VISHν: Flavour-Variant DFSZ Axion Model for Inflation, Neutrino Masses, Dark Matter, and Baryogenesis

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Abstract

The standard flavour-blind DFSZ axion model for solving the strong-CP problem has in the past been extended to account for nonzero neutrino masses and baryogenesis-via-leptogenesis through the Type-I seesaw mechanism, in addition to having viable axion dark matter. Constructing a full and viable cosmological history, however, requires dealing with the cosmological domain wall problem posed by standard DFSZ. In this paper, I report on work with A. Sopov where this challenge is addressed through a flavour-variant model called VISHν that removes the domain wall problem and incorporates successful Higgs-Peccei-Quinn scalar inflation. As part of this, we ensure that the required new high-scale physics does not add to the electroweak naturalness problem.

Keywords: strong-CP problem, axion, domain wall, flavour, inflation, naturalness
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1. THE STRONG-CP PROBLEM

QCD permits the term

$$\mathcal{L}_\theta = \bar{\theta}\frac{g^2}{32\pi^2}G^{\mu\nu}\tilde{G}_{\mu\nu} \tag{1}$$

in the Lagrangian, where $\bar{\theta}$ is a dimensionless parameter, g is the strong gauge coupling constant, $G^{\mu\nu}$ is the gluon field-strength tensor, and $\tilde{G}_{\mu\nu}$ is its dual. This term, however, violates both CP and P conservation and induces a neutron electric dipole moment. Experimental constraints on that quantity require that $\bar{\theta} \lesssim 10^{-10}$. The "strong-CP problem" is the mystery of why this parameter must be so small. Note that the $\bar{\theta} \to 0$ limit is not technically natural in the P- and CP-violating standard model (SM).

A solution to this problem is provided through Peccei-Quinn (PQ) axion models, where a $U(1)_{PQ}$ symmetry is imposed at the classical level such that its current has a colour anomaly, $\partial_\mu J^\mu_{PQ} \propto G^{\mu\nu}\tilde{G}_{\mu\nu}$, at the quantum level [1]. The effective Lagrangian now contains an extended $G\tilde{G}$ term of the form

$$\mathcal{L}_\theta = \left(\frac{a(x)}{f_a} + \bar{\theta}\right)\frac{g^2}{32\pi^2}G^{\mu\nu}\tilde{G}_{\mu\nu}, \tag{2}$$

where $a(x)$ is a pseudoscalar field whose quanta are very light particles called "axions" [2, 3]. The axion potential is minimised when $\langle a\rangle = -\bar{\theta}f_a$, thus removing the CP-violating effects.

2. THE INVISIBLE DFSZ AXION AND TECHNICAL NATURALNESS

One way to extend the SM to incorporate a $U(1)_{PQ}$ anomalous symmetry is to utilise two Higgs doublets, the option taken in Dine-Fischler-Srednicki-Zhitnitskii (DFSZ) type models [4, 5]. To make such models phenomenologically viable, the axion must be made "invisible", meaning very weakly coupled to SM particles. This is achieved by introducing a scalar field S that has a nonzero PQ charge but is a singlet under the SM gauge group. When it develops a nonzero vacuum expectation value (VEV) which is much larger than the electroweak (EW) scale, it becomes the dominant source of spontaneous PQ symmetry breaking. This implies that the axion, which is the pseudo-Nambu-Goldstone boson of $U(1)_{PQ}$, has its admixture dominated by the phase field of S which by construction couples very weakly to SM particles. For cosmological reasons (see later), we favour $\langle S\rangle \sim 10^{10\text{-}11}$ GeV.

In the standard DFSZ model, one of the Higgs doublets $\widetilde{\Phi}_1 \equiv i\sigma_2\Phi_1^*$ Yukawa couples to right-handed (RH) up-type quarks, while the second doublet Φ_2 couples to RH down-type quarks. Depending on how one chooses to Yukawa couple these doublets to leptons, one obtains either a Type-II or flipped two-Higgs-doublet model type of structure. The phenomenological advantage of this flavour-blind choice is, of course, that tree-level Higgs-induced flavour-changing interactions are avoided. We will see later, however, that relaxing this requirement permits an elegant solution of the cosmological domain-wall problem of the standard DFSZ model. But for now, let us continue with a review of this model.

The scalar potential is given by

$$\begin{aligned} V = {} & M_{11}^2\Phi_1^\dagger\Phi_1 + M_{22}^2\Phi_2^\dagger\Phi_2 + M_{SS}^2 S^*S \\ & + \frac{\lambda_1}{2}\left(\Phi_1^\dagger\Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger\Phi_2\right)^2 + \frac{\lambda_S}{2}\left(S^*S\right)^2 \\ & + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ & + \lambda_{1S}\left(\Phi_1^\dagger\Phi_1\right)\left(S^*S\right) + \lambda_{2S}\left(\Phi_2^\dagger\Phi_2\right)\left(S^*S\right) \\ & + \begin{cases} \kappa\Phi_1^\dagger\Phi_2 S + \text{h.c. [VISH}\nu] \\ \epsilon\Phi_1^\dagger\Phi_2 S^2 + \text{h.c. [}\nu\text{DFSZ].} \end{cases} \end{aligned} \tag{3}$$

There is a choice for the last term of either a cubic or a quartic nontrivial interaction between the doublets and S, which serves to relate the PQ charges of all three scalar multiplets. As indicated above, for the νDFSZ variant to be reviewed in the next section, the quartic option is exercised, while for the later VISHν implementation, the cubic choice will be necessary.

For the moment, however, let us adopt the quartic case for definiteness. There is a simple but important observation that needs to be made about the phenomenologically necessary VEV hierarchy $v_S \equiv \langle S\rangle \gg v_{1,2} \equiv \langle\Phi_{1,2}\rangle$ and technical naturalness [6]. The critical points of the above potential satisfy

$$\begin{aligned} M_{11}^2 &= -\frac{1}{2}t_1 v_S^2 - \lambda_1 v_1^2 - \frac{1}{2}\left(\lambda_3+\lambda_4\right)v_2^2, \\ M_{22}^2 &= -\frac{1}{2}t_2 v_S^2 - \lambda_2 v_2^2 - \frac{1}{2}\left(\lambda_3+\lambda_4\right)v_1^2, \\ M_{SS}^2 &= -\frac{1}{2}t_1 v_1^2 - \frac{1}{2}t_2 v_2^2 - \lambda_S v_S^2, \end{aligned} \tag{4}$$

where

$$t_1 \equiv \epsilon\frac{v_2}{v_1} + \lambda_{1S}, \quad t_2 \equiv \epsilon\frac{v_1}{v_2} + \lambda_{2S}. \tag{5}$$

To achieve $v_S \sim 10^{10\text{-}11}$ GeV $\gg v_{1,2} \sim 100$ GeV we set $M_{SS} \gg M_{11}, M_{22}$ which then requires

$$t_{1,2} \lesssim \frac{v_{1,2}^2}{v_S^2} \tag{6}$$

which in turn is assured if

$$\lambda_{1S}, \lambda_{2S}, \epsilon \ll 1. \tag{7}$$

The really nice thing is that this parameter region is technically natural, for an interesting reason. In the limit that $\lambda_{1S,2S}$ and ϵ vanish, S is decoupled from all the SM fields (it becomes a hidden sector) and the flat-space action is separated into the sum of two independent integrals:

$$S = \int d^4x\, \mathcal{L}_{SM}(x) + \int d^4x'\, \mathcal{L}_S(x') \tag{8}$$

with the SM Lagrangian and the S-Lagrangian as the integrands. This action admits *independent* Poincaré transformations with respect to x and x', thus increasing the symmetry of the theory [6, 7]. Hence, the limit of equation (7) is technically natural.

Some comments about the effects of gravity are now pertinent. Any two sectors that are hidden from each other with respect to nongravitational forces will, of course, interact via gravity. This explicitly breaks the Poincaré-squared enhanced symmetry, and may induce Planck-suppressed contributions to, for example, the $\lambda_{1S,2S}$ terms. We may parameterise the coefficients as cM_{SS}^2/M_P^2 where M_P is the Planck mass. This potential contribution will endanger technical naturalness if $c \gtrsim 0.01$. In addition, there is the usual prospect that Planck-scale effects might directly destabilise the EW scale in the SM itself. What the observations in the preceding paragraphs amount to is that a sector that is hidden from the SM by sufficiently weak nongravitational couplings does not provide an *additional* source of EW-scale destabilisation. That is a very worthwhile feature in my opinion, even if concerns about Planck-scale effects remain.[1]

3. THE νDFSZ MODEL: SUCCESSES AND COSMOLOGICAL CHALLENGES

The successes of the DFSZ model are that it solves the strong-CP problem, and when the PQ breaking scale is $10^{10\text{-}11}$ GeV it also provides a viable axion dark matter candidate. Given that energy scale, an obvious extension is to identify it also with the Type-I seesaw [9] scale and thus explain nonzero neutrino masses as well. To do this, heavy neutral leptons N (also called RH neutrinos ν_R) are added to the DFSZ particle content. As a further nice feature, the out-of-equilibrium and CP-violating decays of those heavy neutral leptons can provide baryogenesis via leptogenesis [10]. All of these possibilities were pointed out in the 1980s [11, 12]. The νDFSZ model [13] is a detailed incarnation of these ideas that also addresses a naturalness problem within standard hierarchical, thermal leptogenesis.

When the heavy neutral lepton masses are hierarchical, successful leptogenesis implies that the lightest of these fermions must have its mass M_{N_1} obey

$$M_{N_1} > 5\times 10^8\text{–}2\times 10^9\,\text{GeV}, \tag{9}$$

where the range is due to different assumptions about the initial abundance of N_1. This is usually called the "Davidson-Ibarra (DI) bound" [14, 15]. The reason for a lower bound can be readily explained. The seesaw formula for the light neutrino masses $m_\nu \sim (\lambda_\nu\langle\Phi\rangle)^2/M_N$ shows that the maintenance of a given neutrino mass scale when M_N is made smaller requires correspondingly smaller values for the neutrino Dirac Yukawa coupling constant λ_ν. But the magnitude of the CP violation that plays into the leptogenesis outcome decreases with decreasing λ_ν and eventually becomes too small. Detailed analyses show that the resulting lower bound on the seesaw scale is equation (9).

But this is in tension with an upper bound, first derived by Vissani, from naturalness [16]. The heavy neutral lepton masses contribute at 1-loop to the self-energy of the Higgs doublet in the SM:

$$\delta\mu^2 = \frac{1}{4\pi^2}\frac{1}{\langle\Phi\rangle^2} m_\nu M_N^3, \tag{10}$$

where μ^2 is the coefficient of $\Phi^\dagger\Phi$ in the Higgs potential. Assuming N_1 dominance, setting $m_\nu \sim 0.05$ eV, and demanding that this contribution be smaller than the nominal figure of 1 TeV^2 implies that

$$M_{N_1} < 3\times 10^7\,\text{GeV}, \tag{11}$$

which is smaller than the DI lower bound. Later work established that this tension cannot be removed by considering the full three-family case involving N_1, N_2, and N_3 [17].

To remove the tension, some possibilities are to go supersymmetric, have nonhierarchical M_N, or utilise at least two Higgs doublets. Since the DFSZ model has two Higgs doublets anyway, we follow this third route [18]. Let Φ_2 be the Higgs doublet that Yukawa couples LH lepton doublets to the singlet fermions N. It is the VEV of that Higgs doublet that determines the DI and Vissani bounds. The point is that these bounds scale differently with v_2, as per

$$\begin{aligned} \text{Vissani bound:} \quad & M_{N_1} < 3\times 10^7\,\text{GeV}\left(\frac{v_2}{246\,\text{GeV}}\right)^{2/3}, \\ \text{DI bound:} \quad & M_{N_1} > 5\times 10^8\,\text{GeV}\left(\frac{v_2}{246\,\text{GeV}}\right)^{2}. \end{aligned} \tag{12}$$

The Vissani upper bound is above the DI lower bound for $v_2 \lesssim 30$ GeV, as illustrated by the blue and purple regions in Figure 1.

[1] We note also that Planck-suppressed effects may explicitly break $U(1)_{PQ}$ leading to the "axion quality problem" [8].

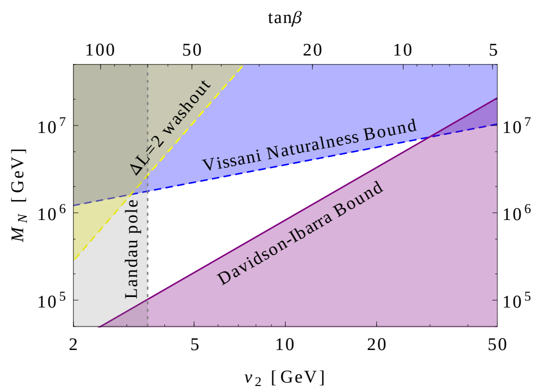

FIGURE 1: The allowed parameter space (white) of the νDFSZ model in the v_2-M_N plane. In addition to the DI and Vissani bounds, the constraint from $\Delta L = 2$ asymmetry washout processes is indicated, as well as the region excluded because of the existence of a low-scale Landau pole. Figure from [13].

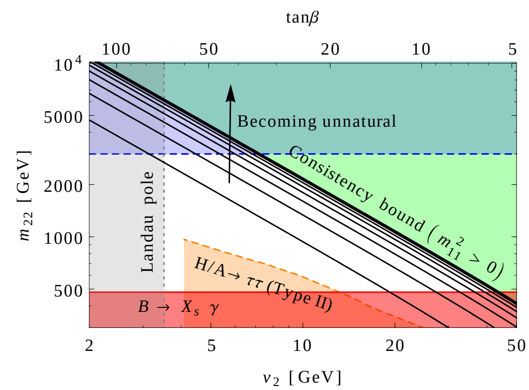

FIGURE 2: The allowed parameter region of the νDFSZ model in the v_2-M_{22} plane. The white region is allowed by collider constraints, naturalness, and the absence of a Landau pole. Figure from [13].

The PQ charges of the fermion and scalar fields in the νDFSZ model are

$$\begin{gathered} q_L \sim 0, \quad u_R \sim \cos^2\beta, \quad d_R \sim \sin^2\beta, \\ \ell_L \sim \tfrac{3}{4} - \cos^2\beta, \quad \nu_R \sim -\tfrac{1}{4}, \quad e_R \sim \left(\tfrac{7}{4} \text{ or } \tfrac{3}{4}\right) - 2\cos^2\beta, \\ \Phi_1 \sim \cos^2\beta, \quad \Phi_2 \sim -\sin^2\beta, \quad S \sim \frac{1}{2}, \end{gathered} \tag{13}$$

where we have renamed N as ν_R and $\tan\beta \equiv \frac{v_1}{v_2}$ with $v_1^2 + v_2^2 \simeq (246\,\text{GeV})^2$. The two possibilities for e_R correspond to the Type-II and Flipped cases, respectively. The seemingly peculiar nature of these charge assignments comes from the requirement to make the PQ current decoupled from the Nambu-Goldstone mode eaten by the Z boson. The Yukawa Lagrangian is

$$\begin{aligned} -\mathcal{L}_Y = {} & y_u \overline{q_L} \widetilde{\Phi}_1 u_R + y_d \overline{q_L} \Phi_2 d_R + y_e \overline{l_L} \Phi_J e_R \\ & + y_\nu \overline{l_L} \widetilde{\Phi}_2 \nu_R + \frac{1}{2} y_N \overline{(\nu_R)^c} S \nu_R + \text{h.c.}, \end{aligned} \tag{14}$$

where $J = 2$ (1) gives a Type-II (Flipped) two-Higgs-doublet model structure. Notice that the PQ scalar S couples to Majorana-type ν_R bilinears, so its large VEV generates large ν_R Majorana masses $M_N = y_N v_S$, implementing a Type-I seesaw mechanism.

Referring to the potential (3), we note that to achieve $v_S \gg v_{1,2}$ we require that M_{SS}^2 is of the order $(10^{10\text{-}11}\,\text{GeV})^2$ and negative, while M_{11}^2 and M_{22}^2 are much smaller. An interesting feature of the νDFSZ model is that its potential can furnish a good reason for why $v_2 \ll v_1$. We choose M_{11}^2 near its SM value of $-(88\,\text{GeV})^2$, but we make M_{22}^2 larger and *positive*, about $(1\,\text{TeV})^2$ or a little larger. On its own, the positive sign for the latter makes v_2 tend towards zero. However, the ϵ term induces a term that is linear in Φ_2 once S and Φ_1 gain nonzero VEVs. This causes v_2 to be nonzero but much smaller than v_1. Implicit is the parameter region of equations (6) and (7) so that technical naturalness holds despite the large hierarchy between the PQ and EW scales.

Figures 1 and 2 illustrate the allowed (white) region of parameter space, the latter depicting the v_2-M_{22} plane. Bounds from collider processes and the absence of a Landau pole and $\Delta L = 2$ asymmetry washout processes are indicated. As M_{22} increases, it too becomes gradually less natural, so our philosophy requires that it be no larger than a few TeV. The successes of the νDFSZ model are (i) solving the strong-CP problem and providing viable axion DM for $v_S \sim 10^{10\text{-}11}$ GeV (inherited from DFSZ), and (ii) nonzero light neutrino masses with successful hierarchical thermal leptogenesis with all the new physics being technically natural.

4. VISHν: TOWARDS MEETING THE COSMOLOGICAL CHALLENGES

However, as foreshadowed above, both the original DFSZ model and its νDFSZ extension pose a cosmological domain wall (DW) problem. This problem has to be solved to produce a variant or variants that have full and viable cosmological histories.

We begin by first reviewing the DW problem, which was pointed out by Sikivie [19]. The picture is that QCD instantons explicitly break the anomalous $U(1)_{PQ}$ symmetry, with a DW problem arising if there is a Z_N subgroup of $U(1)_{PQ}$ that nevertheless remains unbroken at the quantum level. Its eventual dynamical breaking through QCD condensates then causes DWs to form which interpolate between the degenerate and discrete vacua. Sikivie showed that the original DFSZ model has a Z_6 unbroken subgroup. The existence of an unbroken discrete subgroup depends on the size of the colour anomaly, which in turn depends on which coloured fermions contribute and thus the PQ charge assignment.

Among all the ways that have been proposed to remove the DW problem, one stands out for its simplicity: We consider *flavour-dependent* Yukawa interactions in such a way that the colour anomaly/instantons completely break $U(1)_{PQ}$, so there is no unbroken discrete subgroup at the quantum level. This possibility was analysed in some pioneering works in the 1980s [20, 21]. For the sake of simplicity and definiteness, we adopt the most studied of the DW-problem-free DFSZ variants: the "top-specific" model, whose phenomenology has been studied in [22] (see also [23, 24]). In this variant, Higgs doublet Φ_1 is taken to be the only one that couples to the RH top quark, while Φ_2 couples to all other quark and lepton bilinears. As in the νDFSZ model, we arrange for $v_1 \gg v_2$ to obtain, in this new context, a welcome bonus: a rationale for why the top mass is so large.

The top-specific DFSZ model is extended to VISHν (Variant axIon Seesaw Higgs νtrino) [25] by adding three RH neutrinos, Yukawa coupling them to S is the same way as in νDFSZ and for the purpose of inflationary cosmology including nonminimal couplings of the scalar fields to gravity, which is discussed in more detail below. The PQ charge assignment distinguishes the RH top quark u_R^3 from the other two RH charge-2/3 quarks $u_R^{1,2}$, and all other fermions have flavour-blind PQ charges, as per

$$\begin{aligned} &q_L \sim 0, \quad u_R^3 \sim \cos^2\beta, \quad u_R^{1,2} \sim -\sin^2\beta, \quad d_R \sim \sin^2\beta, \\ &\ell_L \sim \tfrac{1}{2} - \cos^2\beta, \quad \nu_R \sim -\tfrac{1}{2}, \quad e_R \sim \tfrac{3}{2} - 2\cos^2\beta, \\ &\Phi_1 \sim \cos^2\beta, \quad \Phi_2 \sim -\sin^2\beta, \quad S \sim 1. \end{aligned} \tag{15}$$

The Yukawa Lagrangian is

$$\begin{aligned} -\mathcal{L}_Y = {} & \overline{q}_L^j y_{u1}^{j3} \widetilde{\Phi}_1 u_R^3 + \overline{q}_L^j y_{u2}^{ja} \widetilde{\Phi}_2 u_R^a + \overline{q}_L^j y_d^{jk} \Phi_2 d_R^k + \overline{\ell}_L^j y_e^{jk} \Phi_2 e_R^k \\ & + \overline{\ell}_L^j y_\nu^{jk} \widetilde{\Phi}_2 \nu_R^k + \frac{1}{2} \overline{\left(\nu_R^j\right)^c} y_N^{jk} S \nu_R^k + \text{h.c.}, \end{aligned} \tag{16}$$

where $a = 1, 2$ and $i, j, k = 1, 2, 3$. The scalar potential is given by equation (3) using the cubic term option with coupling constant κ. Note that this setup features the tree-level Higgs-induced flavour-changing neutral current processes $t \to hc$ and $t \to hu$, where h is the SM-like physical Higgs boson [22].

By construction, VISHν inherits the successes of νDFSZ which, as stated earlier, in turn inherited the successes of standard DFSZ: all up we have a solution to the strong-CP problem, viable axion DM, the generation of small Majorana neutrino masses, and good hierarchical thermal leptogenesis, all with no (nongravitational) naturalness concerns.

But now that the cosmological DW problem has been removed, we may contemplate completing the cosmological history by incorporating a period of inflation. If successful, this would solve the homogeneity and flatness fine-tuning problems of the standard hot big bang while also providing the seeds for large-scale structure formation.

For this, we need an inflaton field. While this could be an additional degree of freedom, we choose to stick with the economical choice of the existing scalar field system of Φ_1, Φ_2, and S and instead employ nonminimal couplings to gravity of the form

$$\frac{\mathcal{L}^{\mathcal{J}}}{\sqrt{-g^{\mathcal{J}}}} \supset \left(\frac{M_P^2}{2} + \xi_1 \Phi_1^\dagger \Phi_1 + \xi_2 \Phi_2^\dagger \Phi_2 + \xi_S S^\dagger S \right) R^{\mathcal{J}}. \tag{17}$$

This approach is in the spirit of "Higgs inflation" [26] and its utilisation in the SMASH extension [27] of the KSVZ axion model [28]. In the above equation, $\mathcal{J}$ denotes the Jordan frame, with $R^{\mathcal{J}}$ being the Ricci scalar and $g^{\mathcal{J}}$ the determinant of the metric tensor in that frame. The dimensionless parameters $\xi_{1,2,S}$ quantify the strengths of the nonminimal interactions between the scalar multiplets and gravity.

It is convenient to adopt modulus and phase decomposition for the complex fields Φ_1^0, Φ_2^0, and S as per

$$\Phi_i^0 = \frac{\rho_i}{\sqrt{2}} e^{i\vartheta_i / v_i}, \quad S = \frac{\sigma}{\sqrt{2}} e^{i\vartheta_S / v_S}. \tag{18}$$

The phase fields do not enter into the nonminimal coupling terms and will thus not admix into the inflaton field.

The utility for inflationary cosmology is revealed by transforming to the Einstein frame $\mathcal{E}$ using a Weyl rescaling of the Jordan-frame metric tensor:

$$\begin{aligned} & g_{\mu\nu}^{\mathcal{J}} \to g_{\mu\nu}^{\mathcal{E}} \\ & = \Omega^2 \left(\rho_1, \rho_2, \sigma\right) g_{\mu\nu}^{\mathcal{J}}, \quad \text{where } \Omega^2 \equiv 1 + \frac{\xi_1 \rho_1^2 + \xi_2 \rho_2^2 + \xi_S \sigma^2}{M_P^2}, \end{aligned} \tag{19}$$

which restores minimal coupling to gravity and flattens the scalar potential at large values of the modulus fields, at the cost of introducing noncanonical kinetic terms. These features are described by the Einstein frame Lagrangian:

$$\frac{\mathcal{L}^{\mathcal{E}}}{\sqrt{-g^{\mathcal{E}}}} \supset \frac{M_P^2}{2} R^{\mathcal{E}} - \frac{1}{2} \sum_{I,J} \mathcal{G}_{IJ}^{\mathcal{E}} \partial_\mu \varphi^I \partial^\mu \varphi^J - V^{\mathcal{E}} \left(\varphi^I\right), \tag{20}$$

where $\varphi^I = (\rho_1, \rho_2, \sigma)$ and $\mathcal{G}_{IJ}^{\mathcal{E}}$ is the nontrivial metric induced by equation (19) on the scalar field space. It is

$$\mathcal{G}_{IJ}^{\mathcal{E}} = \frac{\delta_{IJ}}{\Omega^2} + \frac{3M_P^2}{2} \frac{\partial \ln \Omega^2}{\partial \varphi^I} \frac{\partial \ln \Omega^2}{\partial \varphi^J}. \tag{21}$$

In the large modulus regime ($\Omega^2 \gg 1$), the Einstein frame scalar potential is given by

$$\begin{aligned} V^{\mathcal{E}}(\varphi^I) &= \Omega^{-4}\left(\varphi^I\right) V^{\mathcal{J}}\left(\varphi^I\right) \\ &= \frac{M_P^4}{8} \frac{\lambda_i \rho_i^4 + 2\lambda_{34} \rho_1^2 \rho_2^2 + 2\lambda_{iS} \rho_i^2 \sigma^2 + \lambda_S \sigma^4}{\left(\xi_i \rho_i^2 + \xi_S \sigma^2\right)^2} \\ &\quad \times \left[1 - \mathcal{O}\left(\frac{M_P^2}{\xi_i \rho_i^2 + \xi_S \sigma^2}\right)\right]^2, \end{aligned} \tag{22}$$

where $\lambda_{34} = \lambda_3 + \lambda_4$ and summing over i is implied. The flatness of this potential, a necessary condition for successful inflation, is obvious.

The analysis of the inflationary dynamics begins through the identification of the valleys of equation (22) in the large modulus regime. These valleys act as attractors for a broad range of initial trajectories, thus rendering the dynamics effectively single-field [29, 30]. Before taking account of the parameter space adopted in the VISHν model, one may derive seven such valleys, and thus seven *a priori* possible single-field inflaton directions. The field-space trajectories are in the following general directions:

$$\rho_1 \text{ only}, \rho_2 \text{ only}, \sigma \text{ only}, \quad \text{in the } (\rho_1, \rho_2) \text{ plane}, \tag{23}$$

$$\text{in the } (\rho_1, \sigma) \text{ plane}, \quad \text{in the } (\rho_2, \sigma) \text{ plane}, \tag{24}$$

$$\text{a general direction in } (\rho_1, \rho_2, \sigma) \text{ space}. \tag{25}$$

Each of these valleys exists in certain regions of parameter space. However, it turns out that for the specific parameter space needed in VISHν only the three trajectories of the type specified by equations (24) and (25) produce single-field inflation that is both amenable to analysis and in obvious agreement with the data. In addition to them being inevitable for a large region of parameter space, we are also interested in the single-field scenarios because then there are no concerns over large isocurvature fluctuations and non-Gaussianities. This is not to say that all of the multifield possibilities are ruled out, but considerably more work would be needed to analyse those cases.

In general, the analysis of inflation in this model is complicated because of the nondiagonal 3×3 field-space metric in equations (20) and (21). However, there are relevant regimes where this complicated metric simplifies into either a diagonal form or a block diagonal form comprising a nontrivial 2×2 block only. One regime where the analysis simplifies is at large values of $\xi_{1,2,S}$, a parameter space that is analogous to that of the original Higgs inflation model. The other is where only one of the ξ parameters is significantly different from zero, in which case it need not be large; it may be of order one or smaller [25].

The large-ξ regime has been the subject of concern in regard to the breakdown of unitarity. For self-consistency, unitarity should be preserved at least until the energy scale of inflation, so that the calculations can be trusted. It has been argued that initial claims about the failure of unitarity in Higgs inflation [31, 32] were overstated, because one has to take into account the field-dependence of the unitarity violation scale [33]. We have nothing to add to this debate and shall provisionally assume that there is in fact no real concern. Nevertheless, it would be reassuring if small values of the ξ parameters were permitted, about which there will be further discussion presently.

A detailed analysis produces the following results [25]:

(1) In the large-ξ regime, successful single-field inflation can occur along three trajectories: within the (ρ_1, σ) plane, within the (ρ_2, σ) plane, and along a general direction in (ρ_1, ρ_2, σ) space, as stated above.

(2) In the large-ξ regime, trajectories in the (ρ_1, ρ_2) plane, and along each of the individual axes ρ_1, ρ_2, and σ, feature complications due to the naturalness requirement that $\lambda_{iS} \ll 0$. This parameter space restriction causes the critical trajectories to be either on local maxima (ridges) or saddle points, not valleys, for the generic case of three large-ξ parameters. Setting one or two of the ξ to zero can produce shallow valleys, but then fluctuations orthogonal to the world-be single-field inflaton trajectories have effective masses that are not larger than the Hubble scale. Thus, the evolution is not purely single-field, and further work would be needed to see if there was an acceptable parameter space that was compatible with constraints on isocurvature fluctuations and non-Gaussianities.

(3) In the small-ξ regime where $\xi_s \lesssim 1$ and $\xi_{1,2} \simeq 0$, the (ρ_1, σ), (ρ_2, σ), and (ρ_1, ρ_2, σ) trajectories degenerate into a direction that is very close to being purely along the σ-axis. When the naturalness stipulations $\lambda_{iS} < 10^{-16,-18}$ are imposed, then there may be concerns that orthogonal fluctuations are not sufficiently damped to avoid isocurvature and non-Gaussianity bounds, a topic that requires further analysis. Of course, with $\xi_{1,2} \simeq 0$, there is no possibility of inflation along any EW Higgs direction.

(4) The small-ξ regimes where either ξ_1 or ξ_2 is nonzero and the other two ξ parameters effectively vanish are incompatible with naturalness. It turns out that $\xi_i \neq 0$ requires $\lambda_i < 10^{-8}$ at the inflation scale, which is unnatural.

So, the parameter space that clearly works involves large values of the ξ nonminimal coupling constants. However, viable small ξ inflation is not yet ruled out, because the cases where $\xi_1 \sim \xi_2 \sim \xi_S$ or one ξ parameter is effectively zero and the other two are of similar magnitudes have not been analysed due to the nontrivial kinetic-mixing complication.

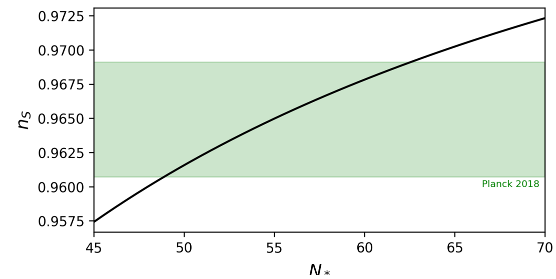

FIGURE 3: The scalar spectral index n_s as a function of the number of e-folds N_*. The green band is the allowed range for n_s. A good fit is obtained for e-folds in the 50–60 range. Figure from [25].

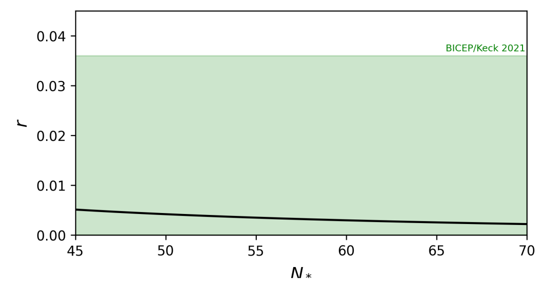

FIGURE 4: The tensor-to-scalar ratio r as a function of the number of e-folds. The upper bound given by the top of the green band is easily accommodated. Figure from [25].

To fit the three single-field large-ξ possibilities to the cosmological data, we first note that the amplitude of scalar perturbations at the horizon exit is set by the magnitude of the Einstein frame potential equation (22) along each trajectory. For (ρ_i, σ) inflation, the relevant quantity has been shown to be [25]

$$\frac{\lambda_{\text{eff}}}{\xi_{\text{eff}}^2} = \frac{\lambda_S \lambda_i}{\lambda_S \xi_i^2 + \lambda_i \xi_S^2}, \tag{26}$$

while for (ρ_1, ρ_2, σ) inflation, it is

$$\frac{\lambda_{\text{eff}}}{\xi_{\text{eff}}^2} = \frac{\lambda_S L}{\lambda_S \left(\lambda_2 \xi_1^2 - 2\lambda_{34}\xi_1\xi_2 + \lambda_1 \xi_2^2\right) + \xi_S^2 L}, \tag{27}$$

where $\lambda_{34} \equiv \lambda_3 + \lambda_4$ and $L \equiv \lambda_1 \lambda_2 - \lambda_{34}^2$. Observationally, we know that

$$\frac{\lambda_{\text{eff}}}{\xi_{\text{eff}}^2} \simeq 8.9 \times 10^{-10}, \tag{28}$$

which can be easily fitted in the allowed parameter space of the theory for all three cases. The scalar spectral index $n_s \simeq 1 - 2/N_*$ and the tensor-to-scalar ratio $r \simeq 12/N_*^2$ are shown in Figures 3 and 4, respectively, as a function of the number of e-folds of inflation N_*. As standard for Higgs-inflation scenarios, very good fits are obtained for N_* in the range of 50–60.

We now turn to predictions for axion DM. The details depend on whether or not the PQ symmetry is restored either during preheating or through reheating. These aspects of VISHν have yet to be quantitatively analysed. The nicest situation arises when PQ symmetry is restored after inflation, because then the axion DM density does not depend on the misalignment angle. Assuming PQ restoration, and incorporating the results of simulations on the contribution of unstable defect networks to axion density [34, 35, 36], we conclude that the observed DM mass density will be reproduced for an axion mass m_A somewhere in the range [25]

$$m_A \sim (40\text{–}500)\,\mu\text{eV} \tag{29}$$

which corresponds to an axion decay constant (PQ breaking scale) in the range 1.1×10^{10}–1.4×10^{11} GeV. The existence of a range of masses is due to uncertainties in the cosmological simulations. Bounds from energy loss constraints for red giant stars [37, 38] provide an upper limit on m_A, which is about 2 meV in the case of VISHν. This figure uses $\tan\beta \gtrsim 8$ from the requirement that leptogenesis avoids the tension between the DI and Vissani bounds. The mass range 40 μeV–2 meV is being probed by a number of axion search experiments; see, for example, [39, 40, 41, 42, 43, 44].

5. CLOSING REMARKS

VISHν and its KSVZ analogue SMASH are interesting, economical models that solve five important problems: strong-CP, dark matter, neutrino masses, baryogenesis, and inflation. In the case of VISHν, an interesting flavour structure for the PQ symmetry is used to avoid a cosmological domain wall problem. The existence of two Higgs doublets is exploited to avoid any tension between the Davidson-Ibarra and Vissani bounds for leptogenesis. In the top-specific model considered here, there is the added bonus of an explanation for why the top-quark mass is so much larger than the masses of the other charged fermions. By extending Higgs inflation to also include the PQ scalar, viable inflation was demonstrated in the regime where the nonminimal couplings to gravity are large. A thorough analysis of preheating and reheating is now required. It would also be interesting to do a full analysis of the small nonminimal couplings regime, because success in that parameter range would robustly ensure the absence of any unitarity violation problem.

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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References

[1] R. D. Peccei and H. R. Quinn. Phys. Rev. Lett. **38**, 1440 (1977).
[2] S. Weinberg. Phys. Rev. Lett. **40**, 223 (1978).
[3] F. Wilczek. Phys. Rev. Lett. **40**, 279 (1978).
[4] A. R. Zhitnitsky. Sov. J. Nucl. Phys. **31**, 260 (1980).
[5] M. Dine, W. Fischler, and M. Srednicki. Phys. Lett. **B104**, 199 (1981).
[6] R. R. Volkas, A. J. Davies, and G. C. Joshi. Phys. Lett. **B215**, 133 (1988).
[7] R. Foot, A. Kobakhidze, K. L. McDonald, and R. R. Volkas. Phys. Rev. **D89**, 115018 (2014).
[8] M. Kamionkowski and J. March-Russell. Phys. Lett. **B282**, 137 (1992); S. M. Barr and D. Seckel. Phys. Rev. **D46**, 539 (1992); S. Ghigna, M. Lusignoli, and M. Roncadelli. Phys. Lett. **B283**, 278 (1992).
[9] P. Minkowski. Phys. Lett. **B67**, 421 (1977); T. Yanagida. in Proceedings of the *Workshop on Unified Theories and Baryon Number in the Universe*, KEK, Japan, 1979, eds O. Sawada and A. Sugamoto (Conf. Proc. **C7902131**, 95 (1979)); M. Gell-Mann, P. Ramond, and R. Slansky. Conf. Proc. **C790927**, 315 (1979); R. N. Mohapatra and G. Senjanovic. Phys. Rev. Lett. **44**, 912 (1980).
[10] M. Fukugita and T. Yanagida. Phys. Lett. **B174**, 45 (1986).
[11] P. Langacker, R. Peccei, and T. Yanagida. Mod. Phys. Lett. **A01**, 541 (1986).
[12] M. Shin. Phys. Rev. Lett. **59**, 2515 (1987).
[13] J. D. Clarke and R. R. Volkas. Phys. Rev. **D93**, 035001 (2016).
[14] S. Davidson and A. Ibarra. Phys. Lett. **B535**, 25 (2002).
[15] G. Giudice et al. Nucl. Phys. **B685**, 89 (2004).
[16] F. Vissani. Phys. Rev. **D57**, 7027 (1998).
[17] J. D. Clarke, R. Foot, and R. R. Volkas. Phys. Rev. **D91**, 073009 (2015).
[18] J. D. Clarke, R. Foot, and R. R. Volkas. Phys. Rev. **D92**, 033006 (2015).
[19] P. Sikivie. Phys. Rev. Lett. **48**, 1156 (1982).
[20] A. Davidson and M. A. H. Vozmediano. Nucl. Phys. **248**, 647 (1984); Phys. Lett. **B141**, 177 (1984).
[21] C. Q. Geng and J. N. Ng. Phys. Rev. **D39**, 1449 (1989); Phys. Rev. **D41**, 3848 (1990).
[22] C.-W. Chiang, H. Fukuda, M. Takeuchi, and T. T. Yanagida. JHEP **11**, 057 (2015); Phys. Rev. **D97**, 035015 (2018).
[23] R. D. Peccei, T. T. Wu, and T. Yanagida. Phys. Lett. **B172**, 435 (1986).
[24] L. M. Krauss and F. Wilczek. Phys. Lett. **B173**, 189 (1986).
[25] A. H. Sopov and R. R. Volkas. arXiv:2206.11598.
[26] F. L. Bezrukov and M. Shaposhnikov. Phys. Lett. **B659**, 703 (2008).
[27] A. Salvio. Phys. Lett. **B743**, 428 (2015); G. Ballesteros, J. Redondo, A. Ringwald, and C. Tamarit. Phys. Rev. Lett. **118**, 071802 (2017); Front. Astron. Space Sci. **6**, 55 (2019); A. Salvio. Phys. Rev. **D99**, 015037 (2019).
[28] J. E. Kim. Phys. Rev. Lett. **43**, 103 (1979); M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov. Nucl. Phys. **B166**, 493 (1980).
[29] D. I. Kaiser and E. I. Sfakianakis. Phys. Rev. Lett. **112**, 011302 (2014).
[30] M. P. DeCross et al. Phys. Rev. **D97**, 023526 (2018).
[31] J. L. F. Barbon and J. R. Espinosa. Phys. Rev. **D79**, 081302 (2009).
[32] C. P. Burgess, H. M. Lee, and M. Trott. JHEP **09**, 103 (2009).
[33] F. Bezrukov, A. Magnin, M. Shaposhnikov, and S. Sibiryakov. JHEP **01**, 016 (2011).
[34] M. Gorghetto, E. Hardy, and G. Villadoro. SciPost Phys. **10**, 50 (2021).

[35] S. Hoof, J. Reiss, and D. J. Marsh. The Open Journal of Astrophysics **5** (2022).
[36] M. Buschmann et al. Nature Commun. **13**, 1049 (2022).
[37] O. Straniero et al. Astron. Astrophys. **644**, A166 (2020).
[38] F. Capozzi and G. Raffelt. Phys. Rev. **D102**, 083007 (2020).
[39] ADMX collaboration. Phys. Rev. Lett. **120**, 151301 (2018); **124**, 101303 (2020); **127**, 261803 (2021).
[40] MADMAX Working Group collaboration. Phys. Rev. Lett. **118**, 091801 (2021).
[41] Y. K. Semertzidis et al. 1910.11591.
[42] HAYSTAC collaboration. Nature **590**, 238 (2021).
[43] M. Lawson et al. Phys. Rev. Lett. **123**, 141802 (2019).
[44] A. P. Quiskamp et al. 2203.12152.