of the relic density obtained with fBE and nBE methods, i.e., $(\Omega h^2)_{\text{fBE}}/(\Omega h^2)_{\text{nBE}}$.

enough that annihilations happens very rarely. Moreover, the upper part of the plot features more tight coupling of the S to the SM plasma, leading to its lowered thermal population and thus a smaller fraction of the total DM is comprised of the end products of its decay. Naturally, this leads to a ratio of fBE to nBE result being close to one. Conversely, a larger fraction of the component coming from decay leads to significant suppression of the relic abundance when calculated at the fBE level.

5. CONCLUSIONS

In this proceedings, we have presented a brief discussion of the results in the literature that highlight the necessity of looking beyond the kinetic equilibrium in the dark matter production processes. The picture that emerges is that although in many cases the standard nBE treatment is a justified and accurate approximation, the number of exceptions is long enough that it is prudent to carefully check if the standard treatment is sufficient for the case at hand.

Then, we focused on new results including additional DM component coming from a decay of a heavier state, which affects the already established thermal population (coming from freeze-out, freeze-in, or other mechanisms). We have shown that when performing the analysis on the level of the full DM distribution function, one can arrive at conclusions that are very different and in fact unexpected compared to the results from a standard treatment. In particular, we have found that injecting additional energetic components can lead to, in the case where the DM annihilates preferentially with higher momenta, a *decrease* in the relic abundance. The effect of this kind is completely invisible and very easy to miss if one does not study the DM distribution evolution.

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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