On Self-Consistency in Quantum Field Theory

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Abstract

A bootstrap approach to the effective action in quantum field theory is discussed which entails the invariance under quantum fluctuations of the effective action describing physical reality (via the *S*-matrix).

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1. INTRODUCTION

These are times of contemplation and reorientation in quantum field theory. With the experimental detection of the Higgs boson in 2012, finally, the finishing stone of the Standard Model of elementary particle physics [1] surfaced. On the theoretical side, the Standard Model is based on the concept of renormalized local quantum field theory. The confidence in this concept originally and primarily relies on the extraordinary success of the centerpiece of the Standard Model, quantum electrodynamics (QED), which exhibits an impressive agreement between theory and experiment (cf., e.g., [2], for more comprehensive reviews, see [3]). The successful application of renormalized local quantum field theory to the other components of the Standard Model, the electroweak theory, to quantum chromodynamics (QCD) has further advanced this confidence. On the other hand, many practicing quantum field theorists are aware of the many shortcomings and deficiencies of the concept of renormalized local quantum field theory which, by the way, has changed and developed in a multifold way in the decades since its birth at the end of the 1940s. To name a few of these issues, we mention here the occurrence of ultraviolet (UV) divergencies, the cosmological constant problem, hierarchy and naturalness problems, and Haag's theorem. (For an instructive illustration of the views of a number of well-known quantum field theorists, see, e.g., the conference volume [4].) It should, however, be pointed out that in the quantum field theory community there is no unified view which of these issues constitute features and which are problematic aspects of renormalized local quantum field theory. Correspondingly, opinions on which direction should be chosen for the future conceptual and technical development of quantum field theory are diverse. (For a recent account of the current situation, see [5].) While many active researchers might favor new ideas which have not been discussed in the past, a certain fraction of the quantum field theory community might be willing to not completely disregard past ideas which have largely been bypassed so far. In the present article, it is our intention to bring together a couple of thoughts and ideas (supplemented by the corresponding references) that have emerged in the past. We hope that the collection of this information in a single place will be beneficial to those readers who consider voices from the past as an inspiration for future research rather than purely as a matter for historians of science.

Let us start by pointing out that with reference to the UV divergency problem in QED some of the very fathers of this theory have repeatedly expressed their dissatisfaction with their own creation up to the end of their lives. So, Richard Feynman stated in 1965 in his Nobel Prize speech quite frankly: "…, I believe there is really no satisfactory quantum electrodynamics, but I'm not sure. …, I think that the renormalization theory is simply a way to sweep the difficulties of the divergences of electrodynamics under the rug." ([6], *Science* p. 707, *Phys. Today* pp. 43/44, *Prix Nobel* p. 189, *Nobel Lectures* p. 176, *Selected Papers* p. 30). Now, one might think that Feynman has later, after the development of the Wilsonian view on renormalization in the early 1970s and the emergence of the effective field theory concept, changed his view. However, this seems not to be the case and one can read in Feynman's popular science book "*QED—The Strange Theory of Light and Matter*" the passage (n and j are the bare counterparts of the physical electron mass m and electron charge e, respectively): "The shell game that we play to find n and j is technically called 'renormalization.' But no matter how clever the word, it is what I would call a dippy process! Having to resort to such hocus-pocus has prevented us from proving that the theory of quantum electrodynamics is mathematically self-consistent. It's surprising that the theory still hasn't been proved self-consistent one way or the other by now; I suspect that renormalization is not mathematically legitimate. What is certain is that we do not have a good mathematical way to describe the theory of quantum electrodynamics: such a bunch of words to describe the connection between n and j and m and e is not good mathematics." ([7], 1st ed. 1985, pp. 128-129, 2nd ed. 2006, pp. 127-128). In a similar way, Paul Dirac stated in a lecture in 1975 (published in 1978): "Hence most physicists are very satisfied with the situation. They say: 'Quantum electrodynamics is a good theory, and we do not have to worry about it any more.' I must say that I am very dissatisfied with the situation, because this so-called 'good theory' does involve neglecting infinities which appear in its equations, neglecting them in an arbitrary way. This is just not sensible mathematics. Sensible mathematics involves neglecting a quantity when it turns out to be small—not neglecting it just because it is infinitely great and you do not want it." ([8], p. 36). A few years later, in 1980 (published in 1983), Dirac repeats his critical view: "Some new relativistic equations are needed; new kinds of interactions must be brought into play. When these new equations and new interactions are thought out, the problems that are now bewildering to us will get automatically explained, and we should no longer have to make use of such illogical processes as infinite renormalization. This is quite nonsense physically, and I have always been opposed to it. It is just a rule of thumb that gives results. In spite of its successes, one should be prepared to abandon it completely and look on all the successes that have been obtained by using the usual forms of quantum electrodynamics with the infinities removed by artificial processes as just accidents when they give the right answers, in the same way as the successes of the Bohr theory are considered merely as accidents when they turn out to be correct." ([9], p. 55). The weight one might be tempted to assign to these views certainly will depend on the scientific taste of each theoretician; however, at least one should take note of them.

It often happens in the course of the development of science that early considerations and ideas are more fundamental than those emerging later. This is easily explainable by the fact that at the early stages of the development of a subject one enters largely unchartered territory and simple and structural ideas are then needed to choose the right road to scientific progress. Sometimes, conflicting ideas are competing with each other. Initial dominance of one idea does not necessarily mean that less successful concepts should be written off. It happens from time to time that these disregarded concepts make a surprising return for one reason or the other. Consequently, a look into the past (science history) may be helpful for shaping the future. For the following considerations, we will depart from such an element of science history.

2. EXTENDING EARLY THOUGHTS OF WOLFGANG PAULI

Let us start our concrete discussion with a statement made by Wolfgang Pauli in a private letter (in German) to Victor Weisskopf (by then, assistant to Wolfgang Pauli at the ETH Zurich) in 1935. The comment of Wolfgang Pauli concerns the self-energy of the electron in QED, a theory that was under development in those days. In the context of the struggle with the UV divergencies of QED, Wolfgang Pauli expresses the following conviction (English translation in brackets: K. S.): "... (Ich glaube allerdings, daß in einer vernünftigen Theorie die Selbstenergie nicht nur endlich, sondern Null sein muß ... [... (However, I believe that in a sensible theory the self-energy has not only to be finite but zero ...]" ([10], p. 779 of: Letter [425b] of December 14, 1935 from Pauli to Weisskopf. Part of: Nachtrag zu Band I: 1919–1929 und II: 1930–1939, pp. 733–826). How can we understand this expectation of a future correct quantum electrodynamical theory expressed by Wolfgang Pauli? If one starts quantizing the theory (in this case, charged fermions interacting with the electromagnetic field) on the basis of a Lagrangian with the physical (i.e., measured) value of the mass of the fermions inserted, all physical processes that can conceivably have an impact on that mass have effectively been taken into account already. Consequently, the total impact of all physical processes taken into account in the (theoretical) process of quantization (i.e., taking into account quantum fluctuations) on the mass of these fermions should vanish (nonrenormalization). This statement can be reformulated by saying that the fermion mass should not receive any radiative corrections under quantization. One can now extend this early point of view of Wolfgang Pauli and consider not only starting quantization with the physical value of the fermion mass in the initial Lagrangian but choosing as initial Lagrangian (in an arbitrary theory now) the (effective) Lagrangian which describes the physical world (with all its—infinitely many—nonlocal and nonpolynomial terms). In principle (in theory, not in practice, of course), this can be read off from scattering experiments. (For the connection between the scattering matrix and the effective action see, e.g., [11], Sec. 2.4, [12].) If one now starts the process of quantization with this "true" Lagrangian, all radiative corrections to it should vanish because any quantum fluctuations described by this Lagrangian have already been taken into account in this Lagrangian. Consequently, the physical (effective) Lagrangian should be invariant under the process of quantization—all radiative corrections should vanish. This view of the process of quantization amounts to bootstrapping the effective action of a theory. Quantities denoted within the standard local renormalizable quantum field theory as bare and renormalized ones, respectively, then coincide.

Before continuing our verbal discussion, let us make the above statements more precise in terms of equations. We consider within a path integral framework Lagrangian quantum field theory in flat (n-dimensional Minkowski) space-time and a (one-component) scalar field theory to pursue the discussion (for the following equations, cf., e.g., [13], Chap. 9). A generalization to more complicated theories is straightforward. The generating functional of Green functions of the scalar field $\phi(x)$ is given by the equation

$$Z[J] = C \int D\phi \, \mathrm{e}^{i\Gamma_0[\phi] + i\int d^n x\, J(x)\phi(x)}, \tag{1}$$

where $\Gamma_0[\phi]$ is the so-called classical action of the theory and C some fixed normalization constant. Then, the generating functional of the connected Green functions is

$$W[J] = -i \ln Z[J]. \tag{2}$$

The effective action $\Gamma[\bar{\phi}]$, which is also the generating functional of the one-particle-irreducible (1PI) Green functions, is obtained as the first Legendre transform of $W[J]$:

$$\Gamma[\bar{\phi}] = W[J] - \int d^n x\, J(x)\bar{\phi}(x). \tag{3}$$

Here

$$\bar{\phi}(x) = \frac{\delta W[J]}{\delta J(x)}, \tag{4}$$

which in turn leads to

$$\frac{\delta \Gamma[\bar{\phi}]}{\delta \bar{\phi}(x)} = -J(x) \tag{5}$$

in analogy to the classical field equation for $\Gamma_0[\phi]$. Equivalently, using the above expressions

$$\mathrm{e}^{i\Gamma[\bar{\phi}]} = C \int D\phi \, \mathrm{e}^{i\Gamma_0[\phi + \bar{\phi}] + i\int d^n x\, J(x)\phi(x)} \tag{6}$$

can be considered as the defining relation for the effective action, where the r.h.s. of the above equation has to be calculated using a current $J(x)$, given by equation (5), which is a functional of $\bar{\phi}$. Equation (1) defines a map, $g_1 : \Gamma_0[\phi] \longrightarrow Z[J]$, from the class of functionals called classical actions to the class of functionals Z. Furthermore, we have mappings, $g_2 : Z[J] \longrightarrow W[J]$ (equation (2)) and $g_3 : W[J] \longrightarrow \Gamma[\bar{\phi}]$ (equation (3)). These three maps together define a map $g_3 \circ g_2 \circ g_1 = f : \Gamma_0[\phi] \longrightarrow \Gamma[\bar{\phi}]$ (equation (6)) from the set of so-called classical actions to the set of effective actions. It is understood that in order to properly define the map a regularization scheme for the scalar field theory is applied. Up to renormalization effects, the classical action $\Gamma_0[\phi]$ determines the effective action $\Gamma[\bar{\phi}]$ uniquely via the map f which encodes quantum principles. In this standard scheme, the (quantum) effective action is built directly from classical physics and exhibits no independence in its own right.

The terms of the difference $\Delta\Gamma[\bar{\phi}] = \Gamma[\bar{\phi}] - \Gamma_0[\bar{\phi}]$ are denoted as "radiative corrections." The above verbal reasoning in generalization of early thoughts of Wolfgang Pauli leads to the equation

$$\Delta\Gamma[\bar{\phi}] = 0, \tag{7}$$

expressing the vanishing of all radiative corrections, i.e.,

$$\Gamma[\bar{\phi}] = \Gamma_0[\bar{\phi}]. \tag{8}$$

The equation for the complete effective action which is equivalent to the fixed point condition for the map f reads (C' is some normalization constant)

$$\mathrm{e}^{i\Gamma[\bar{\phi}]} = C' \int D\phi \, \mathrm{e}^{i\Gamma[\phi + \bar{\phi}] + i\int d^n x \, J(x)\phi(x)}, \tag{9}$$

where

$$J(x) = -\frac{\delta\Gamma[\bar{\phi}]}{\delta\bar{\phi}(x)}. \tag{10}$$

The above self-consistency equation (9) defines the (finite) effective action (including its coupling constants and mass ratios) without any reference to classical physics exclusively via quantum principles encoded in the map f. The fixed points of the map f then describe physical reality. From this perspective, the standard formulation of quantum field theory represented by equation (1) can roughly be understood as the first step of an iterative solution of the nonlinear functional integrodifferential equation (9) by applying the map f to some initial (in this case "classical") action $\Gamma_0[\phi]$. For the first time, equation (9) to be taken as the basis of quantum field theory has been proposed in 1972 by L. V. Prokhorov [14]. Not being aware of the earlier work by Prokhorov, the same proposal has been made by the present author in 1993 [15]. In a somewhat different (Hamiltonian) setting (coupled cluster methods), J. S. Arponen has expressed similar ideas in 1990 ([16], p. 173, paragraph starting with the words: "The possible solution corresponds to a system which suffers no change under quantization.").

3. FURTHER DISCUSSION

Given the mature state of standard renormalizable quantum field theory, the above point of view (defining the effective action as a fixed point of the map f) faces myriads of objections. Some of them may be misunderstandings, others are completely justified concerns, and others yet are possibly prejudices. Misunderstandings can be dealt with most easily—by clarifications. For example, one might ask: Given the crucial role of radiative corrections within the standard formulation of quantum field theory in correctly describing physical reality (for example, in QED) how could one ever possibly think of a theory of physical reality characterized by the vanishing of all radiative corrections (in an interacting theory)? The difficulty here, however, is just a terminological one. Of course, also a modified formulation of quantum field theory needs to deliver those kinds of terms in the effective action we denote as radiative corrections within the established standard approach. While the analytical expression yielded from a modified formulation of quantum field theory may differ from those within the standard formulation, the numerical results for experimentally accessible quantities (e.g., the anomalous magnetic moment of the electron) need to be (almost—within experimental limits) the same. The point is that in the modified view of quantum field theory represented by equation (9) those terms denoted in the standard formulation as radiative corrections are already incorporated in the action to be quantized. But, as the action to be quantized is supposed to be invariant under quantization (according to equation (9)), no new terms may emerge; consequently, in the modified formulation of quantum field theory, no radiative corrections (relative to the initial action to be quantized) occur.

Certainly, one elementary and justified concern with respect to equation (9) consists in the question of whether equation (9) allows any nontrivial (i.e., non-Gaussian) solutions (free field theories, of course, always obey equation (9)). In fact, it has been shown by example in a finite-dimensional Grassmann algebra analog of equation (9) (i.e., within a fermionic zero-dimensional field theory) that equation (9) has exact nontrivial (i.e., non-Gaussian) solutions [17]. For a (bosonic) example from standard analysis, see [18]. Of course, as has been pointed out by Prokhorov [14] from the outset, equation (9) represents a very complicated equation and presently very little can be said about its eventual nontrivial solutions in general. Experience from effective action studies in quantum field theory tells us that nontrivial solutions of equation (9) can be expected to be nonlocal and nonpolynomial functionals of fields. Whether these nonlocal and nonpolynomial actions Γ solving equation (9) preserve unitarity and causality can only be decided once they are found. However, it has been shown for a wide class of nonlocal and nonpolynomial (scalar) quantum field theories in the past [19, 20] that they respect these two principles—a fact from which one may derive certain optimism. It is of course conceivable that the general version of equation (9) (written down for an arbitrary but fixed collection of fluctuating fields) does not have any non-Gaussian solution at all for the certain field content one has chosen. If this was the case, the existence of a non-Gaussian solution to the generalized version of equation (9) could be applied as a theory selection criterion perhaps in the same way as the (stationary) Schrödinger equation selects energy (eigen)values of quantum mechanical systems. In a certain sense, at the end of the day, only non-Gaussian solutions of equation (9) are physical ones because only they provide the interactions for the structures we observe in physical reality. Away from the rigid concept discussed above of the effective action as a fixed point of the map f, non-Gaussian solutions of equation (9) may be considered also interesting within the standard lore. While usually perturbation theory is built around a Gaussian solution of equation (9), choosing a non-Gaussian solution of equation (9) as a starting point for perturbation theory may also be of some interest. For a discussion in this direction, see [21].

From a methodological point of view, the largest difference of the approach represented by equation (9) to the established approach in standard quantum field theory consists in the following. Standard local renormalizable quantum field theory starts (among other things, e.g., choosing the space-time dimensionality) with the choice of the field content of the theory under consideration and the functional form of the (classical/bare) action Γ_0 (cf. equation (1)), a quantity which, in principle, is unobservable (due to the existence of radiative corrections). This is also true in the different versions of the Wilsonian approach to quantum field theory (inspired by the theory of phase transitions in statistical mechanics). While in a statistical mechanical system (e.g., a spin system modelling a certain microscopic condensed matter structure) the structure of the Hamiltonian defined on a lattice with fixed lattice spacing can in principle be linked to experimental data, this is not the case within quantum field theory where the bare action is not related to observation (for a related discussion, see, e.g., [22]). Consequently, in the established standard approach to quantum field theory, the theoretical description of physical reality (i.e., the effective action Γ) is inferred from quantities not accessible to the experiment in principle. In contrast, within the approach represented by equation (9), only the field content of the quantum fluctuations can be chosen, and the functional form of the effective action Γ is self-consistently restricted by its property to be a solution of this equation. Beyond free field theories, i.e., for non-Gaussian solutions of equation (9), this can be expected to be highly restrictive.

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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