Sommerfeld Enhancement of BLSSM-IS Dark Matter

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Abstract

The lightest right-handed (RH) sneutrino in the BLSSM model with inverse seesaw is an intriguing candidate for dark matter. However, its relic abundance typically exceeds the observational limits. We show that when the RH sneutrino mass exceeds 600 GeV and there is a strong coupling with the light Higgs boson, the Sommerfeld Effect (SE) is significantly enhanced. We emphasize that this effect modifies the RH-sneutrino relic abundance and makes it a viable dark matter up to a mass of order 2.5 TeV.

Keywords: BLSSM-IS dark matter, Sommerfeld enhancement of Yukawa potential, lightest right-handed sneutrino dark matter, the relic density with Sommerfeld effect

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1. INTRODUCTION

This article is based on [1], where the Sommerfeld effect was analyzed in the BLSSM model with type I seesaw. Here, we do this analysis in the model of BLSSM with an inverse seesaw.

Nonvanishing neutrino masses and the existence of nonbaryonic dark matter (DM) are new physics beyond the Standard Model (BSM). Since DM decoupling time, the DM candidacy's relic abundance has been a crucial concern for its validity in explaining the DM observation. According to the most recent findings from the Planck satellite's whole mission, the DM relic abundance is given by [2]

$$\Omega_{\rm DM}h^2 = 0.120 \pm 0.001. \tag{1}$$

The parameter space of any BSM with a strong DM candidate is severely constrained by this measurement because of its tiny margin of error. The predicted relic abundance is much greater than the observed limits in equation (1) because the thermal relic abundance of DM is inversely proportional to its annihilation cross section, which is typically quite small because the majority of new particles in BSM models are quite heavy to satisfy the absence of a direct signal.

On the other hand, DM particles can interact with themselves via long range attractive force, as the Yukawa potential before annihilating which can enhance the annihilation cross sections to the observed levels, a phenomenon referred to as the Sommerfeld enhancement (SE).

The Sommerfeld enhancement can arise if the Higgs boson mediator has a mass that satisfies the following condition: $m_\phi \leq \alpha_Y m_{\rm DM}$ [1], where α_Y is the dimensionless coupling between two DM particles and the scalar field ϕ. In case of the scalar DM, α_Y is defined by $\alpha_Y \equiv \frac{Y^2}{16\pi m_{\rm DM}^2}$ [3], where Y is the trilinear coupling between ϕ-DM-DM with dimension mass. Therefore, this scenario entails the following: $m_{\rm DM} \gg m_\phi$.

In this paper, we argue that the lightest right-handed sneutrino, which is a natural candidate for DM in the B-L inverse seesaw extension of the minimal supersymmetric standard model (MSSM), via lightest Higgs boson h' [4, 1] interactions leads to distinctive aspects. As we will show, the thermal relic abundance with Sommerfeld Enhancement (SE) can be consistent with the observed density of the RH-sneutrino dark matter for masses as high as 2.5 TeV, much larger than those usually considered without SE. For dark matter masses above 600 GeV, nonperturbative Sommerfeld corrections [5] to the low-velocity annihilation rate are large. Several authors have recently recognized the potential importance of these corrections to the dark matter relic density calculations [1, 6, 7, 8, 9], which lead to enhanced annihilation cross section in the case of attractive Yukawa potential.

The paper is organized as follows. In Section 2, we review the right-handed sneutrino DM in the BLSSM-IS and highlight that the results of the typical sneutrino relic abundance calculation are outside of the range of the observational limits. In Section 3, through light h' exchange, we investigate the Sommerfeld enhancement of the attractive Hulthen potential. In Section 4, we demonstrate that the severe observational constraints can coincide with the relic abundance with the expected Sommerfeld enhancement. Section 5 presents our conclusion.

2. LIGHTEST RIGHT-HANDED SNEUTRINO DARK MATTER

The BLSSM-IS is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, where the $U(1)_{B-L}$ is spontaneously broken by chiral singlet superfields $\hat{\chi}_{1,2}$ with $B-L$ charge $= \pm 1$. As in conventional $B-L$ model, a gauge boson Z'_{B-L} and three chiral singlet superfields $\hat{\nu}_i^c$ with $B-L$ charge $= -1$ are introduced for the consistency of the model. Finally, three SM singlet fermions S_1 with $B-L$ charge $= +2$ and three singlet fermions S_2 with $B-L$ charge $= -2$ are considered to implement the inverse seesaw mechanism [10].

The superpotential of the BLSSM-IS is given by

$$\begin{aligned} W = {} & Y_u \hat{Q}\hat{H}_2\hat{U}^c + Y_d \hat{Q}\hat{H}_1\hat{D}^c + Y_e hat L \hat{H}_1\hat{E}^c + Y_\nu \hat{L}_i \hat{H}_2 \hat{\nu}^c \\ & + Y_S \hat{\nu}^c \hat{\chi}_1 \hat{S}_2 + \mu \hat{H}_1 \hat{H}_2 + \mu' \hat{\chi}_1 \hat{\chi}_2. \end{aligned} \tag{2}$$

We now turn to consider the right-handed sneutrino spectrum, as the left-handed sneutrino sector almost remains as in the MSSM. If we write $\tilde{\nu}_R$ and $\tilde{S}_2$ (the scalar components of the superfields $\tilde{\nu}_R$ and $\tilde{S}_2$) as $\tilde{\nu}_R = \frac{1}{\sqrt{2}}(\tilde{\nu}_R^+ + \tilde{\nu}_R^-)$, and $\tilde{S}_2 = \frac{1}{\sqrt{2}}(\tilde{S}_2^+ + \tilde{S}_2^-)$, then the CP-even/odd sneutrino mass matrix is given by [11].

$$M_\pm^2 = \begin{pmatrix} M_{\tilde{\nu}_R}^2 + m_D^2 + M_R^2 - \frac{1}{2}M_{Z'}^2\cos 2\beta' & \pm M_R\,(A_S + \mu'\cot\beta') \\ \pm M_R\,(A_S + \mu'\cot\beta') & m_{\tilde{S}}^2 + M_R^2 + M_{Z'}^2\cos 2\beta' \end{pmatrix}, \tag{3}$$

where $M^2_{\tilde{\nu}_R}$ and $m_{\tilde{S}_2}$ are the soft scalar mass matrices, and A_S is the trilinear coupling, which is also a (3×3)-matrix. In our analysis, for simplicity, we assume that these matrices are diagonal. The mass eigenvalues of $M^2_{\pm}$ are given by [12]

$$m^2_{\tilde{\nu}^{\pm}} = \frac{1}{2}\left(M^2_{\tilde{\nu}_R} + m^2_D + m^2_{\tilde{S}_2}\right) + M^2_R + \frac{1}{4}M^2_{Z'}\cos 2\beta' \pm \sqrt{\begin{array}{l}\left(M^2_{\tilde{\nu}_R} + m^2_D - m^2_{\tilde{S}_2} + M^2_R - \frac{3}{2}M^2_{Z'}\cos 2\beta'\right)^2 \\ + 4M_R\left(A_S + \mu'\cot\beta'\right)\end{array}}. \quad (4)$$

The lightest sneutrino $\tilde{\nu}_1$ (either it is CP-even sneutrino, $\tilde{\nu}^R_1$, or CP-odd sneutrino, $\tilde{\nu}^I_1$) can be expressed in terms of $\tilde{\nu}^+_R$ and $\tilde{S}^+_2$ (in case of it is CP-even) as

$$\tilde{\nu}_1 = \sum_{i=1}^{3} Z^R_{1i}\left(\tilde{\nu}^+_R\right)_i + \sum_{j=1}^{3} Z^R_{1j}\left(\tilde{S}^+_2\right)_j, \quad (5)$$

where $Z^R_{1i} = \frac{1}{\sqrt{2}}\{1,0,0\}$ and $Z^R_{1j} = \frac{1}{\sqrt{2}}\{1,0,0\}$.

Since $\triangle m_{\tilde{\nu}_1} \equiv |m_{\tilde{\nu}^I_1} - m_{\tilde{\nu}^R_1}| \simeq 0$ (see Figure 1), both particles $\tilde{\nu}^R_1$ and $\tilde{\nu}^I_1$ could be stable and viable dark matter candidates.

3. THE SOMMERFELD ENHANCEMENT

Dark matter annihilation cross sections in the low-velocity regime can be enhanced by factors depending on the inverse velocity, $1/v_{\rm rel}$, called Sommerfeld enhancement (SE). The Sommerfeld enhancement can arise if the lightest Higgs boson mass satisfies the following condition: $m_{h'} \leq \alpha_{Y} m_{\tilde{\nu}_1}$. [1]. This Sommerfeld enhancement corresponds to the summation of a series of ladder diagrams involving the lightest Higgs h' state repeatedly exchanged (see Figure 2).

The actual annihilation cross section times velocity is given by [1, 5]

$$(\sigma v)_{\rm enh} = S \times (\sigma v), \quad (6)$$

where σv is the tree level cross section times velocity, and the factor S is known as "Sommerfeld enhancement".

In this section, we will compute the Sommerfeld enhancement factor for the attractive Yukawa potential and consider a dark matter particle of mass $m_{\tilde{\nu}_{R_1}}$ whose velocity is $v_{\rm rel}$ and interacts with $V(r) = -\alpha_{Y}\frac{e^{-m_{h'}r}}{r}$, that is, an attractive Yukawa potential mediated by a light Higgs boson of mass $m'_h = 28$, 60, and 90 GeV. Let $\psi(r)$ be the reduced two-body wave function for the S-wave annihilation; in the nonrelativistic limit, it will obey the Schrodinger equation when $\ell = 0$ can be rewritten as

$$\left(-\frac{\bigtriangledown^2}{2\mu_r} + V(r)\right)\psi(r) = E\psi(r), \quad (7)$$

where $\mu_r = m_{\tilde{\nu}_1}/2$ is the reduced mass and the $V(r)$ is the corresponding Yukawa potential, obtained from h'-exchange.

Since the exact analytical solution of the Schrodinger equation with Yukawa potential is not available, approximating the Yukawa potential by a Hulthen potential. Now, it is possible to obtain analytical solutions for the $\ell = 0$ modes of the wavefunctions. It turns out that the S-wave Sommerfeld enhancement is given by [1, 13]

$$S = \left(\frac{2\pi\alpha_{Y}}{v_{\rm rel}}\right)\frac{\sinh\left(\frac{6m_{\tilde{\nu}_1}v_{\rm rel}}{\pi m_{h'}}\right)}{\cosh\left(\frac{6m_{\tilde{\nu}_1}v_{\rm rel}}{\pi m_{h'}}\right) - \cos\left[\sqrt{\frac{24m_{\tilde{\nu}_1}\alpha_{Y}}{m_{h'}} - \frac{36m^2_{\tilde{\nu}_1}v^2_{\rm rel}}{\pi^2 m^2_{h'}}}\right]}, \quad (8)$$

where $v_{\rm rel}$ is the relative velocity between the $\tilde{\nu}_1$ DM particles.

In Figure 3, we display the Sommerfeld enhancement for a Hulthen (Yukawa) potential as a function of the $\tilde{\nu}_1$ and coupling α_{Y} is varied in the given range $[0.079, 0.2]$.

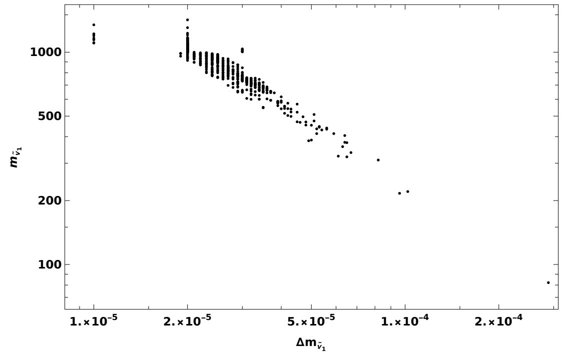

FIGURE 1: RH sneutrino $\tilde{\nu}_1$ DM versus the mass difference of the two eigenstates of real and imaginary RH sneutrino, $\triangle m_{\tilde{\nu}_1}$, for $m_0 \in [600, 1600]$ TeV, $m_{1/2} \in [1.5, 3]$ TeV, $A_0 \in [-4, 4]$ TeV, $\tan\beta \in [10, 60]$, and $\tan\beta' \in [1.03, 1.3]$.

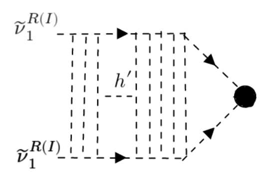

FIGURE 2: The ladder diagram giving rise to the Sommerfeld enhancement for the incoming two $\tilde{\nu}^{R(I)}_1$ states via the exchange of the lightest CP-even Higgs bosons h', and the blob vertex represents all possible S-wave annihilations.

4. CALCULATION OF THE RELIC DENSITY WITH THE SOMMERFELD ENHANCEMENTS

In the BLSSM-IS, the relevant interaction terms of lightest right-handed sneutrino are given by the following Lagrangian [14]:

$$\mathcal{L} = -i\Big[Y\left(\tilde{\nu}^{R(I)}_1\right)^2 h' + \lambda_1\left(\tilde{\nu}^{R(I)}_1\right)^2 h^2 + \lambda_4\left(\tilde{\nu}^{R(I)}_1\right)^2 h'^2 + \lambda_2 h' h^2 + g_{W^{\pm}} h' W^+ W^- + \lambda_3 h' h' h' + g_{ZZ} h' ZZ\Big], \quad (9)$$

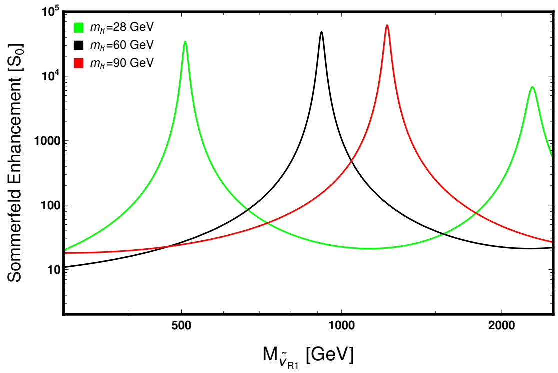

FIGURE 3: Sommerfeld enhancement for a Hulthen potential as a function of $m_{\tilde{\nu}_1}$, with $m_{h'} = 28, 60, 90\,\text{GeV}$ and a coupling strength of $\alpha_Y \in [0.079, 0.2]$.

where the above couplings are defined as follows:

$$
\begin{aligned}
g_{W^\pm} &\simeq -g_2 M_W \left(\Gamma_{32}\sin\beta + \Gamma_{31}\cos\beta\right), \\
g_{ZZ} &\simeq -g_z M_Z \left(\Gamma_{32}\sin\beta + \Gamma_{31}\cos\beta\right), \\
Y &\simeq \Bigg[\frac{g_{BL}^2}{2}\left(v_2'\Gamma_{34} - v_1'\Gamma_{33}\right) + \sqrt{2}\Gamma_{34}\Gamma_{14}^{R(I)}\Gamma_{17}^{R(I)}\mu' Y_S \\
&\quad + \Gamma_{33}\left(\sqrt{2}\Gamma_{14}^{R,I}\Gamma_{17}^{R,I} T_S + v_1' Y_S^2\right)\Bigg], \\
\lambda_1 &\simeq \left[\frac{g_{BL}\tilde{g}}{4}\left(\Gamma_{12}^2 - \Gamma_{11}^2\right) + \Gamma_{12}^2\left(\Gamma_{14}^{R(I)}\right)^2 Y_\nu^2\right], \\
\lambda_4 &\simeq \left[\frac{g_{BL}^2}{2}\left(\Gamma_{34}^2 - \Gamma_{33}^2\right) + \frac{g_{BL}\tilde{g}}{4}\left(\Gamma_{32}^2 - \Gamma_{31}^2\right) + \Gamma_{33}^2 Y_S^2\right], \\
\lambda_3 &\simeq g_{BL}^2\left[v_1'\left(3\Gamma_{33}^3 - 3\Gamma_{33}\Gamma_{34}^2\right) + v_2'\left(-3\Gamma_{33}^2\Gamma_{34} + 3\Gamma_{34}^3\right)\right], \\
\lambda_2 &\simeq -g_{BL}^2\Big[v_1'\left(-3\Gamma_{33}\Gamma_{13}^2 + \Gamma_{33}\Gamma_{14}^2 + 2\Gamma_{34}\Gamma_{13}\Gamma_{14}\right) \\
&\quad + v_2'\left(\Gamma_{34}\Gamma_{13}^2 + 2\Gamma_{33}\Gamma_{14}\Gamma_{13} - 3\Gamma_{34}\Gamma_{14}^2\right)\Big].
\end{aligned}
\tag{10}
$$

Here, Γ and $\Gamma^{R(I)}$ are the matrices that diagonalize the CP-even Higgs mass matrix and the CP-even (odd) sneutrino mass matrix. We assumed that Y_S is diagonal such that $Y_S = Y_S[aa]$, $a = 1, 2, 3$.

However, in our numerical calculations, it is taken into account along with other subdominant interactions. Thus, the Feynman diagrams of the dominant annihilation channels of $\tilde{\nu}_1$ to CP-even Higgs bosons and SM gauge bosons are shown (see Figure 4 without SE, Figure 5 with SE). It is clear that the dominant interaction is the four-point interaction: $\tilde{\nu}_1\tilde{\nu}_1 \to h'h'$, t-channel via $\tilde{\nu}_1$. The values of these annihilation cross sections determine the relic abundance, which is given by

$$
\Omega_{\tilde{\nu}_1} h^2 = \frac{2.1\times 10^{-27}\,\text{cm}^3\text{s}^{-1}}{\left\langle \sigma_{\tilde{\nu}_1}^{\text{an}} v_{\text{rel}}\right\rangle}\left(\frac{x_F}{20}\right)\left(\frac{100}{g_*(T_F)}\right)^{\frac{1}{2}}, \tag{11}
$$

where $\langle \sigma_{\tilde{\nu}_1}^{\text{an}} v_{\text{rel}}\rangle$ is a thermal average for the total cross section of annihilation multiplied by the relative sneutrino velocity, T_F is the freeze out temperature, $x_F = m_{\tilde{\nu}_1}/T_F \simeq \mathcal{O}(20)$, and $g_*(T_F) \simeq \mathcal{O}(100)$ is the number of degrees of freedom at freeze-out. As emphasized in [15], the suppressed annihilation

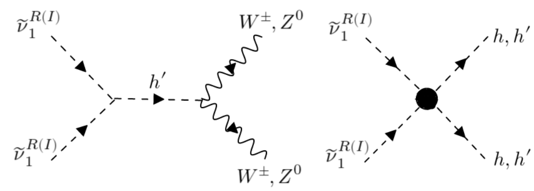

FIGURE 4: The Feynman diagrams of the dominant interaction of the $B-L$ lightest RH sneutrino $\tilde{\nu}_1^{R(I)}$ terms of two real or two imaginary RH sneutrinos without the Sommerfeld enhancements.

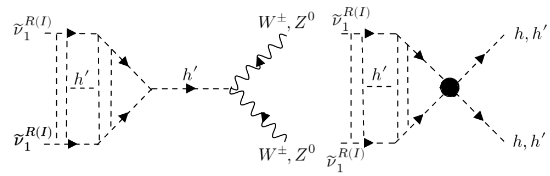

FIGURE 5: All possible annhilation of RH sneutrinos $\tilde{\nu}_1^{R(I)}$ with SE.

g_{BL}	0.51	$\tilde{g}$	-0.14	$\tan\beta$	20
$\tan\beta'$	1.2	v'	7 TeV	Y_S	$\mathcal{O}(3)$
Γ_{17}^R	-0.707	Γ_{14}^R	0.707	Y_ν	0.01
Γ_{31}	-0.001	Γ_{32}	-0.006	Γ_{33}	0.72
Γ_{34}	0.68	Γ_{11}	0.04	Γ_{12}	0.99
Γ_{13}	0.045	Γ_{14}	0.08		
$\mu' \simeq T_S$	17, 17.5, 18 TeV	which leads to			
Y	1.28, 3.285, 4.8 TeV	λ_2	0.6	$g_{w^\pm}$	0.3
λ_4	4.485, 5, 5.36	λ_3	25 GeV	g_{ZZ}	0.4

cross sections lead to a very large relic abundance $\Omega_{\tilde{\nu}_1} h^2$ (typically much larger than one), and for few points in the parameter space, it can be within the 5σ allowed region by the Planck collaboration: $0.11 < \Omega h^2 < 0.13$.

The numerical values of the above parameters are computed by SARAH and SPheno programs [16, 17]. In Figure 6, we present the relic abundance of $\tilde{\nu}_1$ DM with and without Sommerfeld enhancement as function of $\tilde{\nu}_1$ mass for the following three benchmark points.

From Figure 6, it is clear that the relic abundance can only be consistent with the observational limits without the Sommerfeld effect for very special values of right-handed sneutrino mass that render certain fine-tuning. However, with the Sommerfeld effect, these constraints are met for a wider region of $m_{\tilde{\nu}_1}$ and a right-handed sneutrino where the mass of order one TeV remains a viable DM.

5. CONCLUSIONS

We have discussed BLSSM-IS in which the lightest right-handed sneutrino $\tilde{\nu}_1^{R(I)}$ (600 GeV–2.5 TeV) dark matter candidate is present. In particular, we studied BLSSM-IS motivated by light CP-even Higgs h', which is associated with $U(1)_{B-L}$ symmetry breaking. This Higgs boson can be as light as 28, 60, and 90 GeV with a large coupling to the right-handed sneutrino

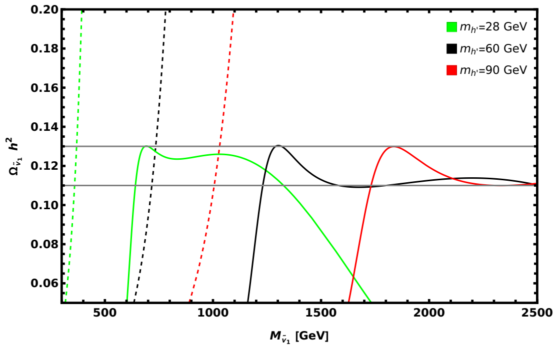

FIGURE 6: Relic density of the right-handed sneutrino as a function of its mass $\tilde{\nu}_1$ without Sommerfeld enhancement (Dashed plot) and with Sommerfeld enhancements (Sold plot) for $Y \sim 2m_{\tilde{\nu}_1}$ (Green, Black, and Red plots), with $m_{h'} = 28, 60, 90\,\text{GeV}$, respectively. The horizontal lines correspond to 5σ of the observed limits for the relic abundance.

($Y_S \approx \mathcal{O}(3)$). Hence, it naturally mediates the lightest RH sneutrino $\tilde{\nu}_1$.

In general and without SE, the relic abundance of right-handed sneutrino is quite large and exceeds the observational limits for most of the parameter space of the BLSSM-IS model.

The lightest right-handed sneutrino DM annihilations in this scenario are considerably enhanced by the Sommerfeld effect. Through this enhancement, the right-handed sneutrino could be a viable DM candidate, with relic abundance within the observational limits: $\Omega h^2 = 0.12 \pm 0.001$, for masses between 600 GeV and 2.5 TeV.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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