Neutrino Oscillation Caused by Spacetime Geometry

Indrajit Ghose, Riya Barick, and Amitabha Lahiri

S. N. Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake, Kolkata 700106, West Bengal, India

Abstract

The effects of gravitational interaction are generally neglected in particle physics. A first-order formulation of gravity is presented to include gravity in the Quantum Mechanical Lagrangian of fermions. It is seen that fermions minimally coupled to gravity give rise to a torsionless effective theory with a quartic interaction. After passing through a thermal background, the most generic form of contortion contributes to neutrino effective mass. This effective mass can change the current oscillation parameters.

Keywords: neutrino oscillation, nonstandard interaction, Einstein-Cartan-Dirac-Sciama-Kibble theory, torsion, chiral torsion, first order formulation of gravity
DOI: 10.31526/LHEP.2023.349

1. INTRODUCTION

In the Standard Model, neutrinos are the neutral partners of the charged leptons of each family. As neutrinos do not have a vertex in the unbroken $U(1)$ gauge group part, they only interact by massive vector boson exchange. As a result, it is very difficult to detect neutrinos even though the flux of solar neutrinos at the Earth's surface is on the order of $10^{11}\ \mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ [1]. After its detection in 1956 [2], Ray Davis Jr. in 1968 [3, 4] tried detecting solar neutrinos following the radiochemical method proposed by Pontecorvo [5]. This led to the solar neutrino anomaly [6]. It is now widely accepted that Pontecorvo's idea of flavor mixing is the explanation for the discrepancy between observed and predicted solar neutrino flux.

The idea of neutrino mixing by Pontecorvo is quite simple—the time evolution operator is not diagonal in the production-detection basis of neutrinos. In the sun, the neutrinos are produced as flavor eigenstates, and the detector can also only detect the flavor eigenstates. However, as the neutrino propagates through space, a linear combination of the flavor eigenstates named mass eigenstates evolves as stationary states. The relation between the two different sets of basis vectors is given by a mixing matrix. They are related as [7]

$$|\nu_\alpha\rangle = \sum_i U^*_{\alpha i} |\nu_i\rangle . \tag{1}$$

$|\nu_i\rangle$'s give the mass basis and $|\nu_\alpha\rangle$'s are the flavor basis.

In the present article, we will study the effect of gravity on neutrinos.[1] The article will be arranged as follows. In Section 2, we discuss the dynamics of fermions in curved spacetime, leading to an effective four-fermion interaction. Using this, we find an additional contribution in the background averaged Lagrangian for neutrinos propagating through matter, in Section 3. Finally, we use this to calculate the refraction probability of neutrinos in flavor space in Section 4.

[1]Based on talk delivered by Indrajit Ghose at the International Conference on Neutrinos and Dark Matter (NuDM-2022), Sharm El-Sheikh, Egypt, 25–28 September, 2022.

2. FERMIONS UNDER GRAVITY

The effects of gravitational interaction are usually neglected in particle physics. However, in scenarios of astroparticle physics, the effects may not be negligible. Let us consider how to write diffeomorphism invariant actions for fermions on curved spacetime. Usually, Quantum Field Theory is defined on a flat Minkowskian manifold. Constructing a theory invariant under general coordinate transformation thus requires defining the theory on the Minkowskian tangent manifold at each point and soldering that to the curved spacetime. In 3+1 dimensional spacetime, this can be done via tetrads or vierbeins, defined by

$$g = e^T \eta e, \tag{2}$$

or in index notation

$$g_{\mu\nu} = e^i_\mu \eta_{ij} e^j_\nu, \tag{3}$$

where g is the spacetime metric and η is the Minkowskian metric. In order to show explicitly that the two kinds of indices live in two different spaces, we denote the spacetime indices by Greek letters and tangent space indices by Latin letters. The inverse tetrad e^μ_i, also called the cotetrad, is defined by

$$e^\mu_i g_{\mu\nu} e^\nu_j = \eta_{ij}. \tag{4}$$

The covariant derivative is taken to be tetrad-compatible (thus metric compatible), which results in the relation sometimes referred to as the tetrad postulate:

$$e^\lambda_i \partial_\mu e^i_\nu + A_\mu{}^i{}_j e^j_\nu e^\lambda_i - \Gamma^\lambda{}_{\mu\nu} = 0. \tag{5}$$

$A_\mu{}^i{}_j$ are components of the spin connection and $\Gamma^\lambda{}_{\mu\nu}$ are the spacetime connection components. In general, $\Gamma^\lambda{}_{\mu\nu}$ are not assumed to be symmetric in the lower indices—the antisymmetric part corresponds to torsion. This enables us to write the Ricci scalar in terms of tetrads and spin connection:

$$R = e^\mu_i e^\nu_j \left(\partial_{[\mu} A_{\nu]}{}^{ij} + A_{[\mu|}{}^i{}_k A_{|\nu]}{}^{kj} \right). \tag{6}$$

The spin connection can be thought of as the gauge potential arising from the local Lorentz invariance of Quantum Field Theory in Curved Spacetime.

The action for free fermions on curved spacetime is

$$S = \int d^4x |e| \left(\frac{1}{2\kappa} R + \sum_f \bar{f} \left(i\not{D} - m_f \right) f \right), \tag{7}$$

with the spinor covariant derivative being defined as [8, 9, 10, 11, 12, 13, 14]

$$D_\mu \psi = \partial_\mu \psi - \frac{i}{4} A_\mu{}^{ab} \sigma_{ab} \psi, \tag{8}$$

where $\sigma_{ab} = i/2[\gamma_a, \gamma_b]$. Then, the action in equation (7) is a diffeomorphism invariant. This is the theory of fermions minimally coupled to gravity through the spin connection. We can vary the action with respect to the spin connection and get the equation of motion, which has the solution

$$A_\mu{}^{ab} = \omega_\mu{}^{ab} + \frac{\kappa}{8} \sum_f \bar{\psi}_f \left\{ \gamma_c, \sigma^{ab} \right\} e^c_\mu \psi_f. \tag{9}$$

Here, $\omega_\mu{}^{ab}$ corresponds to the usual symmetric Levi-Civita connection, while the second term is what is known as contortion $\Lambda_\mu{}^{ab}$, generated fully by the fermion fields. This corresponds to a torsion $T_{\mu\nu\rho} = \Lambda_{\mu ab} e^a_\nu e^b_\rho$, which is completely antisymmetric in all of its indices. For the solution of equation (9), we can write $T_{\mu\nu\rho} = \epsilon_{\mu\nu\rho\lambda} \sum_f \bar{\psi}_f \gamma_a \gamma^5 e^a_\lambda \psi_f$. Thus, the fermions are coupled to the torsion as part of their coupling to the spin connection. However, the symmetries of the Lagrangian also allow a more general form of contortion.

It was recently proposed that the most general form of the contortion is [15]

$$\Lambda_\mu{}^{ab} = \frac{\kappa}{4} \epsilon^{abcd} e_{c\mu} \sum_f \left(\lambda^f_L \bar{f}_L \gamma_d f_L + \lambda^f_R \bar{f}_R \gamma_d f_R \right). \tag{10}$$

Here, λ's are coupling parameters, and the sum runs over all species of fermions. Plugging in this expression of the contortion and rescaling λ's to absorb a factor of κ, we get the fermion part of the Lagrangian as

$$\mathcal{L} = \frac{1}{2\kappa} R + \sum_f \bar{f} \left(i\hat{\not{D}} - m_f \right) f - \sum_f \left(\bar{f} \gamma^\mu \left(\lambda_f + \lambda'_f \gamma^5 \right) f \right)^2, \tag{11}$$

in which $\hat{D}_\mu$ denotes the contortion-free spinor covariant derivative. The coupling constants λ now have mass dimension -1. The four-fermion interaction appearing here causes a modification of neutrino oscillations.

The central question in neutrino oscillation concerns the probability that neutrinos produced in one flavor will be detected later in the same flavor. In flat space, the effective Lagrangian for Dirac neutrinos interacting via weak interactions can be written as [16]

$$\begin{aligned} \mathcal{L}_\nu = &\sum_i \bar{\nu}_i \left(i\not{\partial} - m_i \right) \nu_i \\ &- \sum_\alpha \sum_f \frac{G_F}{\sqrt{2}} \bar{f} \gamma^\mu \left(g_1 - g_2 \gamma^5 \right) f \bar{\nu}_\alpha \gamma_\mu \left(1 - \gamma^5 \right) \nu_\alpha \\ &- \frac{G_F}{\sqrt{2}} \bar{e} \gamma^\mu \left(1 - \gamma^5 \right) e \bar{\nu}_e \gamma_\mu \left(1 - \gamma^5 \right) \nu_e. \end{aligned} \tag{12}$$

Here, i denotes the mass basis of neutrinos, α is the flavor index, and f denotes every other species of fermion. Also, according to standard model $g_1 = T^f_3 - 2Q^f \sin^2 \theta_W$ and $g_2 = T^f_3$ [17]. Q^f is the charge of fermion f, T^f_3 is the third component of isospin, and θ_W is the weak mixing angle. If we now include the four-fermion interaction arising from the neutrinos being on curved spacetime, we get

$$\begin{aligned} \mathcal{L}_\nu = &\sum_i \bar{\nu}_i \left(i\hat{\not{D}} - m_i \right) \nu_i \\ &- \sum_i \left(\lambda_i \bar{\nu}_i \gamma^\mu \mathbb{L} \nu_i \right) \sum_f \left(\bar{f} \gamma_\mu \left(\lambda_f + \lambda'_f \gamma^5 \right) f \right) \\ &- \sum_\alpha \sum_f \frac{G_F}{\sqrt{2}} \bar{f} \gamma^\mu \left(g_1 - g_2 \gamma^5 \right) f \bar{\nu}_\alpha \gamma_\mu \left(1 - \gamma^5 \right) \nu_\alpha \\ &- \frac{G_F}{\sqrt{2}} \bar{e} \gamma^\mu \left(1 - \gamma^5 \right) e \bar{\nu}_e \gamma_\mu \left(1 - \gamma^5 \right) \nu_e. \end{aligned} \tag{13}$$

For a normal matter distribution like the earth and sun, the curvature is not very large. Hence, the Levi-Civita covariant derivative $\hat{D}_\mu$ can be approximated by ordinary partial derivatives. Then, $e^{ip\cdot x}$ are again the free particle states, and the standard techniques of Quantum Field Theory are applicable. Here, we have suppressed the neutrino-neutrino self-interaction as it will not be relevant to our calculations. Equation (13) gives the Lagrangian governing neutrinos, with $\hat{D}_\mu$ being replaced by ∂_μ in regions of small curvature.

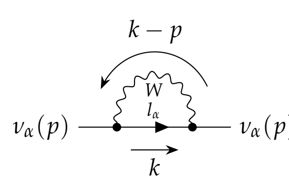
3. NEUTRINOS THROUGH A MEDIUM

Neutrinos propagating through a medium undergo numerous collisions with the thermalized background. S matrix elements for neutrinos must be calculated using thermal field theory. There are different formulations of Finite Temperature Field Theory [18, 19, 20, 21, 22]. We will use the real-time formalism developed by Schwinger as this is convenient for our calculations. In this formalism, the fermion propagator will be changed [23]:

$$D(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon} - 2\pi n_f \left(p^0 \right) \delta \left(p^2 - m^2 \right) (\not{p} + m). \tag{14}$$

Here, n_f is the Fermi-Dirac distribution for the fermion of mass m. Due to the presence of the momentum-conserving delta function, the propagator is still a Green function of the Dirac equation.[2] In the following discussion, we will assume that the temperature is not so high that the gauge boson propagators also receive thermal corrections, which is true in the case of atmospheric and solar neutrinos. Let us now calculate the background averaged Lagrangian after propagation through a neutral medium.

At first, let us consider the charged current mediated processes. The corresponding Feynman diagram is (l_α is the charged lepton corresponding to ν_α)



This diagram will change the neutrino propagator as

$$\frac{i}{\not{p}} (-i\Sigma) \frac{i}{\not{p}}. \tag{15}$$

[2] The authors thank Prof. P. B. Pal for making this clear.

A straightforward calculation yields

$$
\begin{aligned}
-i\Sigma = &-\frac{iG_F}{\sqrt{2}}\int \frac{d^4k}{(2\pi)^4}\gamma^\lambda\left(1-\gamma^5\right) \\
&\times\left[\frac{i(\not{k}+m)}{k^2-m^2+i\epsilon}-2\pi n_f\left(k^0\right)\delta\left(k^2-m^2\right)(\not{k}+m)\right] \\
&\times\gamma_\lambda\left(1-\gamma^5\right).
\end{aligned} \tag{16}
$$

The first term in the parenthesis yields the counterterm for the mass renormalization. This will be infinite, and we are not interested in that. Let us consider that we are working with an already renormalized theory. Then, the second term yields

$$
\begin{aligned}
-i\Sigma = &-\frac{iG_F}{\sqrt{2}}\int \frac{d^4k}{(2\pi)^3 2\omega_k}\gamma^\lambda\left(1-\gamma^5\right)(\not{k}+m)\gamma_\lambda\left(1-\gamma^5\right) \\
&\times\left[\delta\left(k^0-\omega_k\right)+\delta\left(k^0+\omega_k\right)\right]n_f\left(k^0\right) \\
= &-i\sqrt{2}G_F n_e\gamma^0\mathbb{L}.
\end{aligned} \tag{17}
$$

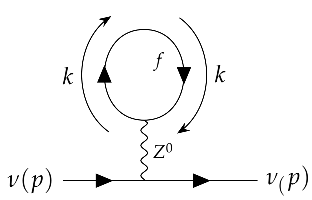
We have used the definition

$$
n_e = 2\int\frac{d^3k}{(2\pi)^3}n_f^e\left(\omega_k\right), \tag{18}
$$

which is the total electron density. Then, the dressed propagator is

$$
\begin{aligned}
&\frac{i}{\not{p}}+\frac{i}{\not{p}}(-i\Sigma)\frac{i}{\not{p}} \\
&=\frac{i}{\not{p}-\Sigma}=\frac{i}{\gamma^0\left(p^0-\sqrt{2}G_F n_e\mathbb{L}\right)-\vec{\gamma}\cdot\vec{p}}.
\end{aligned} \tag{19}
$$

Let us now consider the neutral current mediated process. The corresponding Feynman diagram is (where f is any fermion present in the background)



A calculation similar to the previous one shows that due to the neutral current mediated process the propagator is modified as [23]

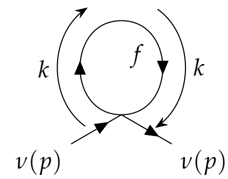
$$
\begin{aligned}
&\frac{i}{\not{p}}+\frac{i}{\not{p}}(-i\Sigma)\frac{i}{\not{p}} \\
&=\frac{i}{\not{p}-\Sigma}=\frac{i}{\gamma^0\left(p^0+\sqrt{2}G_F\frac{n_n}{2}\mathbb{L}\right)-\vec{\gamma}\cdot\vec{p}},
\end{aligned} \tag{20}
$$

where n_n is the total neutron density. Hence, the total change in the propagator pole in the electron neutrino is

$$
\frac{i}{\gamma^0\left(p^0-\left(\sqrt{2}G_F n_e\mathbb{L}-\sqrt{2}G_F\frac{n_n}{2}\mathbb{L}\right)\right)-\vec{\gamma}\cdot\vec{p}}. \tag{21}
$$

This is the familiar result originally derived by Wolfenstein [16].

We can similarly calculate the effective matter potential for the torsional four-fermion interaction. The corresponding Feynman diagram is



Following the same method as in the case of the neutral current mediated process, we can evaluate the shift in the pole of the propagator of type i neutrino as

$$
\frac{i}{\gamma^0\left(p^0-\sum_f\lambda_V^f n_f\lambda_i\mathbb{L}\right)-\vec{\gamma}\cdot\vec{p}}=\frac{i}{\gamma^0\left(p^0-\lambda_i\tilde{n}\mathbb{L}\right)-\vec{\gamma}\cdot\vec{p}}, \tag{22}
$$

where we have defined $\tilde{n}=\sum_f\lambda_f n_f$, the sum running over background fermions, i.e., protons, neutrons, and electrons.

These calculations enable us to write

$$
\begin{aligned}
\mathcal{L}_\nu = &\sum_i\bar{\nu}_i(i\not{\partial}-m_i)\nu_i-\tilde{n}\sum_i\lambda_i\nu_i^\dagger\mathbb{L}\nu_i \\
&+\sqrt{2}G_F\sum_\alpha\left(T_3^\alpha-2Q^\alpha\sin^2\left(\theta_W\right)\right)\nu_\alpha^\dagger\mathbb{L}\nu_\alpha \\
&-\sqrt{2}G_F n_e\nu_e^\dagger\mathbb{L}\nu_e+G_F\frac{n_n}{\sqrt{2}}\sum_\alpha\nu_\alpha^\dagger\mathbb{L}\nu_\alpha.
\end{aligned} \tag{23}
$$

4. NEUTRINO OSCILLATION

Experimentally, it is observed that $\sin\theta_{13}$ is very small. As a result, a two-flavor model is often a very good starting point. We can analyze the solar neutrino problem by considering oscillations between the electron- and muon-type neutrinos. The atmospheric neutrino can also be analyzed by considering muon neutrino to tau neutrino oscillations. Let us therefore briefly discuss the two-flavor model of neutrino oscillations.

From equation (23) we can write [24, 25]

$$
\begin{aligned}
&i\begin{pmatrix}\dot{\nu}_e\\ \dot{\nu}_\mu\end{pmatrix} \\
&=\left[E_0+\frac{1}{4E}\begin{pmatrix}-\Delta m_s^2\cos 2\theta+D & \Delta m_s^2\sin 2\theta\\ \Delta m_s^2\sin 2\theta & \Delta m_s^2\cos 2\theta-D\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix},
\end{aligned} \tag{24}
$$

where we have written $D=2\sqrt{2}G_F n_e E$, defined

$$
E_0=E+\frac{m_1^2+m_2^2}{4E}+\frac{\lambda_1+\lambda_2}{2}\tilde{n}-\frac{G_F}{\sqrt{2}}\left(n_n-n_e\right), \tag{25}
$$

and also defined

$$
\Delta m_s^2=\Delta m^2+2\tilde{n}E\Delta\lambda, \tag{26}
$$

where $\Delta m^2=m_2^2-m_1^2$ and $\Delta\lambda=\lambda_2-\lambda_1$. Let us write θ_M for the mixing angle in matter, modified by the torsional interaction:

$$
\tan 2\theta_M=\frac{\tan 2\theta}{1-\frac{D}{\Delta m_s^2\cos 2\theta}}. \tag{27}
$$

Then, we can diagonalize equation (24). The eigenvalues are $E_0 \mp \frac{\Delta m_M^2}{4E}$, resulting in the survival probability

$$P_{\nu_e \to \nu_e} = 1 - \sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2}{4E}L\right) \tag{28}$$

and the conversion probability

$$P_{\nu_e \to \nu_\mu} = \sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2}{4E}L\right), \tag{29}$$

where we have written

$$\Delta m_M^2 = \sqrt{\left(\Delta m_s^2 \cos 2\theta - D\right)^2 + \left(\Delta m_s^2 \sin 2\theta\right)^2}. \tag{30}$$

Equations (28) and (29) are the simplest examples of neutrino oscillation being affected by the four-fermion interactions induced by spacetime geometry. The $\nu_e \to \nu_\mu$ conversion probability becomes relevant for solar neutrinos, for which, however, a varying matter density should be considered. Probabilities of conversion and survival for other flavors of neutrino become relevant in other processes. Then, the geometrical coupling constants can be found, along with the bare masses m_i of the neutrinos, by fitting the respective formulae to the experimental data. We mention in passing that in principle there could be oscillation among neutrino flavors even when $m_i = 0$ provided that the coupling constants λ_i are not all equal.

5. CONCLUSION AND REMARKS

We have used Einstein-Cartan theory to describe fermions under gravity. In this framework, the torsion couples to the spin of fermions, the most general form of the coupling being a combination of vector and axial currents. The vector part of the contortion contributes to the refractive index of the medium at the lowest order. One remarkable feature of this model is that now there is a part of the refractive index of the medium which survives even if neutrinos are massless or have quasi-degenerate spectra. This means massless or quasi-degenrate neutrinos have a nonvanishing conversion probability, which turns out to be independent of the energy of the incident neutrinos. Our results are expected to affect estimates of the oscillation parameters.

One might think that the coupling constants λ, being of mass dimension $1/M$, should be suppressed by M_{Pl}, which is the natural mass scale of quantum gravity. But that would not be correct. In the case of torsion, the mass scale is not known. Furthermore, the lack of a theory of quantum gravity also means that we do not know if torsion can be described by a renormalizable theory. Hence, the dependence of the coupling parameters with 4-momentum is not known. Thus, the couplings may not necessarily be suppressed by the Planck mass. In the current context, torsion is generated by the fermions, leading to a four-fermion interaction, not determined by any other theory. Thus, the couplings λ can be fixed only by appealing to experimental data.

This model can be easily extended to oscillations among three neutrino flavors. We can include the effects of torsion into the Hamiltonian for that as well and find the conversion probability in presence of torsion. Details will be presented elsewhere.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] J. N. Bahcall, "Neutrinos from the Sun," Scientific American July **221(1)** pp. 28–37 (1969).

[2] C. L. Cowan Jr., F. Reines, F. B. Harrison, H. W. Kruse, and A. D. McGuire, "Detection of the free neutrino: a confirmation" Science **124(3212)** 103-104 (1956).

[3] R. Davis Jr., D. S. Harmer, and K. C. Hoffman, "Search for neutrinos from the sun," Phys. Rev. Lett. **20**, 1205–1209 (1968).

[4] B. T. Cleveland, T. Daily, R. Davis Jr., J. R. Distel, K. Lande, C. K. Lee, P. S. Wildenhain, and J. Ullman, "Measurement of the solar electron neutrino flux with the Homestake chlorine detector," Astrophys. J. **496**, 505–526 (1998).

[5] B. Pontecorvo, "Inverse beta process," Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. **1**, 25–31 (1991) PD-205.

[6] J. N. Bahcall and R. Davis Jr, "Solar neutrinos: a scientific puzzle", Science **191(4224)** 264–267 (1976).

[7] P. A. Zyla et al. [Particle Data Group], "Review of Particle Physics," PTEP **2020**, no.8, 083C01 (2020).

[8] T. W. B. Kibble, "Lorentz invariance and the gravitational field," J. Math. Phys. **2**, 212 (1961).

[9] D. W. Sciama, "The Physical structure of general relativity," Rev. Mod. Phys. **36**, 463 (1964) Erratum: [Rev. Mod. Phys. **36**, 1103 (1964)].

[10] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick, and J. M. Nester, "General Relativity with Spin and Torsion: Foundations and Prospects," Rev. Mod. Phys. **48**, 393 (1976).

[11] F. W. Hehl, G. D. Kerlick, and P. Von Der Heyde, "General relativity with spin and torsion and its deviations from einstein's theory," Phys. Rev. D **10**, 1066–1069 (1974).

[12] R. T. Hammond, "Torsion gravity," Rept. Prog. Phys. **65**, 599–649 (2002).

[13] F. W. Hehl and Y. N. Obukhov, "Elie Cartan's torsion in geometry and in field theory, an essay," Annales Fond. Broglie **32**, 157–194 (2007) [arXiv:0711.1535 [gr-qc]].

[14] M. Gasperini, "Theory of Gravitational Interactions," Cham, Springer (2013).

[15] S. Chakrabarty and A. Lahiri, "Geometrical contribution to neutrino mass matrix," Eur. Phys. J. C **79**, no.8, 697 (2019) [arXiv:1904.06036 [hep-ph]].

[16] L. Wolfenstein, "Neutrino Oscillations in Matter," Phys. Rev. D **17**, 2369–2374 (1978).

[17] S. Weinberg, "A Model of Leptons," Phys. Rev. Lett. **19**, 1264–1266 (1967).

[18] M. L. Bellac, "Thermal Field Theory," Cambridge University Press, 2011, ISBN 978-0-511-88506-8, 978-0-521-65477-7.

[19] A. J. Niemi and G. W. Semenoff, "Finite Temperature Quantum Field Theory in Minkowski Space," Annals Phys. **152**, 105 (1984).

[20] J. I. Kapusta and C. Gale, "Finite-temperature field theory: Principles and applications," Cambridge University Press, 2011, ISBN 978-0-521-17322-3, 978-0-521-82082-0, 978-0-511-22280-1.

[21] A. K. Das, "Finite Temperature Field Theory," World Scientific, 1997, ISBN 978-981-02-2856-9, 978-981-4498-23-4.
[22] A. K. Das, "Topics in finite temperature field theory," [arXiv:hep-ph/0004125 [hep-ph]].
[23] P. B. Pal and T. N. Pham, "Field Theoretic Derivation of Wolfenstein's Matter Oscillation Formula," Phys. Rev. D **40**, 259 (1989) doi:10.1103/PhysRevD.40.259.
[24] R. N. Mohapatra and P. B. Pal, Massive Neutrinos in Physics and Astrophysics, World Scientific Lecture Notes in Physics (World Scientific, Singapore, 1991) vol. 41.
[25] C. W. Kim and A. Pevsner, Neutrinos in Physics and Astrophysics, Contemporary Concepts in Physics (Harwood Academic Press, Chur, Switzerland, 1993) vol. 8.