

# Majorana Neutrinos and Clockworked Yukawa Couplings Contribution to Nonobservation of the Rare Leptonic Decay $l_i \rightarrow l_j \gamma$ , Clockwork Photon and Clockwork Graviton

Gayatri Ghosh<sup>1,2</sup>

<sup>1</sup>*Department of Physics, Gauhati University, Jalukbari, Assam 781015, India*

<sup>2</sup>*Department of Physics, Pandit Deendayal Upadhyay Mahavidyalaya, Karimganj, Assam 788720, India*

## Abstract

The clockwork is an extra-dimensional setup for generating light particles with exponentially suppressed or hierarchical couplings of light particles with  $N$  massive states having comparable masses near the threshold scale of the mechanism in theories which contain no small parameters at the fundamental level. We explore the prospect of charged lepton flavor violation (cLFV) in a clockwork framework which encompasses Dirac mass terms as well as Majorana mass terms for the new clockwork fermions. We derive the masses of the nonzero clockwork Majorana masses, and new particles in a clockwork framework and for their Yukawa couplings to the lepton doublets, in the framework where the clockwork parameters are universal. When the new clockwork Majorana masses are nonzero, neutrino masses are generated as a result of the exchange of heavy messenger particles such as right-handed isosinglet neutrinos or isotriplet scalar bosons known as the seesaw mechanism. In the case of nonzero clockwork Majorana masses, owing to the sizable effective Yukawa couplings of the higher mass modes, neutrino masses can only be made tiny by conjecturing a large Majorana mass in the teraelectron volt range for all the clockwork gears. This is apparent from the constraints on the mass scale of the clockwork fermions due to the nonobservation of the rare cLFV decay  $\mu \rightarrow e\gamma$ ,  $\tau \rightarrow \mu\gamma$ ,  $\tau \rightarrow e\gamma$ . A general description of the clockwork mechanism valid for fermions, gauge bosons, and gravitons is discussed here. This mechanism can be implemented with a discrete set of new fields or, in its continuum version, through an extra spatial dimension. In both cases, the clockwork emerges as a useful tool for model-building applications. Notably, the continuum clockwork offers a solution to the Higgs naturalness problem, which turns out to be the same as in linear dilaton duals of Little String Theory.

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## 1. INTRODUCTION

Beyond Standard Model Physics are best explained at high energies as renormalizable field operators which become non-renormalizable site operators involving Standard Model fields at low energy. There is a strong association between the interaction scales and mass scales like the dimension 5 Weinberg operator LLHH which generates the lepton number violating coupling. Interaction scales and mass scales are totally distinct quantities. Interaction scales and mass scales inherently hold different units of measure. Masses are consistent with the product of scales and Yukawa couplings. In natural units, the distinction between mass and interaction scales has very less fall out if the couplings are of the order 1. But the dynamics get totally changed in presence of tiny Yukawa couplings [1, 2, 3] which are small in natural units since now the dynamics of interaction scales can enter low-scale energies. In this work, we discuss lepton flavor violation in a generalization of clockwork models where large effective interaction scales or ultraviolet completion enters at energy regimes exponentially smaller than set forth by a given interaction strength.

In this work, our theory gives rise to small numbers with couplings of the order 1, and thus, dynamics of large effective interaction scales enter much lower energies. In such mechanisms, the full ultraviolet completion occurs at energies exponentially suppressed than the scale of a given interaction

strength. The clockwork mechanism which is very malleable generates exponentially incited large interaction scales for light or massless scalars, fermions, vectors, and even gravitons. This clockwork is a mechanism where  $O(1)$  couplings exponentiate generating tiny Yukawa couplings from a theory with no exponentially small parameters at the fundamental level. This new model building includes axion, neutrino, flavor, weak scale, and even gravitational physics. Let us consider a system having a massless particle  $P$ , because of a symmetry  $S$ . Let us enlarge this simple setup to  $N + 1$  version of the actual system, which can be contemplated as sites of a one-dimensional lattice in crystallography theory.  $N + 1$  version of the actual system contains a symmetry  $F$ , which owns at least the product of the individual symmetries ( $F \supseteq S^{N+1}$ ), outlining  $N + 1$  massless particles  $P$ . At a mass scale  $m$ , spontaneous soft explicit symmetry breaking of  $F$  happens which is local gauge theory through a mass mixing matrix of its nearest neighbor interaction of the  $N + 1$  lattice sites. This explicit breaking of symmetry  $F$  possesses a parameter  $q$  that interacts with the site  $j + 1$  and the site  $j$  ( $j = 0, 1, 2, 3, 4, \dots, N - 1$ ) asymmetrically. Thus, the clockwork mechanism generates tiny couplings in theories coupled to the Standard Model which acts as a linear quiver model with no large hierarchies in the theory parameters, which incites site-dependent exponentially suppressed couplings to the zero mode of the clockwork gear generating lepton flavor violating charged current, neutral current, and Higgs boson interactions. It is generic to Abelian Goldstone Bosons, fermions, vectors, and other fields. The interaction of the clockwork fields to the SM particles is site-dependent since the breaking of  $F$  symme-

try is exponentially squeezed out by factor  $\frac{1}{q^N}$  at each lattice point  $N$ . In a 1D lattice in crystal theory space, one tends to produce only  $N$  links out of the  $N + 1$  crystal sites. One symmetry group  $S_0$  remains after the soft breaking of  $F$ , which tends to keep one linear combination of original particles massless. This massless particle is exponentially localized toward one of the boundaries of the 1D lattice. At the opposite boundary, the component of the massless particle is exponentially squeezed and is given by  $\frac{1}{q^N}$ , and now, if we couple this site of clockwork sector to the SM, we attain tiny couplings of  $\frac{1}{q^N}$  of the massless particle to ordinary SM particles, without inciting small parameters, multiple mass scales, or exponentially large field multiplicities in the fundamental theory. A clockwork fermion thus elucidates a very small Dirac neutrino mass or explains the hierarchical nature of quark and lepton masses. The clockwork graviton explains an innovative answer to the pragmatic naturalness problem of the electroweak scale, leading to the dynamical explanation for the weak Gravity conjecture.

Neutrino's tiny mass is an open challenge in Fundamental Physics and points toward new laws of particle physics and reshape theories of cosmology. One probable answer to this enigma is provided by the seesaw mechanism, where tiny neutrino masses are unraveled by the lepton number violation at a very high-energy scale by two units  $\Delta_L = 2$  [4, 5, 6, 7]. Tiny Yukawa couplings of the neutrino, on the other hand, couple to the Standard Model Higgs in models with lepton number conservation. The occurrence of tiny Yukawa couplings is a phenomenologically feasible possibility and can be worked out in further extensions of the model. A new mechanism of producing small couplings in theories coupled to the Standard Model has been introduced [1, 2, 3, 8, 9, 10, 11, 12, 13, 14, 15]. The implicative mechanism, of deconstruction models, is a linear model with no large hierarchies in the site-dependent field theory parameters, which essentially arouses site-dependent suppressed couplings to the zero mode [16]. Originally, the clockwork theory was proposed for a quiver of axions; it is now generalized to a spin-0 scalar, spin- $\frac{1}{2}$  fermion, spin-1 boson, or spin-2 graviton and other fields [17]. A clockwork fermion elucidates a very small Dirac neutrino mass or explains the hierarchical pattern of quark and lepton masses. With a clockwork gauge boson, the presence of tiny gauge charges is justified. Clockwork graviton solves the naturalness problem of the electroweak scale, syndicating an effective explanation for the weakness of gravity. Independently of the specific implementation of the clockwork, the theory proposes the existence of  $N$  particles at the universal mass scale  $m$ , which are known as the "clockwork gears", as they are the degrees of freedom. The gears have universal properties, and their exploration of the clockwork will also lead us to consider the limit  $N$  that tends to infinity and eventually the continuum limit, in which the 1D space lattice in the field space is elucidated as a physical spatial dimension. Some studies on the applications and generalizations of clockwork mechanism are done [18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34].

In this work, we explore the impact of Majorana neutrinos in the case of charged lepton flavor violation in the fermionic clockwork model and the constraints on the masses of the clockwork gears. Concretely, we identify the masses of the clockwork gears with the zero modes of a clockwork sector, which fits the neutrino masses and charged lepton masses, such that tiny Yukawa couplings are naturally generated and there-

fore tiny neutrino masses. We establish the clockwork framework for the right-handed neutrinos by counting Majorana mass terms. We show how in the clockwork theory the suppression of the Yukawa couplings by field-dependent exponential factors is not altered by the manifestation of the Majorana mass terms. In fact, the combination of the clockwork "suppression" and the Majorana "seesaw" sets now the mass constraints on the clockwork gears. The clockwork mechanism suppresses the zero mode couplings, and the Yukawa couplings of the higher modes induce, via loops, potentially large sizable rates for the leptonic rare decays. The rest of the paper is organized as follows. In Section 2, we present the most general phenomenology for clockwork neutrinos with Dirac and Majorana mass terms. In Section 3, we discuss the impact of Majorana neutrinos on lepton flavor violation in the clockwork scenario and calculate constraints on the clockwork gear masses. We close with a summary.

## 2. CLOCKWORK FERMION

Fermions may be massless due to chiral symmetry. Fermion realization of the clockwork model requires a single chiral symmetry which is split among a number of field sites in the underlying model. The left-out chiral symmetry drags the massless fermion exponentially to one edge of the clockwork. To this climax, one introduces  $N + 1$  chiral fermions  $\psi_{Rj}$  ( $j = 0, \dots, N$ ) together with  $N$  fermions  $\psi_{Li}$  ( $i = 0, \dots, N - 1$ ) of opposite chirality. The global chiral symmetry is spontaneously broken down by  $N$  mass parameters  $m_i$  which pair up the fields in  $N$  massive Dirac fermions, leaving a single massless chiral component or zero mode. Let us signify  $U(1)_{Rj}$  and  $U(1)_{Li}$  Abelian factors under which  $N + 1$  chiral fermions  $\psi_{Rj}$  and  $N$  fermions  $\psi_{Li}$  ( $i = 0, \dots, N - 1$ ) have charge 1, respectively. Then,  $m_j$  get charges  $(1, -1)$  under  $U(1)_{Lj} \times U(1)_{Rj}$ , and  $m_q$  get charges  $(1, -1)$  under  $U(1)_{Lj} \times U(1)_{Rj}$ . One Abelian factor of the chiral symmetry or the zero mode in the  $\psi_R$  sector remains unbroken by both  $m$  and  $m_q$ .

Under the Standard Model gauge group, we expand the Standard Model with  $N$  left-handed and  $N + 1$  right-handed chiral fermions, singlets, which we represent as  $\psi_{Li}$  ( $i = 0, \dots, N - 1$ ) and  $\psi_{Ri}$  ( $i = 0, \dots, N$ ), respectively. The Lagrangian of the model interprets as

$$L = L_{\text{SM}} + L_{\text{CLOCKWORK}} + L_{\text{INT}}. \quad (1)$$

Here,  $L_{\text{SM}}$  is the Standard Model Lagrangian,  $L_{\text{CLOCKWORK}}$  is the part of the Lagrangian involving only the new right-handed chiral fermions, singlets, and  $L_{\text{INT}}$  is the interaction term of the new fields, right-handed chiral fermions, singlets with the Standard Model fields. The Standard Model fields only share coupling to the last one of the fermionic clockwork, right-handed chiral fermion.

$$L_{\text{INT}} = -Y\tilde{H}L_l\psi_{RN}, \quad (2)$$

where  $\tilde{H} = i\tau_2 H^*$ .  $H$  is the Standard Model Higgs doublet and  $L_l$  is the Standard Model left-handed leptonic fields.

The Lagrangian for the fermion fields is

$$L = L_{\text{kin}} - l, \quad (3)$$

$$l = \sum_{j=0}^{n-1} \left( m_i \bar{\psi}_{Lj} \psi_{Rj} - m'_i \bar{\psi}_{Lj} \psi_{Rj+1} + h.c \right) - \sum_{j=0}^{n-1} \left( \frac{1}{2} M_{Lj} \bar{\psi}_{Lj} \psi_{Lj} \right) - \sum_{j=0}^n \left( \frac{1}{2} M_{Rj} \bar{\psi}_{Rj} \psi_{Rj} \right),$$

where  $L_{\text{kin}}$  is the kinetic term for all fermions. We take universal values for  $m$  and  $q$ .  $m$  is real with a chiral rotation of the fermions, the parameters  $q$  are in general complex, but we choose them as real for simplicity.  $M$  is a  $(2N+1) \times (2N+1)$  mass matrix.  $L_{\text{kin}}$  is symmetric with respect to the global gauge group  $U(N)_L \times U(N+1)_R$ . The mass terms  $m_i$  break the global symmetry  $U(N)_L \times U(N+1)_R \rightarrow \prod_{i=0}^{N-1} U(1)_i$ , where  $U(1)_i$  acts as  $\psi_{L,i} \rightarrow e^{i\beta_i} \psi_{L,i}$ ,  $\psi_{L,i} \rightarrow e^{i\beta_i} \psi_{L,i}$ , which when combined with the mass terms  $m_i$  automatically breaks the global symmetry  $U(N)_L \times U(N+1)_R \rightarrow U(1)_{\text{CW}}$ , where under  $U(1)_{\text{CW}}$  clockwork fermion fields transform as  $\psi_{L,i} \rightarrow e^{i\beta} \psi_{L,i}$ ,  $\psi_{L,i} \rightarrow e^{i\beta} \psi_{L,i}$  for all  $i$ .  $M_{Lj}$  and  $M_{Rj}$  are the Majorana masses for the left- and right-handed singlet fields. For simplicity, we assume universal Dirac masses, Majorana masses, and nearest neighbor interactions as  $m_i = m$ ,  $m'_i = mq$ ,  $M_{Lj} = M_{Rj} = m\bar{q}$  for all  $i$ . This conjecture leads to the mass matrix:

$$M_\psi = m \begin{pmatrix} \bar{q} & 0 & \cdots & 0 & 1 & -q & \cdots & 0 \\ 0 & \bar{q} & \cdots & 0 & 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & \bar{q} & 0 & 0 & 0 & -q \\ 1 & 0 & \cdots & 0 & \bar{q} & 0 & \cdots & 0 \\ -\bar{q} & 1 & \cdots & 0 & 0 & \bar{q} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & -\bar{q} & 0 & 0 & 0 & \bar{q} \end{pmatrix}. \quad (4)$$

The eigen-values of the mass matrix are

$$M_0 = m\bar{q}, \quad (5)$$

$$M_k = m\bar{q} - m\sqrt{\kappa_r}, \quad r = 1, \dots, N, \quad (6)$$

$$M_{n+k} = m\bar{q} + m\sqrt{\kappa_r}, \quad r = 1, \dots, N, \quad (7)$$

where  $\kappa_r$  is defined as

$$\kappa_r = q^2 + 1 - 2q \cos \frac{r\pi}{N+1}. \quad (8)$$

Some studies on different aspects of Lepton Flavor Violation have been extensively done in [35]. Also,  $A_4$  symmetry which gives the corrections to the tribimaximal (TBM) form for the leading order neutrino mixing matrix is studied in [36]. The predictions of vanishing  $\theta_{13}$  by tribimaximal mixing (TBM) are owing to its invariance under  $\mu$ - $\tau$  exchange symmetry [35]. Small explicit breaking of  $\mu$ - $\tau$  symmetry can generate large Dirac CP violating phase in the context of neutrino oscillations [37].

### 3. LEPTON FLAVOR VIOLATION IN CASE OF $M_{Lj}, M_{Rj} \neq 0$

The Yukawa couplings for the zero mode of the clockwork gears are suppressed, thereby explaining why the neutrino

masses are small, but the Yukawa couplings for the higher modes of the clockwork gears are unsuppressed which leads to observable effects like lepton flavor violation at low energies. We find in this work that if the clockwork gear masses are low, then signatures of lepton flavor violation existing in the Yukawa couplings of the higher modes through quantum effects generate rare leptonic decays such as  $l_j \rightarrow l_j + \gamma$  or  $\mu$ - $e$  conversion processes in nuclei, induced by clockwork fermions, with decay rates which could be accessible at next run of LHC.

We calculate the rate  $l_j \rightarrow l_j + \gamma$  from [4]. For  $n$  clockwork generations, we have

$$B(\mu \rightarrow e\gamma) \simeq \frac{3\alpha_{em}v^4}{8\pi} \left| \sum_{\alpha=1}^n \sum_{k=1}^{N_\alpha} \frac{Y_k^{e\alpha} Y_k^{\mu\alpha}}{M_k^{\alpha^2}} F(x_k^\alpha) \right|^2, \quad (9)$$

where  $\alpha_{em}$  denotes the fine structure constant.  $N_\alpha$  is the number of clockwork gears in the  $\alpha$ -th generation.  $M_k^\alpha$  is the mass of the  $k$ -th clockwork mode in the  $\alpha$ -th generation ( $k = 1, \dots, N_\alpha$ ). Here,

$$x_k^\alpha \equiv \frac{M_k^{\alpha^2}}{M_W^2}. \quad (10)$$

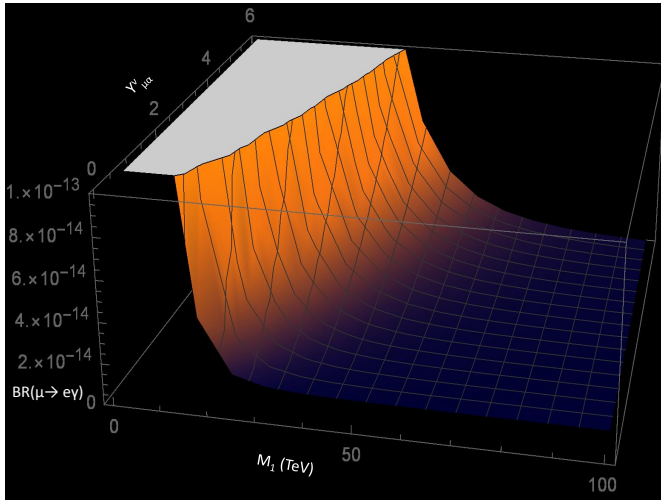
We define the loop function  $F(x)$  as

$$F(x) \equiv \frac{1}{6(1-x)^4} \left( 10 - 43x + 78x^2 - 49x^3 + 4x^4 - 18x^3 \log x \right), \quad (11)$$

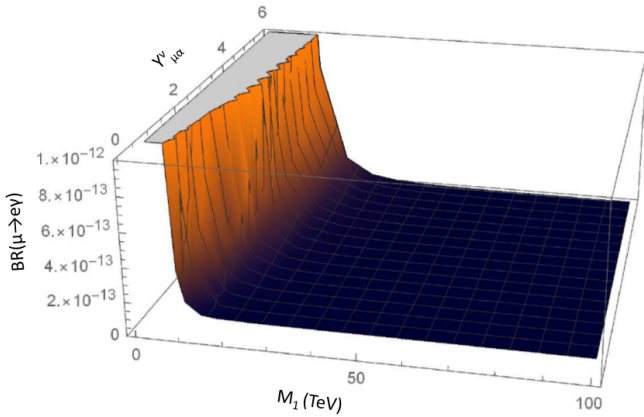
which has limits  $F(0) = \frac{5}{3}$  and  $F(\infty) = \frac{2}{3}$ . The two extreme cases of  $F(x)$  are  $F(0) = \frac{5}{3}$  and  $F(\infty) = \frac{2}{3}$ .

For  $N$  clockwork generations of clockwork gears, we calculate the rate for  $l_j \rightarrow l_j + \gamma$  from [38, 39, 40].

The present stringent upper bound  $\text{BR}(\mu \rightarrow e + \gamma) \leq 4.2 \times 10^{-13}$  from the MEG experiment poses firm constraints on the mass scale of the clockwork gears. In Figures 1, 2, 3, and 4, we present the branching ratio expected for points reproducing the measured neutrino masses as well as charged lepton masses, assuming two clockwork generations, as shown in the figures presented in Section 3, as a function of the mass of the first clockwork gear. It is observed from Figure 1 that if the Yukawa modes  $Y_{\mu\alpha}^\nu$  are of the order 0 to 6, then the clockwork gears must be larger than 10 TeV and then the clockwork gears must be larger than 10 TeV in order to elude the experimental constraints on  $\text{BR}(\mu \rightarrow e + \gamma)$  as apparent from Figure 1, except the fact that very fine cancellations occur among all contributions to this process. Similarly, for observation of  $\text{BR}(\tau \rightarrow e + \gamma)$  for a large number of clockwork generations, we observe even more restricted lower limits on the lightest gear mass, due to more number of particles in the loop. As evident from Figure 2, it follows from the figure that the clockwork gears must be larger than  $\sim 0.2$  TeV and must be less than 1 TeV for Yukawa corresponding to 0.5 as restricted by the experimental constraints on  $\text{BR}(\tau \rightarrow e + \gamma) \leq 3.3 \times 10^{-8}$ , unless very fine cancellations occur among all contributions to the process  $\text{BR}(\tau \rightarrow e + \gamma)$ . Also for Yukawas resembling the values around 2.4238, the values of the mass of the first clockwork gear must be less than 2.5 TeV and greater than 0.2 TeV, if the Yukawa coupling modes  $Y_{\tau\alpha}^\nu$  are of the order 0.4 to 2.4238. It follows from Figure 3 that the clockwork gears must be larger than 2 TeV and must be less than 5 TeV as restricted by the experimental constraints  $\text{BR}(\tau \rightarrow \mu + \gamma) \leq 4.4 \times 10^{-8}$ , for Yukawa  $Y_{\tau\alpha}^\nu$  corresponding to 1 unless very fine cancellations occur among all contributions



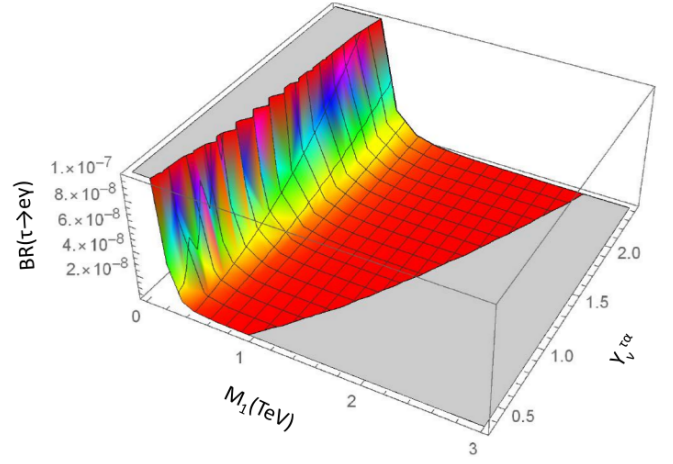
(a)



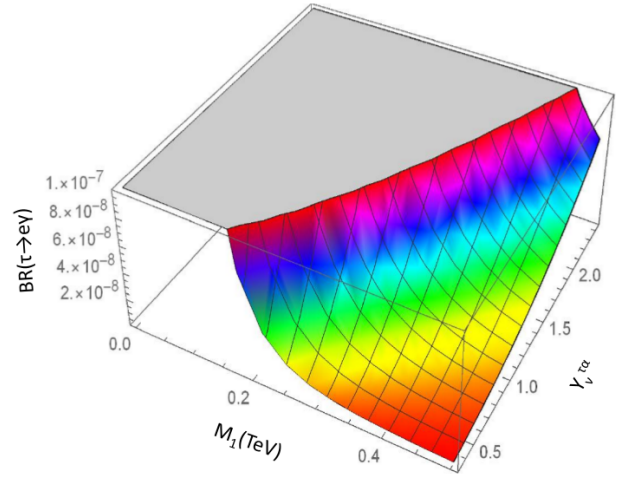
(b)

**FIGURE 1:** (a) The figure in the top panel shows the predicted value of  $BR(\mu \rightarrow e + \gamma)$  for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear and the Yukawas satisfying the charged lepton masses. (b) The figure in the bottom panel shows the predicted value of  $BR(\mu \rightarrow e + \gamma)$  for points of the parameter space reproducing the observed neutrino oscillation parameters as a function of the mass of the first clockwork gear and the Yukawas satisfying the charged lepton masses. It follows from the figure that the clockwork gears must be larger than  $\sim 10$  TeV in order to evade the experimental constraints, unless very fine cancellations occur among all contributions to the process  $BR(\mu \rightarrow e + \gamma)$ .

to the process  $BR(\tau \rightarrow \mu + \gamma)$  and the clockwork gears must be larger than  $\sim 2$  TeV and must be less than 40 TeV restricted by the experimental constraints  $BR(\tau \rightarrow \mu + \gamma) \leq 4.4 \times 10^{-8}$ , for Yukawa  $Y_V^{\tau\alpha}$  corresponding to 7.67. It is found from Figure 4 that the clockwork gears must be larger than  $\sim 90$  TeV and must be less than 200 TeV as constrained by the experimental upper bound on  $BR(\tau \rightarrow e + \gamma) \leq 4.4 \times 10^{-8}$ , for Yukawa  $Y_V^{\tau\alpha}$  corresponding to 1 to 2 unless very fine cancellations occurs among all contributions to the process  $BR(\tau \rightarrow \mu + \gamma)$  and the clockwork gears must be greater than  $\sim 90$  TeV for Yukawa  $Y_V^{\tau\alpha}$  corresponding to 2 to 7.67 as constrained by experimental limit.



(a)

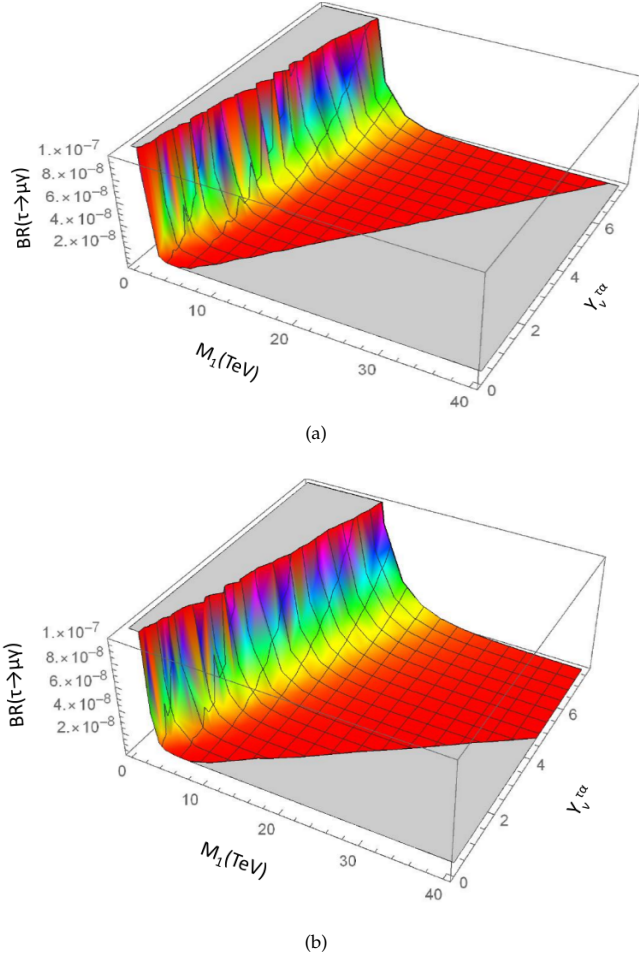


(b)

**FIGURE 2:** (a) The figure in the top panel shows the predicted value of  $BR(\tau \rightarrow e + \gamma)$  for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear. It follows from the figure that the clockwork gears must be larger than  $\sim 0.2$  TeV and must be less than 1 TeV for Yukawas corresponding to 0.5 as restricted by the experimental constraints on  $BR(\tau \rightarrow e + \gamma) \leq 3.3 \times 10^{-8}$  unless very fine cancellations occur among all contributions to the process  $BR(\tau \rightarrow e + \gamma)$ . Also for Yukawas resembling the values around 2.42338, the values of the mass of the first clockwork gear must be less than 2.5 TeV and greater than 0.2 TeV. (b) The figure in the bottom panel shows the predicted value of  $BR(\tau \rightarrow e + \gamma)$  for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear and the Yukawas satisfying the charged lepton masses.

#### 4. CLOCKWORK PHOTON

A clockwork photon can be acquired by clockworking the invariance of gauge symmetry. Let us take  $N + 1$   $U(1)$  gauge groups with the same gauge coupling  $g$  and  $N$  complex scalars,  $\psi_j$ , where  $j = 0, \dots, N - 1$  having charge  $(1, -q)$ . The underlying gauge symmetry is  $U(1)_j \times U(1)_{j+1}$ , given these scalars acquire a negative mass-squared, which generates vacuum ex-



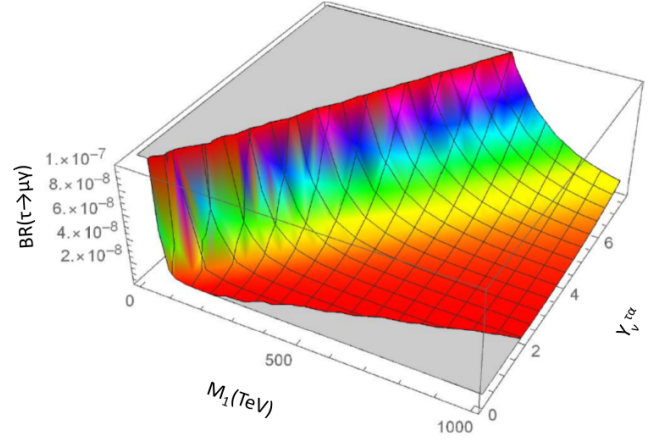
**FIGURE 3:** (a) The figure in the top panel shows the predicted value of  $\text{BR}(\tau \rightarrow \mu + \gamma)$  for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear. It follows from the figure that the clockwork gears must be larger than 2 TeV and must be less than 5 TeV restricted by the experimental constraints  $\text{BR}(\tau \rightarrow \mu + \gamma) \leq 4.4 \times 10^{-8}$ , for Yukawa  $Y_v^{\tau\alpha}$  corresponding to 1 unless very fine cancellations occur among all contributions to the process  $\text{BR}(\tau \rightarrow \mu + \gamma)$  and the clockwork gears must be larger than  $\sim 2$  TeV and must be less than 40 TeV restricted by the experimental constraints  $\text{BR}(\tau \rightarrow \mu + \gamma) \leq 4.4 \times 10^{-8}$ , for Yukawa  $Y_v^{\tau\alpha}$  corresponding to 7.67. (b) Predicted value of  $\text{BR}(\tau \rightarrow \mu + \gamma)$  for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear.

peptation values which are at the same scale  $\lambda$ . The Lagrangian is

$$L = - \sum_{j=0}^N \frac{1}{4} F_{\alpha\beta}^j F^{j\alpha\beta} - \sum_{j=0}^{N-1} \left[ |D_\alpha \psi_j|^2 + \lambda \left( |\psi_j|^2 - \frac{1}{2} \lambda^2 \right) \right], \quad (12)$$

where

$$D_\alpha \psi_j = \left[ \partial_\alpha - ig \left( A_\alpha^j - q A_\alpha^{j+1} \right) \right] \psi_j. \quad (13)$$



**FIGURE 4:** Predicted value of  $\text{BR}(\tau \rightarrow \mu + \gamma)$  for points of the parameter space reproducing the observed neutrino oscillation parameters, as a function of the mass of the first clockwork gear. It follows from the figure that the clockwork gears must be larger than  $\sim 90$  TeV and must be less than 200 TeV as constrained by the experimental upper bound on  $\text{BR}(\tau \rightarrow e + \gamma) \leq 4.4 \times 10^{-8}$ , for Yukawa  $Y_v^{\tau\alpha}$  corresponding to 1 to 2 unless very fine cancellations occur among all contributions to the process  $\text{BR}(\tau \rightarrow \mu + \gamma)$  and the clockwork gears must be greater than  $\sim 90$  TeV for Yukawa  $Y_v^{\tau\alpha}$  corresponding to 2 to 7.67 as constrained by experimental limit.

The spontaneous symmetry breaking is of the form  $U(1)^{N+1} \rightarrow U(1)$

$$L = - \sum_{j=0}^N \frac{1}{4} F_{\alpha\beta}^j F^{j\alpha\beta} - \sum_{j=0}^{N-1} \frac{g^2 f^2}{2} \left( A_\alpha^j - q A_\alpha^{j+1} \right)^2. \quad (14)$$

The above implicit Lagrangian is below the scale  $\lambda$  while working in the unitary gauge field. The mass matrix is the same as of the clockwork form, where the heavy gauge bosons are the photon gear  $A_\alpha$  and one photon remains massless. The clockwork photon has important implications. When matter gets charged only under the Abelian factor identical to the last site of the clockwork gear, then the clockwork phenomenon will produce exponentially small couplings to the massless photon which generates visible particles with millicharges. Heavy photon gears could be attainable in the next run of collider searches.

## 5. CLOCKWORK GRAVITON

For  $N + 1$  copies of general relativity, there are  $N + 1$  analogous massless gravitons. Each graviton can be described through perturbation of the metric around flat space-time,

$$g_i^{\mu\nu} = \varepsilon_i^{\mu\nu} + \frac{2}{M_i^2} h_i^{\mu\nu}. \quad (15)$$

The clockwork mechanism breaks  $N + 1$  copies of diffeomorphism invariance to one diffeomorphism invariance at the linear level through near-neighbor Pauli-Fierz term attire for massive gravitons.

$$L = - \frac{m^2}{2} \sum_{i=0}^{N-1} \left( \left[ h_i^{\mu\nu} - q h_{i+1}^{\mu\nu} \right] - \left[ \varepsilon_{\mu\nu} \left( h_i^{\mu\nu} - q h_{i+1}^{\mu\nu} \right) \right]^2 \right). \quad (16)$$

The massive Lagrangian in equation (16) is invariant under the gauge symmetry

$$h_i^{\mu\nu} \rightarrow h_i^{\mu\nu} + \frac{1}{q^i} (\delta^\mu A^\nu + \delta^\nu A^\mu), \quad (17)$$

where  $A^\mu$  is a space-time vector. This gauge symmetry imposes the masslessness of the clockwork graviton and is maintained by the clockwork form of the mass terms. The mass matrix is of the clockwork structure, with the massive gravitons being the gears and one remaining as zero mass graviton. If the energy-momentum tensor  $T^{\mu\nu}$  couples only to the last site of the clockwork gear at Planck alike scale  $M_N$ , then the coupling to the zero mass graviton is

$$-\frac{1}{M_N} h_N^{\mu\nu} T_{\mu\nu} \rightarrow -\frac{1}{M_P} \tilde{h}_0^{\mu\nu} T_{\mu\nu}, \quad (18)$$

where  $M_N$  and  $M_P$  are related by  $M_P = \frac{q^N M^N}{N_0}$ . The effective Planck scale  $M_P$ , which realizes the strength of gravity in the low-energy scale of the theory, is exponentially larger than the fundamental gravity scale  $M_N$  amplified by a factor  $q^N$ . This provides the clockwork mechanism as a result of the hierarchy problem where new physics, like quantum gravity, sets near the weak scale, in total resemblance as afforded by Large Extra-Dimensions [41] and the warped extra-dimensional model of Randall-Sundrum [42]. The metric of the clockwork space-time is similar to a 5D metric known as the linear dilaton model which resembles the dual of Little String Theory [43], after compressing additional dimensions. This counts as a signature of hint toward an untrodden connection between string theory and the continuum clockwork theory, as it distinguishes the clockwork solution of the hierarchy nature from the Little String Theory solution.

## 6. CONCLUSION

The clockwork is a significant mechanism for promoting exponentially suppressed interactions within a quantum field theory containing only  $O(1)$  parameters, in natural units, and a definite number of clockwork fields. As a result, one can stir up exponentially large interaction scales, even when no new physics materializes at this high energy scale. Clockwork holds in models of inflation or relaxation to prompt apparent super Planckian field excursions.

We have scrutinized in detail the impact of the Majorana masses in the clockwork sector in the generation of the lepton flavor violation. When the Majorana masses are nonvanishing, the zero mode of the clockwork gear is no longer massless. But, the corresponding Yukawa coupling still in spite of that has the clockwork structure. In this case, small neutrino masses arise from the interaction between the standard seesaw mechanism and the ‘‘clockwork’’ Yukawa couplings and need very large Majorana masses to generate the small neutrino mass scale inferred as concluded from oscillation experiments. The Standard Model leptons couple to the fermions of the clockwork sector with a site-dependent field strength, which prompts lepton flavor violating charged current, neutral current, and Higgs boson interactions. From the nonfindings of the rare leptonic decay scale of the clockwork fermions due to the nonobservation of the rare cLFV decay  $\mu \rightarrow e\gamma$ ,  $\tau \rightarrow \mu\gamma$ ,  $\tau \rightarrow e\gamma$ , our results stir up that the lightest particle of the clockwork sector must

have a mass greater than 10 TeV, 0.2 TeV to 2.5 TeV, and 2 TeV to 40 TeV, respectively if the Yukawa couplings of exponentially suppressed interactions within a quantum field theory contain only  $O(1)$  parameters.

## CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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