Review on Dark Energy Problem and Modified Gravity Theories

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Abstract

We review the issue of dark energy, namely, the late-time cosmic acceleration. There are two representative approaches to resolve it. One is to introduce an unknown energy component, the so-called dark energy. The other is to consider the modification of the gravity theory, i.e., general relativity, on large scales. In addition, we shortly mention our recent related work of an autonomous system analysis for homogeneous and anisotropic Bianchi-I spacetimes in $f(R)$ gravity. It is shown that only for the case of R^2, there exists a stable de Sitter solution (the solution of the Starobinsky inflation).

Keywords: dark enerty, modified gravity theories
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1. INTRODUCTION

According to recent various cosmological observations including Supernovae Ia (SNe Ia) [1], cosmic microwave background (CMB) radiation [2, 3], large-scale structure (LSS) [4], baryon acoustic oscillations (BAO) [5], and weak lensing [6], it has been known that not only at the inflationary stage in the early universe [7, 8, 9, 10] but also at the present (late) time, the expansion of the universe is accelerating.

There have been proposed two representative approaches to explain the late-time cosmic acceleration. The first is to introuce an unknown energy component, the so-called dark energy, such as the cosmological constant, in the framework of general relativity. The second is to modifiy (extend) the gravitional theory itself from general relativity in the large scale. There are a number of modified gravity theories. As one of the simplest theory, $f(R)$ gravity is known. Here, $f(R)$ is an arbitrary function of the scalar curvature R. In addition, cosmolgical fluid escriptions have been proposed as a possible mechanism of the cosmic acceleration. There have been written various reviews in terms of the issue of dark energy and modified gravity theories account for the mechanism of the late-time cosmic acceleration, for example, [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]).

The article is organized as follows. In Section 2, we explain scalar field models for dark energy. In Section 3, we discuss modified gravity theories. Among these various theories, as one of the simplest modified gravity theories, in Section 4, we describe $f(R)$ gravity. In Section 5, as our recent work in terms of modified gravity theories, we briefly introduce an autonomous system analysis by using new expansion-normalized variables for homogeneous and anisotropic Bianchi-I spacetimes in $f(R)$ gravity in the presence of anisotropic matter. Finally, we summarize our considerations in Section 6. Here, we use units of $k_{\mathrm{B}} = c = \hbar = 1$ and represent the gravitational constant $8\pi G$ as $\kappa^2 \equiv 8\pi/{M_{\mathrm{Pl}}}^2$ with the Planck mass of $M_{\mathrm{Pl}} = G^{-1/2} = 1.2 \times 10^{19}$ GeV.

2. SCALAR FIELD MODELS FOR DARK ENERGY

There are several approaches to account for the origin of the energy component of dark energy within the framework of general relativity: (i) cosmological constant Λ (for instance, [27]); (ii) scalar field models such as X matter [28], Quintessence [29] (reference [30] can be regarded as a pioneering work), Phantom field with a wrong sign kinetic term [31], K-essence with non canonical kinetic term [32, 33], and Tachyon field predicted by string theories [34]; and (iii) cosmic fluids including Chaplygin gas [35, 36] with the pressure times the energy density being a negative constant and viscous fluids [37, 38]. Furthermore, holographic dark energy models have been proposed [39, 40, 41, 42].

In this section, we explain scalar field models for dark energy in general relativity. The action of scalar field theories in general relativity is expressed as

$$S = \int d^4x \sqrt{-g} \left(\frac{R}{2\kappa^2} - \frac{1}{2}\omega(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right), \qquad (1)$$

where R is the Ricci scalar, g is the determinant of the metric tensor $g_{\mu\nu}$, $\omega(\phi)$ is a function of a scalar field ϕ, and $V(\phi)$ is the potential of ϕ.

For $\omega(\phi) = 0$ and $V(\phi) = \Lambda/\kappa^2$ with Λ being a cosmological constant, this action describes the Λ cold dark matter (CDM) model. Moreover, for $\omega(\phi) = +1$, this action corresponds to a quintessence model with a canonical kinetic term, while for $\omega(\phi) = -1$, this action denotes a phantom model.

We consider the case in which the scalar field ϕ is a spatially homogeneous one; i.e., it depends only on time t.

In the following, we consider the 4-dimensional spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) metric describing the homogeneous and isotropic universe:

$$ds^2 = -dt^2 + a^2(t) \sum_{i=1,2,3} \left(dx^i \right)^2, \qquad (2)$$

where $a(t)$ is the scale factor.

In the FLRW background (2), the Friedmann equations are given by

$$H^2 = \frac{\kappa^2}{3}\rho_\phi, \qquad (3)$$

$$\dot{H} = -\frac{\kappa^2}{2}\left(\rho_\phi + P_\phi\right), \qquad (4)$$

where $H \equiv \dot{a}/a$ is the Hubble paramer and the dot denotes the derivative with respect to the cosmic time t. Furthermore, ρ_ϕ and P_ϕ are the energy density and pressure of the scalar field ϕ, respectively, given by

$$\rho_\phi = \frac{1}{2}\omega(\phi)\dot{\phi}^2 + V(\phi), \tag{5}$$

$$P_\phi = \frac{1}{2}\omega(\phi)\dot{\phi}^2 - V(\phi). \tag{6}$$

The equation of $a(t)$ is represented as

$$\ddot{a}/a = -\left(\kappa^2/6\right)(1+3w)\rho_\phi, \tag{7}$$

where $w \equiv P/\rho$ is the equation of state (EoS), defined by the ratio of the pressure P to the energy density ρ. To realize the acceleated expansiton, namely, $\ddot{a} > 0$, we need the condition of $w < -1/3$. According to the Planck data [43], the value of the EoS parameter of dark energy at the present time is $w = 1.03 \pm 0.03$. This means that the current cosmic expansion is accelerating.

3. MODIFIED GRAVITY THEORIES

In this section, we discuss modified gravity theories. There are a number of proposals for the modifications of gravity theory from general relativity on large scales to account for the late-time cosmic acceleration as well as inflation in the early universe. We raise representative modified gravity theories: $f(R)$ gravity, where $f(R)$ is an appropriate function of the Ricci scalar R [44, 45, 46, 47] (for applications to inflationary cosmology, see [10, 48, 49]); scalar-tensor theories with explicit coupling between a function of scalar fields and R [50, 51] such as the Brans-Dicke theories [52]; the ghost condensate scenario [53]; theories with a higher-order curvature term, e.g., the Gauss-Bonnet term $\mathcal{G}$ with a coupling to a scalar field [54]; $f(\mathcal{G})$ gravity with an arbitrary function of $\mathcal{G}$ [55]; the DGP (Dvali-Gabadadze-Porrati) braneworld scenario [56, 57]; $f(T)$ gravity, where $f(T)$ is an extended teleparallel Lagrangian density described by the torsion scalar T [58, 59]; in teleparallelism (teleparallel gravity), one could use the Weitzenböck connection, which has no curvature but torsion, rather than the curvature defined by the Levi-Civita connection [60, 61]; Galileon gravity [62, 63], in which the equations of motion are invariant under the Galilean shift and therefore the equations of motion can be kept up to the second order; this property is welcome to avoid the appearance of an extra degree of freedom associated with ghosts; Horndeski theory [64, 65], which corresponds to the generalization of Galileon gravity; Degenerate Higher-Order Scalar-Tensor (DHOST) theories [26]; non-local gravity [66]; Hořava-Liftshiz gravity [67]; massive gravity [68, 69, 70, 71]; bigravity [72, 73]; extended Proca-Nuevo theory [74].

4. $F(R)$ GRAVITY

In this section, as one of the most simplest modified gravity theories, we describe $f(R)$ gravity (which can also be regarded as a kind of scalar-tensor theories).

The action describing $f(R)$ gravity with matter is given by

$$S = \int d^4x\sqrt{-g}\frac{f(R)}{2\kappa^2} + \int d^4x\mathcal{L}_{\mathrm{M}}, \tag{8}$$

where $\mathcal{L}_{\mathrm{M}}$ is the Lagrangian of matter.

The viability conditions for $f(R)$ gravity have been explored:

(a) the positivity of the effective gravitational coupling,

(b) the stability of cosmological perturbations [46, 75],

(c) the asymptotic behavior to the standard ΛCDM model in the large curvature regime,

(d) the stability of the late-time de Sitter point [76],

(e) the constraints from the equivalence principle,

(f) the solar-system constraints [77].

The following four viable models are known: (1) Hu-Sawicki [78], $f_{\mathrm{HS}} \equiv R - [c_1 R_{\mathrm{HS}}(R/R_{\mathrm{HS}})^p]/[c_2(R/R_{\mathrm{HS}})^p + 1]$, where c_1, c_2 $p(>0)$, and $R_{\mathrm{HS}}(>0)$ are constant parameters (for an extended model, see [79, 80]); (2) Starobinsky [81], $f_{\mathrm{S}} \equiv R + \lambda R_{\mathrm{S}}[(1 + R^2/R_{\mathrm{S}}^2)^{-n} - 1]$ with $\lambda(>0)$, $n(>0)$, and R_{S} being constant parameters; (3) Tsujikawa [82], $f_{\mathrm{T}} \equiv R - \mu R_{\mathrm{T}} \tanh(R/R_{\mathrm{T}})$ with $\mu(>0)$, and $R_{\mathrm{T}}(>0)$ being constant parameters; and (4) Exponential gravity [83, 84, 85], $f_{\mathrm{E}} \equiv R - \beta R_{\mathrm{E}}[1 - \exp(-R/R_{\mathrm{E}})]$, where β and R_{E} are constant parameters. It is mentioned that the crossing of the phantom divide can be realized in the above viable $f(R)$ models on the past and the future [86]. The crossings of the phantom divide have also been reconstructed analytically [87] and numerically [88].

5. AN IMPLICATION TO R^2 GRAVITY

In this section, we briefly introduce an autonomous system analysis by using new expansion-normalized variables for homogeneous and anisotropic Bianchi-I spacetimes in $f(R)$ gravity in the presence of anisotropic matter.

In our recent work [89], we extend the so-called expansion-normalized variables to write down the dynamical equations of $f(R)$ gravity for a homogeneous and anisotropic Bianchi-I metric in the presence of an anisotropic fluid, as a 5-dimensional system of ordinary differential equations. We show that some further assumptions may lead to considerable simplifications in the equations, and for several examples, we end up with analytically soluble systems. For the sake of illustration, we consider explicitly the case of $f(R) = R^{1+\delta}$. We demonstrate that the formulation of [90, 91] is recovered in the isotropic matter limit. Moreover, in a simpler and more direct way, we rederive some uniqueness and stability properties of Starobinsky's isotropic inflationary scenario in R^2 gravity [92, 93, 94], which is consistent with the Planck 2018 results [43, 95].

6. SUMMARY

In the present article, we have reviewed the late-time cosmic acceleration, namely, dark energy problem, and reviewed candidates for dark energy and modified gravity.

In [89], we have analyzed the cosmological solutions for homogeneous and anisotropic Bianchi-I spacetimes in $f(R)$ gravity under the existence of anisotropic matter. It has been demonstrated that Einstein's equations are reduced to an autonomous 5-dimensional system of ordinary differential equations for new variables. By making the autonomous system analysis of the vacuum solutions for the power-law forms of $f(R)$, we have shown that the dynamics can be solved exactly, and that only for the case of R^2, there exists a stable de Sitter solution (the solution of the Starobinsky inflation).

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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References

[1] S. Perlmutter et al. [SNCP Collaboration], Astrophys. J. **517**, 565 (1999) [arXiv:astro-ph/9812133]; A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. **116**, 1009 (1998) [arXiv:astro-ph/9805201].
[2] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. **148**, 175 (2003) [arXiv:astro-ph/0302209]; Astrophys. J. Suppl. **170**, 377 (2007) [arXiv:astro-ph/0603449]; E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. **180**, 330 (2009) [arXiv:0803.0547 [astro-ph]].
[3] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. **192**, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].
[4] M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D **69**, 103501 (2004) [arXiv:astro-ph/0310723]; U. Seljak et al. [SDSS Collaboration], Phys. Rev. D **71**, 103515 (2005) [arXiv:astro-ph/0407372].
[5] D. J. Eisenstein et al. [SDSS Collaboration], Astrophys. J. **633**, 560 (2005) [arXiv:astro-ph/0501171].
[6] B. Jain and A. Taylor, Phys. Rev. Lett. **91**, 141302 (2003) [arXiv:astro-ph/0306046].
[7] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[8] K. Sato, Mon. Not. Roy. Astron. Soc. **195**, 467–479 (1981) NORDITA-80-29.
[9] A. D. Linde, Phys. Lett. **108B**, 389 (1982).
[10] A. A. Starobinsky, Phys. Lett. **91B**, 99 (1980).
[11] T. Padmanabhan, Gen. Rel. Grav. **40**, (2008) 529–564 [arXiv:0705.2533 [gr-qc]].
[12] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D **15**, (2006) 1753–1936 [arXiv:hep-th/0603057 [hep-th]].
[13] R. Durrer and R. Maartens, Gen. Rel. Grav. **40**, (2008) 301–328 [arXiv:0711.0077 [astro-ph]].
[14] S. Capozziello and V. Faraoni, *Beyond Einstein Gravity* (Springer, 2010).
[15] T. Clifton, P. G. Ferreira, A. Padilla and C. Skordis, Phys. Rept. **513**, (2012) 1–189 [arXiv:1106.2476 [astro-ph.CO]].
[16] K. Bamba, S. Capozziello, S. Nojiri and S. D. Odintsov, Astrophys. Space Sci. **342**, (2012) 155–228 [arXiv:1205.3421 [gr-qc]].
[17] S. Capozziello and M. De Laurentis, Phys. Rept. **509**, (2011) 167–321 [arXiv:1108.6266 [gr-qc]].
[18] S. Nojiri and S. D. Odintsov, Phys. Rept. **505**, (2011) 59–144 [arXiv:1011.0544 [gr-qc]].
[19] S. Nojiri, S. D. Odintsov and V. K. Oikonomou, Phys. Rept. **692**, (2017) 1–104 [arXiv:1705.11098 [gr-qc]].
[20] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. **82**, (2010) 451–497 [arXiv:0805.1726 [gr-qc]].
[21] A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, (2010) 3 [arXiv:1002.4928 [gr-qc]].
[22] A. Joyce, B. Jain, J. Khoury and M. Trodden, Phys. Rept. **568**, 1–98 (2015) [arXiv:1407.0059 [astro-ph.CO]].
[23] K. Bamba and S. D. Odintsov, Symmetry **7**, 220–240 (2015) [arXiv:1503.00442 [hep-th]].
[24] Y. F. Cai, S. Capozziello, M. De Laurentis and E. N. Saridakis, Rept. Prog. Phys. **79**, no.10, 106901 (2016) [arXiv:1511.07586 [gr-qc]].
[25] R. Kase and S. Tsujikawa, Int. J. Mod. Phys. D **28**, no.05, 1942005 (2019) [arXiv:1809.08735 [gr-qc]].
[26] D. Langlois, Int. J. Mod. Phys. D **28**, no.05, 1942006 (2019) [arXiv:1811.06271 [gr-qc]].
[27] S. Weinberg, Rev. Mod. Phys. **61** (1989) 1.
[28] T. Chiba, N. Sugiyama and T. Nakamura, Mon. Not. Roy. Astron. Soc. **289**, L5 (1997) [astro-ph/9704199].
[29] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998) [astro-ph/9708069].
[30] Y. Fujii, Phys. Rev. D **26**, 2580 (1982).
[31] R. R. Caldwell, Phys. Lett. B **545**, 23 (2002) [astro-ph/9908168].
[32] T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D **62**, 023511 (2000) [arXiv:astro-ph/9912463 [astro-ph]].
[33] C. Armendariz-Picon, V. F. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. **85**, 4438–4441 (2000) [arXiv:astro-ph/0004134 [astro-ph]].
[34] T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002) [hep-th/0204150].
[35] A. Y. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B **511**, 265 (2001) [arXiv:gr-qc/0103004].
[36] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. D **66**, 043507 (2002) [arXiv:gr-qc/0202064].
[37] I. H. Brevik and O. Gorbunova, Gen. Rel. Grav. **37**, 2039–2045 (2005) doi:10.1007/s10714-005-0178-9 [arXiv:gr-qc/0504001 [gr-qc]].
[38] I. Brevik, Ø. Grøn, J. de Haro, S. D. Odintsov and E. N. Saridakis, Int. J. Mod. Phys. D **26**, no.14, 1730024 (2017) doi:10.1142/S0218271817300245 [arXiv:1706.02543 [gr-qc]].
[39] M. Li, Phys. Lett. B **603** (2004) 1 [arXiv:hep-th/0403127].
[40] E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D **71** (2005) 103504 [arXiv:hep-th/0502082].
[41] M. Tavayef, A. Sheykhi, K. Bamba and H. Moradpour, Phys. Lett. B **781**, 195–200 (2018) [arXiv:1804.02983 [gr-qc]].
[42] E. N. Saridakis, K. Bamba, R. Myrzakulov and F. K. Anagnostopoulos, JCAP **12**, 012 (2018) [arXiv:1806.01301 [gr-qc]].
[43] N. Aghanim et al. [Planck], Astron. Astrophys. **641**, A6 (2020) [erratum: Astron. Astrophys. **652**, C4 (2021)] [arXiv:1807.06209 [astro-ph.CO]].
[44] S. Capozziello, Int. J. Mod. Phys. D **11**, 483–492 (2002) doi:10.1142/S0218271802002025 [arXiv:gr-qc/0201033 [gr-qc]].
[45] S. Capozziello, S. Carloni and A. Troisi, Recent Res. Dev. Astron. Astrophys. **1**, 625 (2003) [arXiv:astro-ph/0303041

[astro-ph]].
[46] S. Nojiri and S. D. Odintsov, Phys. Rev. D **68**, 123512 (2003). [arXiv:hep-th/0307288].
[47] S. M. Carroll, V. Duvvuri, M. Trodden and M. S. Turner, Phys. Rev. D **70**, 043528 (2004) [arXiv:astro-ph/0306438 [astro-ph]].
[48] M. He, A. A. Starobinsky and J. Yokoyama, JCAP **05**, 064 (2018) [arXiv:1804.00409 [astro-ph.CO]].
[49] A. S. Koshelev, K. S. Kumar and A. A. Starobinsky, [arXiv:2209.02515 [hep-th]].
[50] B. Boisseau, G. Esposito-Farese, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. **85**, 2236 (2000) [arXiv:gr-qc/0001066 [gr-qc]].
[51] R. Gannouji, D. Polarski, A. Ranquet and A. A. Starobinsky, JCAP **09**, 016 (2006) [arXiv:astro-ph/0606287 [astro-ph]].
[52] C. Brans and R. H. Dicke, Phys. Rev. **124**, 925–935 (1961).
[53] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, JHEP **05**, 074 (2004) [arXiv:hep-th/0312099 [hep-th]].
[54] S. Nojiri, S. D. Odintsov and M. Sasaki, Phys. Rev. D **71**, 123509 (2005) [arXiv:hep-th/0504052 [hep-th]].
[55] S. Nojiri and S. D. Odintsov, Phys. Lett. B **631**, 1–6 (2005) [arXiv:hep-th/0508049 [hep-th]].
[56] G. R. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B **485**, 208–214 (2000) [arXiv:hep-th/0005016 [hep-th]].
[57] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002) [arXiv:astro-ph/0105068 [astro-ph]].
[58] G. R. Bengochea and R. Ferraro, Phys. Rev. D **79**, 124019 (2009) [arXiv:0812.1205 [astro-ph]].
[59] E. V. Linder, Phys. Rev. D **81**, 127301 (2010) [Erratum-ibid. D **82**, 109902 (2010)] [arXiv:1005.3039 [astro-ph.CO]].
[60] F. W. Hehl, P. Von Der Heyde, G. D. Kerlick and J. M. Nester, Rev. Mod. Phys. **48**, 393 (1976).
[61] K. Hayashi and T. Shirafuji, Phys. Rev. D **19**, 3524 (1979) [Addendum-ibid. D **24**, 3312 (1982)].
[62] A. Nicolis, R. Rattazzi and E. Trincherini, Phys. Rev. D **79**, 064036 (2009) [arXiv:0811.2197 [hep-th]].
[63] S. Tsujikawa, Lect. Notes Phys. **800**, 99–145 (2010) [arXiv:1101.0191 [gr-qc]].
[64] G. W. Horndeski, Int. J. Theor. Phys. **10**, 363–384 (1974).
[65] T. Kobayashi, M. Yamaguchi and J. Yokoyama, Prog. Theor. Phys. **126**, 511–529 (2011) [arXiv:1105.5723 [hep-th]].
[66] S. Deser and R. P. Woodard, Phys. Rev. Lett. **99**, 111301 (2007) [arXiv:0706.2151 [astro-ph]].
[67] P. Horava, Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].
[68] H. van Dam and M. J. G. Veltman, Nucl. Phys. B **22**, 397–411 (1970).
[69] V. I. Zakharov, JETP Lett. **12**, 312 (1970).
[70] C. de Rham and G. Gabadadze, Phys. Rev. D **82**, 044020 (2010) [arXiv:1007.0443 [hep-th]].
[71] C. de Rham, G. Gabadadze and A. J. Tolley, Phys. Rev. Lett. **106**, 231101 (2011) [arXiv:1011.1232 [hep-th]].
[72] S. F. Hassan and R. A. Rosen, Phys. Rev. Lett. **108**, 041101 (2012) [arXiv:1106.3344 [hep-th]].
[73] S. F. Hassan and R. A. Rosen, JHEP **02**, 126 (2012) [arXiv:1109.3515 [hep-th]].
[74] C. de Rham, S. Garcia-Saenz, L. Heisenberg and V. Pozsgay, JCAP **03**, 053 (2022) [arXiv:2110.14327 [hep-th]].
[75] A. D. Dolgov and M. Kawasaki, Phys. Lett. B **573**, 1 (2003).
[76] V. Muller, H. J. Schmidt and A. A. Starobinsky, Phys. Lett. B **202**, 198 (1988).
[77] T. Chiba, Phys. Lett. B **575**, 1 (2003) [arXiv:astro-ph/0307338].
[78] W. Hu and I. Sawicki, Phys. Rev. D **76**, 064004 (2007).
[79] S. Nojiri and S. D. Odintsov, Phys. Lett. B **657**, 238 (2007) [arXiv:0707.1941 [hep-th]].
[80] S. Nojiri and S. D. Odintsov, Phys. Rev. D **77**, 026007 (2008) [arXiv:0710.1738 [hep-th]].
[81] A. A. Starobinsky, JETP Lett. **86**, 157 (2007).
[82] S. Tsujikawa, Phys. Rev. D **77**, 023507 (2008).
[83] G. Cognola, E. Elizalde, S. Nojiri, S. D. Odintsov, L. Sebastiani and S. Zerbini, Phys. Rev. D **77**, 046009 (2008) [arXiv:0712.4017 [hep-th]].
[84] E. V. Linder, Phys. Rev. D **80**, 123528 (2009) [arXiv:0905.2962 [astro-ph.CO]].
[85] K. Bamba, C. Q. Geng and C. C. Lee, JCAP **1008**, 021 (2010) arXiv:1005.4574 [astro-ph.CO].
[86] K. Bamba, C. Q. Geng and C. C. Lee, JCAP **11**, 001 (2010) [arXiv:1007.0482 [astro-ph.CO]].
[87] K. Bamba, C. Q. Geng, S. Nojiri and S. D. Odintsov, Phys. Rev. D **79**, 083014 (2009) [arXiv:0810.4296 [hep-th]].
[88] K. Bamba and C. Q. Geng, Prog. Theor. Phys. **122**, 1267 (2009) [arXiv:0909.1249 [astro-ph.CO]].
[89] S. Chakraborty, K. Bamba and A. Saa, Phys. Rev. D **99**, 064048 (2019) [arXiv:1805.03237 [gr-qc]].
[90] J. A. Leach, S. Carloni and P. K. S. Dunsby, Class. Quant. Grav. **23**, 4915 (2006) [gr-qc/0603012].
[91] N. Goheer, J. A. Leach and P. K. S. Dunsby, Class. Quant. Grav. **24**, 5689 (2007) [arXiv:0710.0814].
[92] J.D. Barrow. and A.C. Ottewill, J. Phys. A**16**, 2757 (1983).
[93] K. Maeda, Phys. Rev. D **37**, 858 (1988).
[94] J. D. Barrow and S. Hervik, Phys. Rev. D **74**, 124017 (2006) [gr-qc/0610013].
[95] Y. Akrami et al. [Planck], Astron. Astrophys. **641**, A10 (2020) [arXiv:1807.06211 [astro-ph.CO]].