Neutrino Oscillations Induced by Chiral Torsion

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Abstract

Neutrino mixing is caused by the fact that neutrino flavors are not eigenstates of the free Hamiltonian. This causes oscillations among different neutrino flavors. When neutrinos pass through a medium, weak interactions produce different effective masses for neutrinos of different flavors, leading to a modification of the mixing parameters. In curved spacetime, there is an additional contribution to neutrino Hamiltonian from a torsion-induced four-fermion interaction, which also causes neutrino mixing while propagating through fermionic matter. We provide an outline of the calculation of this effect on neutrino oscillation.

Keywords: chiral torsion, NSI, neutrino mixing
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1. INTRODUCTION

The Standard Model says that neutrinos are massless and purely left-handed, described by two component Weyl spinors. On the other hand, the disappearance or appearance of flavors among neutrinos from various sources—top of the atmosphere [1], the Sun [2], reactors [3], accelerators [4], and in many other experiments [5, 6]—point to neutrino flavor mixing and oscillation, which is explained by the existence of neutrino masses. This clearly indicates the presence of physics beyond the Standard Model. There are many ways to extend the Standard Model to include massive neutrinos [7]. Different mechanisms will have different consequences for neutrino oscillations [8, 9].

One kind of BSM physics that is not usually considered is the interaction of fermions with spacetime geometry. Fermions in a curved spacetime generate a spacetime torsion, which is non-dynamical and can thus be integrated out, leaving a quartic interaction term [10, 11]. Although gravity is a very weak force, the freedom to choose coupling constants means that the contribution of spacetime geometry on neutrino oscillations may not be negligible. In what follows, we present a brief outline of a calculation for three flavors of neutrinos.[1]

The layout of the paper is as follows. In Section 2, we provide a very brief review of how fermions behave in spacetime and then write how an effective quartic interaction arises. In Section 3, we calculate the modified mixing matrix for two neutrino flavors, and then, we do the same calculations for three neutrino flavors in Section 4.

2. FERMIONS IN SPACETIME

Usually, the geometry of spacetime is affected in the presence of matter. The effect of matter on bosons can be neglected for small curvature, as in most physical phenomena. But the case is different for a fermionic field. When a fermion passes through a fermionic matter, a four-fermi interaction happens there, which couples with fermions with different coupling constants which are fixed by experiments.

Torsion ${\Lambda_\mu}^{ab}$ is added to the Levi-Civita connection ${\omega_\mu}^{ab}$ in the 1st order formulation of Gravity, which uses tetrad fields, through the following relation [12, 13]:

$$ {A_\mu}^{ab} = {\omega_\mu}^{ab} + {\Lambda_\mu}^{ab}, \tag{1} $$

where ${A_\mu}^{ab}$ is called a spin connection. If we assume that Λ couples chirally to fermions, the equation of motion for Λ is

$$ {\Lambda_\mu}^{ab} = \frac{\kappa}{4}\epsilon^{abcd} e_{c\mu} \sum_i \left(-\lambda_L^i \bar{\psi}_{iL}\gamma_d \psi_{iL} + \lambda_R^i \bar{\psi}_{iR}\gamma_d \psi_{iR} \right), \tag{2} $$

which is clearly also its solution. Here, $\kappa = 8\pi G$, while e^a_μ are the tetrad fields and e^μ_a are their inverse fields, defined by $\eta_{ab} e^a_\mu e^b_\nu = g_{\mu\nu}$. The tetrads can be combined into one 4×4 matrix, which has a determinant equal to the square root of the metric determinant, $|e| = \sqrt{|g|}$. Then, we can put Λ back into the action to get an effective quartic interaction term [10]

$$ -\frac{1}{2}\left(\sum_i \left(-\lambda_L^i \bar{\psi}_L^i \gamma_a \psi_L^i + \lambda_R^i \bar{\psi}_R^i \gamma_a \psi_R^i \right) \right)^2, \tag{3} $$

where the sum is over all species of fermions, and we have also redefined the λ by absorbing $\sqrt{\frac{3\kappa}{8}}$. We identify this term as the **torsional interaction** term, which is usually independent of the background metric, but can modify it through Einstein equations. This results in a curvature which is generally small enough so that we can take it as a flat spacetime and do normal QFT calculations. We emphasize that we do not get this term by extending or modifying GR, but this is how ordinary fermions behave in a spacetime that is not flat. Along with this interaction, Standard Model interactions are always present—both will contribute to the calculations of neutrino mixing.

In this paper, we mainly concern ourselves about how neutrino oscillations will be affected in the presence of torsional four-fermion interaction. We generally believe that matter effects suppress the effects of curvature on neutrino oscillations, even in regions of strong gravity such as supernovae [14]. Our approach here is completely different from ordinary gravitational effect because of the dimensionful coupling constants $\lambda^i_{L,R}$ which are not universal but can be fixed only from experimental observations.

3. TWO FLAVORS OF NEUTRINO

Let us first begin with two species of neutrinos passing through normal matter of uniform density. It is the field in the mass ba-

[1] Based on talk delivered by Riya Barick at the International Conference on Neutrinos and Dark Matter (NuDM-2022), Sharm El-Sheikh, Egypt, 25–28 September, 2022.

sis that couples to torsion, since torsion appears with the geometric connection. Interaction of neutrinos with background is

$$-\left(\sum_{i=1,2}\left(-\lambda_i^L\bar{\nu}_i\gamma_a L\nu_i+\lambda_i^R\bar{\nu}_i\gamma_a R\nu_i\right)\right)\times\left(\sum_{f=e,p,n}\left(\lambda_f^V\bar{f}\gamma_a f+\lambda_f^A\bar{f}\gamma_a\gamma^5 f\right)\right). \quad (4)$$

The sum includes only e ,p, and n because other fermions have negligible presence in the atmosphere or the Sun. In most situations, the density of matter is not sufficient to cause high curvature, so we can neglect the $\omega_\mu{}^{ab}$ term and do the calculations as in case of a flat background. Like weak interactions, the background factor can be replaced by its average value by considering the forward scattering of neutrinos:

$$\sum_{f=e,p,n}\left\langle\lambda_f^V\bar{f}\gamma_a f+\lambda_f^A\bar{f}\gamma_a\gamma^5 f\right\rangle. \quad (5)$$

If the background consists of nonrelativistic fermions, the average becomes the number density. Thus, the interaction term is

$$-\left(\sum_{i=1,2}\left(-\lambda_i^L\bar{\nu}_i\gamma_0 L\nu_i+\lambda_i^R\bar{\nu}_i\gamma_0 R\nu_i\right)\right)\tilde{n}, \quad (6)$$

where $\tilde{n}$ is the weighted number density of the background matter, $\tilde{n}=\sum\lambda_f^V n_f$. We also consider maximal chirality violation, so $\lambda_i^R=0$ for neutrinos. Then, the contribution to the effective Hamiltonian is

$$\sum_{i=1,2}\left(\lambda_i\nu_i^\dagger L\nu_i\right)\tilde{n}. \quad (7)$$

The flavor eigenstates $|\nu_\alpha\rangle$ can be written in terms of the mass eigenstates $|\nu_i\rangle$ as

$$|\nu_\alpha\rangle=\sum_i U_{\alpha i}^*|\nu_i\rangle, \quad (8)$$

where the mixing matrix $U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$. Following [15], we can now write the Schrödinger equation for the neutrinos:

$$i\frac{d}{dx}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}=\left[E\mathbb{I}+\frac{1}{2E}\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}+\begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix}\tilde{n}-\frac{G_F}{\sqrt{2}}(n_n-n_e)+\frac{G_F}{\sqrt{2}}U^T\begin{pmatrix}n_e & 0\\ 0 & -n_e\end{pmatrix}U^*\right]\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}, \quad (9)$$

where the effect of weak interaction has also been included. Let us define a torsionally modified mass-squared difference Δm_s^2 as

$$\Delta m_s^2=\Delta m^2+2\tilde{n}E\Delta\lambda, \quad (10)$$

where $\Delta m^2=m_2^2-m_1^2$ and $\Delta\lambda=\lambda_2-\lambda_1$. It can be shown easily that the mixing angle in matter, modified by the torsional four-fermion interaction, is given by [16]

$$\tan 2\theta_M=\frac{\tan 2\theta}{1-\frac{D}{\Delta m_s^2\cos 2\theta}}, \quad (11)$$

where $D=2\sqrt{2}G_F n_e E$. By diagonalizing equation (9), we can find the $\nu_e\to\nu_\mu$ conversion probability

$$P_{\nu_e\to\nu_\mu}=\sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2}{4E}L\right), \quad (12)$$

and the ν_e survival probability

$$P_{\nu_e\to\nu_e}=1-\sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2}{4E}L\right), \quad (13)$$

where for convenience we have written

$$\Delta m_M^2=\sqrt{\left(\Delta m_s^2\cos 2\theta-D\right)^2+\left(\Delta m_s^2\sin 2\theta\right)^2}. \quad (14)$$

Therefore, we see that spacetime geometry modifies the mass-squared differences via equation (10) and mixing angle via equation (11) and thus modifies the oscillation probabilities through equations (12) and (13).

4. THREE FLAVORS OF NEUTRINO

In nature, we have three species of neutrinos. Let us now consider the effect of spacetime geometry, and thus torsional four-fermion interaction, on mixing between three flavors of neutrinos. It is known that if more than two families of neutrino exist, CP and T can be broken via complex elements of mixing matrix [17, 18]. We follow the conventions of Particle Data Group (PDG) and write the mixing matrix as [19]

$$U=\begin{pmatrix}c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta}\\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13}\\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{pmatrix}, \quad (15)$$

where $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$, where θ_{ij} are the mixing angles, δ is the CP-violating phase, and we have ignored Majorana phases as we are dealing with Dirac neutrinos. The angles θ_{ij} are in the first quadrant and the CP-violation phase δ is taken to be between 0 and 2π. Then, U is conveniently expressed as a product of rotation matrices $\mathcal{O}_{ij}$ for rotation in the ij-plane as [20]

$$U=\mathcal{O}_{23}\mathcal{U}_\delta\mathcal{O}_{13}\mathcal{U}_\delta^\dagger\mathcal{O}_{12}, \quad (16)$$

where $U_\delta=\mathrm{diag}(1,1,e^{i\delta})$. The Schrödinger equation is written in the mass basis, similar to equation (9),

$$i\frac{d}{dx}\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}=\left[E+\frac{1}{2E}\begin{pmatrix}m_1^2 & 0 & 0\\ 0 & m_2^2 & 0\\ 0 & 0 & m_3^2\end{pmatrix}+\begin{pmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{pmatrix}\tilde{n}-\frac{G_F}{\sqrt{2}}n_n+U^T\begin{pmatrix}A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}U^*\right]\begin{pmatrix}\nu_1\\ \nu_2\\ \nu_3\end{pmatrix}, \quad (17)$$

where $A=\sqrt{2}G_F n_e$. For uniform matter density or slowly varying matter, we can write on the flavor basis

$$i\begin{pmatrix}\dot{\nu}_e\\ \dot{\nu}_\mu\\ \dot{\nu}_\tau\end{pmatrix}=\left[E_0'\mathbb{I}+\frac{1}{2E}U^*\begin{pmatrix}0 & 0 & 0\\ 0 & \Delta\tilde{m}_{21}^2 & 0\\ 0 & 0 & \Delta\tilde{m}_{31}^2\end{pmatrix}U^T+\begin{pmatrix}A & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\right]\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}. \quad (18)$$

We have used the definition

$$E_0' = E + \frac{m_1^2 + 2\lambda_1 \tilde{n} E}{2E} - \frac{G_F}{\sqrt{2}} n_n. \tag{19}$$

In order to find the eigenvalues of the resulting Hamiltonian, we take the help of perturbation theory using a small parameter. For this, we first define

$$\Delta\tilde{m}_{ij}^2 := \Delta m_{ij}^2 + 2\tilde{n}E\Delta\lambda_{ij}, \tag{20}$$

where $\Delta m_{ij}^2 = m_i^2 - m_j^2$ and $\Delta\lambda_{ij} = \lambda_i - \lambda_j$. Then, equation (18) takes the form

$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{\Delta\tilde{m}_{31}^2}{2E}\mathcal{O}_{23}\mathcal{U}_\delta^* M \mathcal{U}_\delta^T \mathcal{O}_{23}^T \begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}. \tag{21}$$

Here, the matrices proportional to the identity matrix have been suppressed as they will contribute to a common phase for all neutrinos and thus have no effect on oscillation probabilities. We have also written

$$M = \mathcal{O}_{13}\mathcal{O}_{12}\begin{pmatrix}0 & 0 & 0\\ 0 & \alpha & 0\\ 0 & 0 & 1\end{pmatrix}\mathcal{O}_{12}^T\mathcal{O}_{13}^T + \begin{pmatrix}\hat{A} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{22}$$

In these expressions, $\hat{A} = 2AE/\Delta\tilde{m}_{31}^2$ and $\alpha = \Delta\tilde{m}_{21}^2/\Delta\tilde{m}_{31}^2$ are dimensionless quantities. Now, our main focus is to find out the eigenvalues and eigenvectors of the resulting Hamiltonian. Exactly diagonalizing a 3×3 matrix is quite difficult and thus we need to use some approximations at this point [20, 21, 22]. We will assume that s_{13} and α are small parameters and then find the transition probabilities in the second order of these parameters.

In order to find the mixing matrix, we will diagonalize M by finding the eigenvalues and eigenvectors using perturbation theory. Let us first decompose M into three parts containing different powers of the small parameters α and s_{13}:

$$\begin{aligned}
M^{(0)} &= \begin{pmatrix}\hat{A} & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 1\end{pmatrix},\\
M^{(1)} &= \begin{pmatrix}\alpha s_{12}^2 & \alpha s_{12}c_{12} & s_{13}\\ \alpha s_{12}c_{12} & \alpha c_{12}^2 & 0\\ s_{13} & 0 & 0\end{pmatrix},\\
M^{(2)} &= \begin{pmatrix}s_{13}^2 & 0 & -\alpha s_{13}s_{12}^2\\ 0 & 0 & -\alpha s_{13}s_{12}c_{12}\\ -\alpha s_{13}s_{12}^2 & -\alpha s_{13}s_{12}c_{12} & -s_{13}^2\end{pmatrix}.
\end{aligned} \tag{23}$$

Using perturbation theory, it is straightforward to calculate the eigenvalues μ and the eigenvectors v. Keeping terms up to the second order in the small parameters α and s_{13}, we find the eigenvalues

$$\mu_1 = \hat{A} + \alpha s_{12}^2 + s_{13}^2\frac{\hat{A}}{\hat{A}-1} + \frac{\alpha^2\sin^2(2\theta_{12})}{4\hat{A}}, \tag{24}$$

$$\mu_2 = \alpha c_{12}^2 - \frac{\alpha^2\sin^2(2\theta_{12})}{4\hat{A}}, \tag{25}$$

$$\mu_3 = 1 - s_{13}^2\frac{\hat{A}}{\hat{A}-1}. \tag{26}$$

The Hamiltonian in equation (18) is related to M by a unitary transformation as seen in equation (21). Hence, the energy eigenvalues of the Hamiltonian are

$$E_i = \frac{\Delta\tilde{m}_{31}^2}{2E}\mu_i. \tag{27}$$

The zeroth-order eigenvectors are the basis vectors $\hat{e}_i = \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$. By using perturbation theory, we can calculate the higher-order corrections to the eigenvectors as

$$\begin{aligned}
v_i^{(1)} &= \sum_{i\neq j}\frac{M_{ij}^{(1)}}{\mu_i^{(0)}-\mu_j^{(0)}}\hat{e}_j,\\
v_i^{(2)} &= \sum_{j\neq i}\frac{1}{\mu_i^{(0)}-\mu_j^{(0)}}\left(M_{ij}^{(2)} + \left(M^{(1)}v_i^{(1)}\right)_j - \mu_i^{(1)}\left(v_i^{(1)}\right)_j\right)\hat{e}_j.
\end{aligned} \tag{28}$$

We will rewrite equation (21) in terms of the diagonal matrix $\hat{M}$ and the diagonalizing matrix W, which diagonalizes M into $\hat{M}$.

$$\begin{aligned}
i\frac{d}{dx}\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} &= \frac{\Delta\tilde{m}_{31}^2}{2E}\mathcal{O}_{23}\mathcal{U}_\delta^* W\hat{M}W^T\mathcal{U}_\delta^T\mathcal{O}_{23}^T\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}\\
&= \frac{\Delta\tilde{m}_{31}^2}{2E}U'^*\hat{M}U'^T\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{\Delta\tilde{m}_{31}^2}{2E}U'^*HU'^T\begin{pmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{pmatrix}.
\end{aligned} \tag{29}$$

Here, U', the new mixing matrix, is defined as $U' = \mathcal{O}_{23}\mathcal{U}_\delta W$, which is calculated to be

$$U' = \begin{pmatrix}
\begin{array}{c}1 - \frac{1}{2}\frac{\alpha^2}{\hat{A}^2}c_{12}^2s_{12}^2\\ -\frac{1}{2}\frac{s_{13}^2}{(\hat{A}-1)^2}\end{array} & -\frac{\alpha}{\hat{A}}s_{12}c_{12}\left(1+\frac{\alpha}{\hat{A}}\cos(2\theta_{12})\right) & -\frac{s_{13}}{\hat{A}-1}\left(1-\frac{\alpha\hat{A}}{\hat{A}-1}s_{12}^2\right)\\[2ex]
\begin{array}{c}\frac{\alpha}{\hat{A}}c_{12}s_{12}\left(1+\frac{\alpha}{\hat{A}}\cos(2\theta_{12})\right)c_{23}\\ +\frac{s_{13}}{\hat{A}-1}\left(1-\frac{\alpha\hat{A}}{\hat{A}-1}s_{12}^2\right)e^{i\delta}s_{23}\end{array} & \begin{array}{c}\left(1-\frac{1}{2}\frac{\alpha^2}{\hat{A}^2}s_{12}^2c_{12}^2\right)c_{23}\\ +\alpha s_{13}s_{12}c_{12}\left(1+\frac{1}{\hat{A}}\right)e^{i\delta}s_{23}\end{array} & \begin{array}{c}-\frac{\alpha s_{13}}{\hat{A}-1}\hat{A}s_{12}c_{12}c_{23}\\ +\left(1-\frac{1}{2}\frac{s_{13}^2}{(\hat{A}-1)^2}\right)e^{i\delta}s_{23}\end{array}\\[2ex]
\begin{array}{c}-\frac{\alpha}{\hat{A}}c_{12}s_{12}\left(1+\frac{\alpha}{\hat{A}}\cos(2\theta_{12})\right)s_{23}\\ +\frac{s_{13}}{\hat{A}-1}\left(1-\frac{\alpha\hat{A}}{\hat{A}-1}s_{12}^2\right)e^{i\delta}c_{23}\end{array} & \begin{array}{c}-\left(1-\frac{1}{2}\frac{\alpha^2}{\hat{A}^2}s_{12}^2c_{12}^2\right)s_{23}\\ +\alpha s_{13}s_{12}c_{12}\left(1+\frac{1}{\hat{A}}\right)e^{i\delta}c_{23}\end{array} & \begin{array}{c}+\frac{\alpha s_{13}}{\hat{A}-1}\hat{A}s_{12}c_{12}s_{23}\\ +\left(1-\frac{1}{2}\frac{s_{13}^2}{(\hat{A}-1)^2}\right)e^{i\delta}c_{23}\end{array}
\end{pmatrix}. \tag{30}$$

Using this mixing matrix, various transition probabilities can be calculated quite easily. For example, the expression for the amplitude of conversion of ν_e to ν_μ is

$$\begin{aligned} A_{\nu_e\to\nu_\mu} &= \langle\nu_\mu|\, e^{-iHt}\,|\nu_e\rangle \\ &= \left(\frac{\alpha}{\hat{A}}s_{12}c_{12}\left(1+\frac{\alpha}{\hat{A}}\cos\left(2\theta_{12}\right)\right)c_{23}\right. \\ &\qquad \left. +\frac{s_{13}}{\hat{A}-1}\left(1-\frac{\alpha\hat{A}}{\hat{A}-1}s_{12}^2\right)e^{i\delta}s_{23}\right)e^{-iE_1t} \\ &\quad +\left(-\frac{\alpha}{\hat{A}}s_{12}c_{12}\left(1+\frac{\alpha}{\hat{A}}\cos\left(2\theta_{12}\right)\right)c_{23}\right)e^{-iE_2t} \\ &\quad +\left(-\frac{s_{13}}{\hat{A}-1}\left(1-\frac{\alpha\hat{A}}{\hat{A}-1}s_{12}^2\right)e^{i\delta}s_{23}\right)e^{-iE_3t}. \end{aligned} \tag{31}$$

The conversion probability is given by $P_{\nu_e\to\nu_\mu} = |A_{\nu_e\to\nu_\mu}|^2$. Other conversion amplitudes (e.g., $\nu_e \to \nu_\tau$) can be similarly calculated, leading to the corresponding conversion probabilities as well as survival probability ($P_{\nu_e\to\nu_e}$). Our result exactly matches those found in [23, 24] if the torsional interaction is set to zero. The amplitudes and probabilities for $\nu_\mu \to \nu_e$, $\nu_\mu \to \nu_\tau$, and $\nu_\mu \to \nu_\mu$ were shown elsewhere recently [25], along with the difference from $\lambda = 0$ results. Fitting these results to neutrino data should produce an estimate of the coupling parameters λ.

It is important to recognize that it is not possible to obtain bounds on λ from purely theoretical considerations. Since the four-fermion interaction appears from the spin connection, it is enticing to think of it as fundamentally gravitational and thus expect the couplings to be of size $\sim \frac{1}{M_p}$. This is a red herring. Four-fermion interactions arise in effective gauge theories when the boson propagator $\sim \frac{1}{q^2-M^2}$ in an exchange process is replaced by $-\frac{1}{M^2}$ in the low energy limit $|q^2| \ll M^2$. This is what happens in Fermi's theory of weak interactions, for example. Here, the geometrical interactions do not appear in the low energy limit of any theory—the contorsion field Λ_μ^{ab}, which was eliminated to produce these interactions, is not dynamical and does not have a propagator. We have not quantized gravity or spacetime, so the scale of quantum gravity is not relevant here. The couplings should not change even at very high energy scales. When the couplings λ were redefined to absorb a factor of κ, it was only for bookkeeping purposes, there was no reason to take them to be small dimensionless numbers before that. This torsional interaction is one kind of Non-Standard Interaction (NSI), so known bounds on various NSI coupling parameters should be compared with the parameters which appear here.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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