An Overview of Standard Model Calculations for Higgs Boson Production and Decay

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Abstract

We present an overview of theoretical predictions for the production and decay of the Standard Model Higgs boson, focusing on fixed-order perturbative corrections. Developments in the decade since the Higgs boson discovery are reviewed and current state-of-the-art calculations are compared to the current and projected experimental precision at the LHC and the HL-LHC.

Keywords: Higgs boson, LHC, QCD, collider phenomenology, standard model
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1. INTRODUCTION

The Higgs field and the Brout-Englert-Higgs mechanism [1, 2, 3, 4, 5, 6] are cornerstones of the Standard Model (SM) of particle physics. The SM posits that elementary particles acquire mass by interacting with the (complex scalar) Higgs field and that there is a quantum excitation of this field, known as the Higgs boson.

The discovery of a 125 GeV resonance compatible with the predicted properties of the Higgs boson during Run 1 (2009–2013) of the LHC [7, 8], and the continued agreement with the SM predictions observed during Run 2 (2015-2018), simultaneously secured the SM as our current best understanding of fundamental particle physics and underscored the many open questions its discovery leaves regarding the Higgs sector, electroweak symmetry breaking, and the nature of dark matter. We leave the discussion of some of these open questions to a companion review. In the absence of clear Beyond Standard Model (BSM) signals pointing to a resolution of the open questions, a detailed exploration of the experimentally accessible properties of the Higgs boson is justified as an important avenue for improving our understanding of fundamental particle physics.

Alongside a broad search for new physics, a key goal of the LHC, HL-LHC, and future high-energy collider projects is to precisely measure the coupling of the Higgs boson to SM particles. Measurements of Higgs boson production and decay represent a concrete deliverable for future collider projects and will either unearth new physics in the Higgs sector, in the form of discrepancies between SM predictions and experiments, or constrain BSM models. The experimental program must be matched, and partly guided, by theoretical input, including precise predictions of the null hypothesis (i.e., precision SM calculations).

In recognition and in celebration of the 10-year anniversary of the discovery of the Higgs boson at the LHC, the goal of this short invited review is threefold:

(1) To recall the status of theoretical calculations for Higgs Boson production/decay prior to its discovery,
(2) To review the impressive progress in SM calculations since the discovery and what we have learned from them, with a focus on significant and recent developments,
(3) To outline why the ongoing drive for precise predictions is crucially important for investigating the Higgs sector, and highlight avenues for further progress.

This review is organized as follows. In Section 2, we introduce the main production and decay modes of the SM Higgs boson and summarise the current status of experimental measurements; in Section 3, we review the evolution of theory calculations and the current state of the art; in Section 4, we discuss the need for precision SM calculations for ongoing and planned experiments. We conclude in Section 5.

2. OVERVIEW OF PRODUCTION AND DECAY

By early 2012, prior experimental results and the experimental reach of the LHC led theorists to focus on producing predictions for Higgs boson production and decay for $100\,\mathrm{GeV} \lesssim M_H \lesssim 1\,\mathrm{TeV}$, with M_H being the mass of the Higgs boson. For this range of masses, the dominant production channels in the SM are, ordered from largest total cross section to the smallest, gluon fusion (ggF), vector-boson fusion (VBF), and associated production with vector bosons (VH), top quark pairs ($t\bar{t}H$), or a single top quark (tH). Example Feynman diagrams for each of these production channels are shown in Figure 1. We also show an example diagram for di-Higgs boson production via gluon fusion, a rarer process that has not yet been observed at the LHC.

Unlike the production cross sections, the branching ratios for Higgs boson decay depend strongly on the Higgs boson mass. The $H \to b\bar{b}$ decay channel dominates for $m_H \sim 100\,\mathrm{GeV}$ and $H \to W^+W^-$ dominates for $m_H \gtrsim 160\,\mathrm{GeV}$. In Figure 2, we display the Higgs boson branching ratios as a function of the Higgs boson mass. Interestingly, the observed value of the Higgs boson mass means that there is a very rich variety of potential decay channels including $H \to b\bar{b}/c\bar{c}$, $H \to WW^*/ZZ^*$, $H \to gg$, $H \to \tau^+\tau^-/\mu^+\mu^-$, $H \to \gamma\gamma$, and $H \to \gamma Z$.

The performance of the experiments at the LHC has been exceptional, thanks in no small part to the diligence and ingenuity of the physicists and engineers involved. In Run 1, the Higgs boson was discovered primarily through its decay to bosons, $H \to ZZ \to 4f$ and $H \to \gamma\gamma$ (combined with $H \to WW \to 4f$) using data from all major production modes. At the end of Run 2, with $139\,\mathrm{fb}^{-1}$ of data collected by the ATLAS detector and $138\,\mathrm{fb}^{-1}$ of data from CMS, the experiments have observed Higgs boson production in all of the major production channels and have measured its coupling to photons,

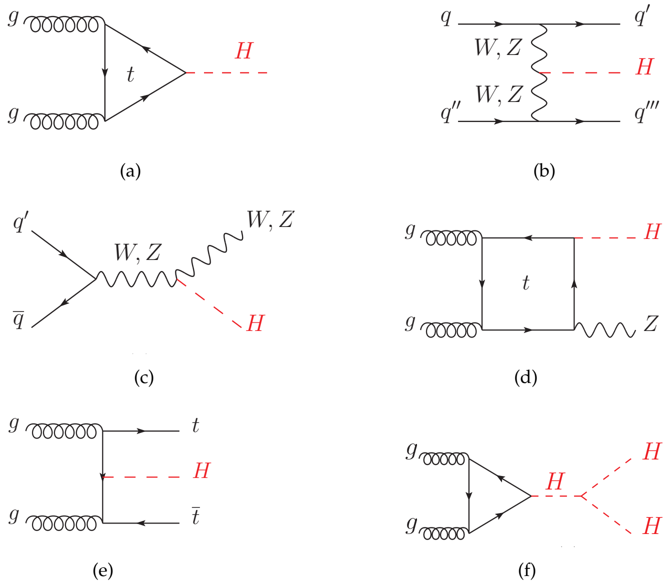

FIGURE 1: Example Feynman diagrams for the dominant Higgs boson production channels: (a) gluon fusion, (b) vector-boson fusion, (c, d) associated production with a vector boson, (e) associated production with top quarks, and (f) di-Higgs boson production via gluon fusion. Figure adapted from [9].

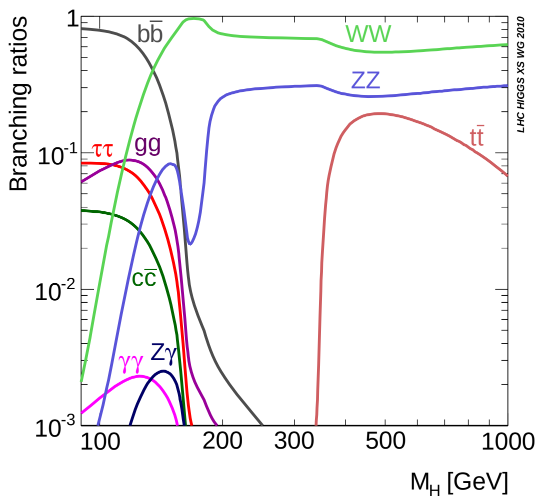

FIGURE 2: Theory predictions of the SM Higgs boson branching ratios as a function of its mass, M_H. Figure reproduced from [10].

Z/W bosons, and τ leptons at the 5–10% level. The Run 2 ATLAS [11] and CMS [12] results for the signal strength (the ratio of the experimentally measured Higgs boson yield to the SM theory prediction) are,

$$\begin{aligned}\mu_{\text{ATLAS}} &= 1.05 \pm 0.03(\text{stat.}) \pm 0.03(\text{exp.}) \\ &\quad \pm 0.04(\text{sig.th.}) \pm 0.02(\text{bkg.th.}), \\ \mu_{\text{CMS}} &= 1.002 \pm 0.029(\text{stat.}) \pm 0.033(\text{exp.}) \pm 0.036(\text{sig.th.}).\end{aligned} \tag{1}$$

The quoted errors are statistical uncertainties (stat.), experimental systematic uncertainties (exp.), and theoretical uncertainties in the signal (sig.th.) and background modeling (bkg.th.). The experimental and theoretical uncertainties are almost a factor of 2 lower than the Run 1 result. For the Run 2 results, current theory uncertainties are already similar to, or larger than, experimental and statistical errors. A detailed review of the experimental performance is left to a companion review. With Run 3 of the LHC now underway and the HL-LHC approved, it is clear that work to improve the precision of theoretical calculations for Higgs boson production and decay is well motivated.

3. CALCULATIONS FOR PRODUCTION AND DECAY

In this section, we will discuss each of the Higgs boson production channels in turn, followed by the decay channels. For each process, we begin by briefly reviewing the state of the art of SM calculations just prior to its discovery; for more detail, see [10, 13]; then, we will discuss the impressive precision theory progress that has been made since the discovery. Here, we omit the discussion of Higgs production in bottom quark fusion (known to N^{3}LO [14, 15]), $H+\geq 3$ jets, $tH+\bar{t}H$, and off-shell Higgs boson production. Detailed reviews of recent progress in precision theory for Higgs boson production and decay can be found in [16, 17, 18, 19]. We also point the interested reader to the reports of the LHC Higgs Working Group (LHCHWG)[1] on which much of this review is based [20]. A detailed list of recently completed fixed-order calculations and a wishlist of future calculations is produced biennially at the Les Houches: Physics at TeV Colliders workshop, see [19].

Throughout this review, we will use the following nomenclature to describe (next-to) leading order perturbative QCD and EW fixed-order corrections:

$$\begin{aligned} d\sigma_X &= d\sigma_X^{\text{LO}} \Bigg(1 + \sum_{k=1} \alpha_s^k d\sigma_X^{\delta \text{N}^k\text{LO}_{\text{QCD}}} \\ &\quad + \sum_{k=1} \alpha^k d\sigma_X^{\delta \text{N}^k\text{LO}_{\text{EW}}} + \sum_{k,l=1} \alpha_s^k \alpha^l d\sigma_X^{\delta \text{N}^{(k,l)}\text{LO}_{\text{QCD}\otimes\text{EW}}} \Bigg). \end{aligned} \tag{2}$$

We also use the notation N^{k}LL to indicate when a (next-to) leading logarithmic accuracy resummation has been applied to the fixed-order result.

The main sources of theoretical uncertainty common to all Higgs production and decay channels are the theoretical scale uncertainty, the parton distribution function (PDF) uncertainty, and the α_s uncertainty. Conventionally, the scale uncertainty is assessed by recomputing the prediction with the (arbitrary) renormalisation and factorisation scales increased or decreased by a factor of 2. The PDF uncertainty is assessed by recomputing the prediction using PDF "error sets" which are provided along with the PDF fit; this propagates the experimental and/or theoretical uncertainty associated with the underlying PDF to the observable in question. Similarly, the α_s uncertainty is assessed by recomputing the prediction with different values of the strong coupling.

[1] Formerly known as the LHC Higgs Cross Section Working Group (LHCHXSWG).

Importantly, we should always note that as we improve the precision of our measurements or our theoretical predictions, previously negligible sources of uncertainty can become important or even dominate our uncertainty. We will shortly see this truth in action in gluon fusion production, Higgs boson production in association with vector bosons, and di-Higgs production.

The focus of this review is on progress in precision fixed-order calculations. However, to compare with differential and fiducial experimental results, we often wish to combine fixed-order with parton showers or resummation. For many Higgs production processes, we are at or approaching the point where it is this combination and the precision of parton showers themselves that limits our ability to make predictions. The development of parton showers and their matching to fixed-order calculations is a very active area, with several groups pursuing NNLO + Parton Shower matching; see, for example, GENEVA [21, 22, 23, 24, 25, 26, 27], UNNLOPS [28, 29, 30], MINNLO [31, 32, 33, 32], and in improving the intrinsic accuracy of parton showers themselves, e.g., the PanScales project [34, 35, 36, 37, 38, 39].

In Table 1, we list the theory predictions available for various Higgs boson production channels, along with the (approximate) current experimental uncertainty and the projected uncertainty at the end of the HL-LHC.

3.1. *Gluon Fusion (ggF)*

The dominant gluon fusion production mode proceeds at leading order via a top quark loop which couples the incoming gluons to the Higgs boson. At LO, the process is proportional to α_s^2, the square of the QCD strong coupling, and therefore depends sensitively on the value of the coupling. Using the current world average of $\alpha_s(M_Z^2) = 0.1179 \pm 0.0009$ leads to an uncertainty of 2-3% on the gluon fusion total cross section.

Prior to the discovery of the Higgs boson, the gluon fusion mode was known at $\mathrm{NNLL} + \mathrm{NNLO}_{\mathrm{HTL}} + \mathrm{NLO}_{\mathrm{EW}}$ accuracy. The $\mathrm{NNLO}_{\mathrm{HTL}}$ correction [41, 42, 43, 44] was computed in the heavy top quark limit (HTL), an effective approximation in which the top quark mass is taken to be large in comparison to all other masses/scales. The NLO contribution [45, 46, 47, 48, 49] for this process is very large, increasing the cross section by 80–100% with the $\mathrm{NNLO}_{\mathrm{HTL}}$ corrections further enhancing it by 25%. The $\mathrm{NLO}_{\mathrm{EW}}$ corrections [50, 51, 52, 53] increase the cross section by 5% for $M_H = 125\,\mathrm{GeV}$. Without full knowledge of the mixed QCD-EW corrections, it was not clear how to combine the QCD and EW corrections; the dominant mixed corrections due to light-quarks were known in the $M_H \ll M_W$ limit [54] and indicated that the EW corrections should be multiplied onto the full QCD result (and not just the LO piece).

The higher-order perturbative corrections reduce the theoretical scale uncertainty and improve its reliability as an estimate of the true uncertainty. At low orders in perturbation theory, the conventional scale uncertainty is an unreliable estimate of the theory uncertainty. For example, the LO gluon fusion prediction has a conventional scale uncertainty of $\pm 25\%$ but does not overlap the NLO scale uncertainty band, while the NLO prediction has an uncertainty of $\pm 20\%$ and overlaps the NNLO band of $\pm 8\%$ but does not encompass the central prediction.

Since the discovery of the Higgs boson, the $\mathrm{N^3LO}_{\mathrm{HTL}}$ corrections have been obtained in a landmark calculation that redefined the frontier of what could be computed in perturbation theory [55, 56, 57]. The $\mathrm{N^3LO}_{\mathrm{HTL}}$ corrections enhance the cross section by 3% and reduce the scale uncertainty to just $\pm 2\%$. Efforts to compute the $\mathrm{N^4LO}_{\mathrm{HTL}}$ are ongoing [58], with approximate results computed using the soft-virtual approximation in the large-N_c limit already available [59]. With such a small QCD scale uncertainty, other sources of uncertainty need to be examined carefully, specifically the use of the HTL, the missing mixed QCD-EW corrections, and uncertainties related to the PDFs and α_s. Recently, an $\mathrm{NNLO}_{\mathrm{QCD}}$ calculation was completed including the top quark mass [60, 61]; the result reduces the total cross section by about 0.3% and significantly reduces the uncertainty related to the use of the HTL. In [62], the dominant gluon-induced light-quark $\mathrm{N^{(1,1)}LO}_{\mathrm{QCD}\otimes\mathrm{EW}}$ corrections were computed without the $M_H \ll M_W$ approximation, again favouring the complete factorisation of the EW corrections.

In Figure 3 we show a breakdown of the various sources of theoretical uncertainty on the gluon fusion total cross section as of the last LHCHWG recommendation. The $\delta(\mathrm{EW})$ (related to the missing mixed QCD-EW corrections) and $\delta(1/m_t)$ (related to the missing $\mathrm{NNLO}_{\mathrm{QCD}}$ calculation) uncertainties will be significantly reduced in the next update, thanks to the results described above. The uncertainty $\delta(\mathrm{PDF} + \alpha_s)$, due to our imprecise knowledge of the underlying PDFs and strong coupling, is large, and a more precise determination of these quantities would have a significant impact on the uncertainty. Finally, $\delta(\text{PDF-TH})$ parametrises the uncertainty due to the missing $\mathrm{N^3LO}$ PDFs; it could be eliminated by the computation of the 4-loop DGLAP splitting functions, the 3-loop transition matrix elements, and DIS coefficient functions as well as $\mathrm{N^3LO}$ matrix elements for hadronic cross sections. Approximate $\mathrm{N^3LO}$ PDFs produced using the available $\mathrm{N^3LO}$ data were presented in [63].

Beyond the total cross section, $\mathrm{N^3LO}_{\mathrm{HTL}}$ corrections are known fully differentially [64, 65, 66, 67, 68] and $\mathrm{N^3LL}' + \mathrm{N^3LO}_{\mathrm{HTL}}$ results are known for the Higgs p_T spectrum [69]. These results allow precise predictions to be made for fiducial cross sections (i.e., including experimental cuts) as well as for differential observables, which are being measured with increasing precision. We note that, at this level of precision, fixed-order observables can be very sensitive to the precise choice of experimental cuts which must be chosen to avoid sensitivity to infrared effects [70].

One differential observable of particular interest is high-p_T (boosted) Higgs boson production [71]. By demanding that the Higgs boson recoils against a hard jet, it is possible to resolve the structure of the loop connecting the gluons to the Higgs boson; heavy new particles beyond the SM have the potential to alter the tail of the p_T distribution. Above the top quark threshold, the HTL does not always provide reliable predictions and it is necessary to study the effects of a finite top quark mass. Results are known for Higgs boson production with an additional jet at $\mathrm{NLO}_{\mathrm{QCD}}$ accuracy, including the top quark mass [72, 73, 74, 75]. The NLO corrections are large and enhance the cross section by 66% (with a $p_T > 30\,\mathrm{GeV}$ cut applied to the jet), and the associated scale uncertainty is $\pm 16\%$. The dependence of the $\mathrm{NLO}_{\mathrm{QCD}}$ result on the renormalisation scheme used to define the top quark mass was studied in [76] and was found to be comparable to the scale uncertainty.

Process	Theory	σ_{th} [pb]	δ_{th} [%]	δ_{PDF} [%]	δ_{α_s} [%]	Run 2 δ_μ^{tot} [%]	HL-LHC δ_μ^{tot} (δ_μ^{th}) [%]
ggF	$\text{N}^3\text{LO}_{\text{HTL}} + \text{NLO}_{\text{EW}}$ $^\dagger\text{NNLO}_{\text{QCD}}^{(t)}$ $\text{NNLO}_{\text{HTL}}^{(1/m_T)}$ NLO_{QCD} $^\dagger\text{N}^{(1,1)}\text{LO}_{\text{QCD}\otimes\text{EW}}$	48.61	$^{+4.27}_{-6.49}$	±1.85	$^{+2.59}_{-2.62}$	±8	±1.6 (±1.2)
VBF	$\text{N}^3\text{LO}_{\text{QCD}}^{(\text{VBF}^*)}$ (incl.) $\text{NNLO}_{\text{QCD}}^{(\text{VBF}^*)}$ $\text{NLO}_{\text{EW}}^{(\text{VBF})}$	3.766	$^{+0.43}_{-0.33}$	±2.1	—	±15	±3.1 (±2.1)
WH	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	1.358	$^{+0.51}_{-0.51}$	±1.35	—	±18	±5.7 (±4)
ZH	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $\text{NLO}_{gg\to HZ}^{(\text{HTL})}$ $^\dagger\text{NLO}_{gg\to HZ}^{(t,b)}$	0.880	$^{+3.50}_{-2.68}$	±1.65	—	±18	±4.2 (±3.1)
$t\bar{t}H$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ $^\dagger\text{NNLO}_{\text{QCD}}^{(\text{sv})}$	0.5065	$^{+5.8}_{-9.2}$	±3.0	±2.0	±21	±4.3 (±3.7)
$ggF\ HH$	$\text{N}^3\text{LO}_{\text{HTL}}$ NLO_{QCD}	0.03105	$^{+2.2}_{-5.0}$	±2.1	±2.1	$-1.0 < \lambda/\lambda_{\text{SM}} < 6.6$	$0.5 < \lambda/\lambda_{\text{SM}} < 1.5$

TABLE 1: A comparison of theoretical and experimental results for Higgs boson production at $\sqrt{s} = 13$ TeV for $M_H = 125.09$ GeV. The uncertainties δ_{th}, δ_{PDF} and δ_{α_s} refer to the theoretical scale uncertainty, PDF uncertainty, and uncertainty due to α_s, on the total cross section, respectively. Where δ_{α_s} is omitted the uncertainty due to α_s has been included in δ_{PDF}. The predicted total cross section (σ_{th}) and theory uncertainty along with the projected HL-LHC uncertainty (HL-LHC δ_μ^{tot}) for the signal strength, μ, are taken from [77]. The total uncertainty on the signal strength (δ_μ^{tot}) includes both experimental and theoretical sources of uncertainty, the theoretical uncertainty (δ_μ^{th}) is reported separately for the HL-LHC projection. As shown in Figure 4, the HL-LHC δ_μ^{tot} error is dominated by the projected theory uncertainty for all Higgs production processes. The current uncertainties for the signal strength (Run 2 δ_μ^{tot}) are estimated from [11, 12], they include the theoretical uncertainties on the signal and background processes, which have a non-negligible contribution to the reported uncertainty. The subscripts/superscripts indicate partial or approximated calculations: HTL (Heavy Top Limit) results are obtained using the infinite top quark mass limit, $1/m_T$ results are expanded around large top quark mass, (t) results include only top quark corrections, (VBF) results have vector-boson fusion cuts applied and (VBF$*$) results additionally use the structure-function approximation, (sv) results are obtained using the soft-virtual approximation and $(gg \to ZH)$ results include only the loop induced gluon-fusion contribution. Theory results marked with † are not yet included in LHCHWG recommendations or in the numbers reported in the table. For the $ggF\ HH$ result, a large uncertainty of $^{+4\%}_{-18\%}$ due to the mass scheme uncertainty is not shown in the table. The $ggF\ HH$ δ_μ columns report the observed/expected limit on the Higgs boson trilinear self-coupling, λ, [78, 79] rather than the uncertainty on the signal strength.

3.2. *Vector-Boson Fusion (VBF)*

In the VBF process, a pair of W or Z bosons radiated off initial-state quarks fuse to form a Higgs boson. Experimentally, the Higgs boson is produced along with two forward jets which can be used to identify such events. The channel is sensitive to the coupling of the Higgs boson to the electroweak gauge bosons. A Higgs boson can also be produced with two or more jets via gluon fusion, which is, therefore, a background to the VBF signal.

Prior to the discovery of the Higgs boson, the VBF process was known at $\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ accuracy. The full NLO_{QCD} correction [80, 81, 82, 83, 84] decreases the total cross section by 3%, after VBF cuts are applied, with a $\pm1\%$ scale uncertainty. The NNLO_{QCD} corrections [85, 86] were computed using the structure-function approximation, in which the QCD corrections to W/Z emission from each quark line are computed separately and gluon exchange between the two quark lines is ignored [85, 86]; they reduce the cross section by 1% and reduce the scale uncertainty to $\pm0.5\%$. The NLO_{EW} corrections [87, 88, 89, 90] are around -5% inclusively but can be as large as -10% differentially.

In the years since the discovery, several substantial calculations have been completed. The $\text{N}^3\text{LO}_{\text{QCD}}$ correction is now known [91] and confirms the NNLO_{QCD} result, reducing the residual scale uncertainty to $\pm0.2\%$. The NNLO_{QCD} corrections were computed differentially in [92, 93]; these corrections were found to be very large in comparison to the inclusive correction, modifying distributions at the level of 4–7% and arguably motivating an $\text{N}^3\text{LO}_{\text{QCD}}$ differential calculation. The nonfactorisable QCD corrections (i.e., those neglected in the structure-function approach) have been computed using the eikonal approximation [94, 95]; after VBF selection cuts are applied, they are found to mostly be small and are usually contained within the scale uncertainty band of the structure-function calculation. The uncertainty of $\pm2\%$ due to the imprecise knowledge of the PDFs and α_s is large in comparison to the scale uncertainty. The uncertainty associated with the missing N^3LO PDFs of around 1% [91] is also not entirely negligible.

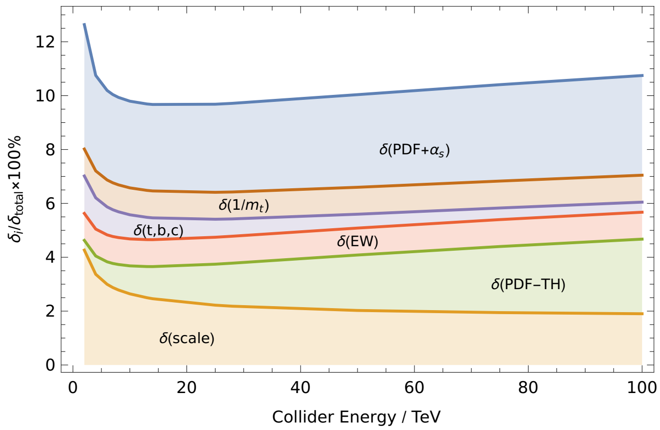

FIGURE 3: Breakdown of the current sources of theoretical uncertainty for the Higgs boson production via gluon fusion total cross section. Figure reproduced from [40].

The gluon fusion background can account for around 20–30% of the events passing VBF cuts but is known only at $\mathrm{NLO_{HTL}}$ (reweighted by $\mathrm{LO_{QCD}}$); see, e.g., [96, 97, 75]; it has a relatively large-scale uncertainty of $\pm 25\%$. A comparison of the existing VBF calculations, including irreducible backgrounds, was performed in [98].

Results for Higgs boson production with ≥ 2 jets have also been obtained beyond fixed-order, including the leading $\ln(s/p_T^2)$, using the "High Energy Jets" framework [99, 100, 101]. Parton shower effects for VBF can be sizable, with uncertainties at the level of $\pm 10\%$ for NLO accurate observables and $\pm 20\%$ for LO accurate observables; see, for example, [102].

3.3. *Associated Production with a Vector Boson (VH)*

In VH production, a Higgs boson is produced alongside a W or Z boson; at LO, this occurs via an off-shell W/Z boson radiating a Higgs boson. The decay products of the vector boson can be used to experimentally tag the event independently of the Higgs boson, which makes the channel useful in searches where reconstruction of the Higgs boson is difficult, for example, invisible/hadronic/$b\bar{b}$ Higgs decays.

For the dominant Drell-Yan-like piece, results have recently been computed at $\mathrm{N^3LO_{QCD}}$ accuracy for the inclusive cross section [103, 104]. Differentially, results are known at $\mathrm{NNLO_{QCD}}$ [41, 105, 106, 107, 108] and have been combined with $\mathrm{NLO_{EW}}$ corrections [109]. The full $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ corrections are also known [90, 110, 111]. The $\mathrm{NLO_{QCD}}$ results enhance the total cross section by 17% and leave a $\pm 1\%$ scale uncertainty. The $\mathrm{NNLO_{QCD}}$ corrections are at the 1% level but are largely canceled inclusively by the $\mathrm{N^3LO_{QCD}}$ corrections, leaving a residual scale uncertainty of $\pm 0.3\%$. The $\mathrm{NLO_{EW}}$ corrections reduce the cross section by 5–10% and impact the differential distributions substantially. Differential $\mathrm{NNLO_{QCD}}$ results have been obtained with the Higgs boson decaying to bottom quarks [112, 113, 114, 115], and results are also known at $\mathrm{NNLO_{QCD}}$ for VH production with an additional jet [116].

For the ZH production channel, there is an additional loop-induced $gg \to ZH$ contribution which enters at NNLO. Due to the large gluon luminosity at the LHC, it accounts for 10% of the total cross section. The $\mathrm{NLO_{QCD}}$ corrections to this loop-induced contribution were recently computed [117, 118, 119] and found to be large, enhancing the $gg \to ZH$ cross section by 100% and reducing the scale uncertainty to $\pm 15\%$. The uncertainty related to the top quark mass renormalisation scheme was studied in [118] and found to be as large as the scale uncertainty. The $gg \to ZH$ contribution is one of the largest sources of theoretical uncertainty for ZH production.

3.4. *Associated Production with Top Quarks ($t\bar{t}H$)*

In the $t\bar{t}H$ process, a Higgs boson is produced along with a $t\bar{t}$ pair. The process is directly sensitive to the top quark Yukawa coupling but is experimentally challenging to measure due to its small production cross section and complicated final state.

The $t\bar{t}H$ process is known at $\mathrm{NNLO_{QCD}^{(sv)}} + \mathrm{NLO_{EW}}$ accuracy. The $\mathrm{NLO_{QCD}}$ corrections were computed for the on-shell top quarks long ago [120, 121, 122, 123, 124] and found to enhance the cross section by 25% with a scale uncertainty of $\pm 7\%$. The $\mathrm{NLO_{EW}}$ corrections [125, 126] are negative and decrease the inclusive cross section by 1-2% and by 10% for boosted Higgs production; they have been combined with the $\mathrm{NLO_{QCD}}$ corrections in [127]. Results are known also for off-shell top quarks [128] and with Higgs decays [129]; they have been combined with $\mathrm{NLO_{EW}}$ corrections in [130]. Very recently, the flavour off-diagonal channels for of $t\bar{t}H$ were computed at $\mathrm{NNLO_{QCD}}$ [131]; the corrections were found to be a few per mille. The remaining channels were added in [132], using the soft Higgs boson approximation to estimate the unknown loop amplitudes. The $\mathrm{NNLO_{QCD}^{(sv)}}$ corrections increase the NLO result by 4% and reduce the scale uncertainty to $\pm 1.6\%$. Usually, the top quark mass in $t\bar{t}H$ is renormalised in the on-shell scheme, and the impact of this mass scheme choice was studied at $\mathrm{NLO_{QCD}}$ in [133] and found to be small in comparison to the NLO scale uncertainty.

The $t\bar{t}b\bar{t}$ process is an important irreducible background to $t\bar{t}H(\to b\bar{b})$. The full $\mathrm{NLO_{QCD}}$ corrections to the complicated $(2 \to 8)$ off-shell background process were computed in [134] and compared to the double-pole approximation.

At the HL-LHC, the statistical error on the measurement of $t\bar{t}H$ is expected to shrink to the level of a few percent, leaving a systematically dominated experimental uncertainty. However, the systematic uncertainties are currently dominated by the modeling of the signal and backgrounds, something that future theory input may help to improve.

3.5. *Di-Higgs Production (HH)*

The di-Higgs production processes are similar to those for single Higgs boson production, except that their cross sections are around 1000 times smaller. Each Higgs boson may decay via any of the usual channels, resulting in many possible final state combinations. Constraining HH production gives a direct probe of the Higgs boson trilinear self-coupling which, along with the quartic self-coupling, describes the shape of the Higgs potential. The physics of di-Higgs boson production is at least as rich as that of single Higgs boson production and, in the interest of brevity, we refrain from providing a thorough review of each channel and instead focus on the dominant gluon fusion channel. For a detailed overview, we refer the interested reader to [135, 20, 136].

The loop-induced gluon fusion di-Higgs production process receives contributions from two classes of diagrams at LO, the box-type diagrams, in which both Higgs bosons are radiated from a massive quark line, and the triangle-type diagrams in which a single off-shell Higgs boson splits into two Higgs bosons. The triangle-type diagrams depend on the Higgs boson trilinear self-coupling; in the SM, they interfere destructively with the box-type diagrams suppressing the total cross section. Unlike inclusive Higgs boson production, the HTL approximation is not reliable for HH production as the pair of Higgs bosons can be produced with a large invariant mass allowing the intermediate top quark loop to go on-shell. Results are known at $\mathrm{N^3LO_{HTL}}$ accuracy [137, 138, 139] and have been reweighted by the full two-loop $\mathrm{NLO_{QCD}}$ results [140, 141, 142, 143, 144]. The $\mathrm{NLO_{QCD}}$ corrections are large, enhancing the cross section by 66% with a scale uncertainty of $\pm 13\%$. The $\mathrm{NNLO_{HTL}}$ results [145] have been reweighted by the $\mathrm{NLO_{QCD}}$ result and supplemented with real radiation computed retaining the top quark mass [146]; they further enhance the cross section by 12% and reduce the scale uncertainty to $\pm 3.5\%$. The $\mathrm{N^3LO_{HTL}}$ results give a further 8% enhancement with a scale uncertainty of just $\pm 1.7\%$. The uncertainty associated with our imprecise knowledge of the PDFs and α_s is $\pm 3\%$.

The impact of the top quark mass is sizable for di-Higgs boson production. In [142, 144, 147], the impact of the uncertainty related to the top quark mass renormalisation scheme was studied and found to be similar in size to the $\mathrm{NLO_{QCD}}$ scale uncertainty. For the HTL-improved results, the mass scheme uncertainty is the largest source of theoretical uncertainty.

Searches for di-Higgs boson production measurements of the Higgs boson trilinear coupling are ongoing at the LHC. The HL-LHC is projected to obtain a 50% uncertainty on the Higgs boson trilinear self-coupling [79], while a future $\sqrt{s} = 100\,\mathrm{TeV}$ collider could achieve an accuracy of 5%; see, e.g., [148].

3.6. Decays

We now consider theory predictions for the SM Higgs boson decay channels. As discussed above, and shown in Figure 2, the mass of the Higgs boson, $M_H = 125.09 \pm 0.21(\mathrm{stat.}) \pm 0.11(\mathrm{exp.})\,\mathrm{GeV}$ [149], means that the SM Higgs boson has many possible decay channels. This makes for a very rich experimental program and, correspondingly, there is a need for precise calculations for many different processes.

In Table 2, we list the decay channels (ordered from the largest branching ratio to the smallest) and the available theory predictions for each channel [150]. The table also displays the corresponding QCD scale uncertainty, EW uncertainty, and an estimate of the total uncertainty. As for the production channels, many impressive calculations have been completed both prior to the discovery of the Higgs boson and in the intervening decade. Below, we will discuss only a few of the selected highlights and we refer the interested reader to more thorough reviews; see, for example, [16, 150].

The Higgs boson decays dominantly to $b\bar{b}$ quarks with a $H \to b\bar{b}$ branching ratio of 58%. The full $\mathrm{NLO_{QCD}}$ corrections, including the mass dependence, are known [151, 152, 153, 154, 155], and the leading mass effects are known up to $\mathrm{N^4LO_{QCD}}$ [156], leaving a small residual scale uncertainty. In the last few years, $H \to b\bar{b}$ decay has been computed at $\mathrm{N^3LO_{QCD}}$ fully differentially [14].

The Higgs boson decays $H \to WW^*$ and $H \to ZZ^*$ have branching ratios of 21% and 3%, respectively. The most significant $\mathrm{NLO_{QCD}} + \mathrm{NLO_{EW}}$ radiative corrections have been available for some time and can be evaluated using the Prophecy4f [157, 158] tool.

The $H \to gg$ decay accounts for 8% of SM Higgs boson decay. The full $\mathrm{NLO_{QCD}}$ corrections are large [159, 160, 45] and enhance the partial decay width by 70%, with a significant scale uncertainty. Results are known up to $\mathrm{N^4LO_{HTL}}$ in the HTL [161] and provide a further 20% enhancement as well as significantly reducing the scale uncertainty to around $\pm 1\%$. The scale uncertainty is smaller than the uncertainty of $\pm 2.5\%$ due to imprecise knowledge of α_s.

Higgs boson decay to a pair of photons has a branching ratio of just 0.2%. However, the $H \to \gamma\gamma$ decay has a distinctive experimental signature making it one of the Higgs boson discovery channels. The full two-loop $\mathrm{NLO_{QCD}}$ results are known and enhance the partial decay width by 2% with a small associated scale uncertainty. Results at 3-loop [162], expanded in M_H^2/M_t^2, and even at 4-loop in the HTL [163], are also known.

The $H \to Z\gamma$ and Dalitz decays, $H \to f\bar{f}\gamma$, have a branching ratio of 0.2%. The two-loop $\mathrm{NLO_{QCD}}$ corrections are known [164, 165, 166] and are found to be negligibly small. The dominant uncertainty, therefore, comes from the missing EW corrections which are complicated to evaluate in part due to the decay of the off-shell Z boson.

For Higgs boson decays, the most relevant radiative corrections are made available in the programs HDECAY [167, 168] and Prophecy4f [157, 158], which are used almost universally by the experiments.

Partial Width	Theory	δ_{QCD} [%]	δ_{EW} [%]	Tot. [%]
$b\bar{b}/c\bar{c}$	$\mathrm{N^4LO_{QCD}}$ $\mathrm{NLO_{EW}}$	0.2	0.5	0.5
WW/ZZ	$\mathrm{NLO_{QCD}}$ $\mathrm{NLO_{EW}}$	<0.5	0.5	0.5
$\tau^+\tau^-/\mu^+\mu^-$	— $\mathrm{NLO_{EW}}$	—	0.5	0.5
gg	$\mathrm{N^3LO_{QCD}}$ $\mathrm{NLO_{EW}}$	3	1	3
$\gamma\gamma$	$\mathrm{NLO_{QCD}}$ $\mathrm{NLO_{EW}}$	<1	<1	1
$Z\gamma$	$\mathrm{LO_{QCD}}$ $\mathrm{LO_{EW}}$	<1	5	5

TABLE 2: Available theory results for Higgs boson decay and the associated theoretical uncertainty due to missing higher-order QCD (δ_{QCD}) and EW (δ_{QCD}) corrections, along with the estimated total theoretical uncertainty. Table adapted from [150].

4. OUTLOOK

With just 5% of the data expected from the LHC and HL-LHC experimental programs analysed, already now the theoretical uncertainty on Higgs production and decay processes is non-negligible.

Run 3 of the LHC and the HL-LHC project, which aims to collect $3000\,\mathrm{fb^{-1}}$ of data, will continue to probe the Higgs sector of the SM with increasing precision. Over the coming years, we can expect improved fiducial cross section measurements, differential measurements probing the high-energy tails of Higgs production processes, and more precise determinations of the Higgs boson couplings. These measurements have the potential to uncover various BSM scenarios involving the Higgs sector or to provide improved constraints on the BSM parameter

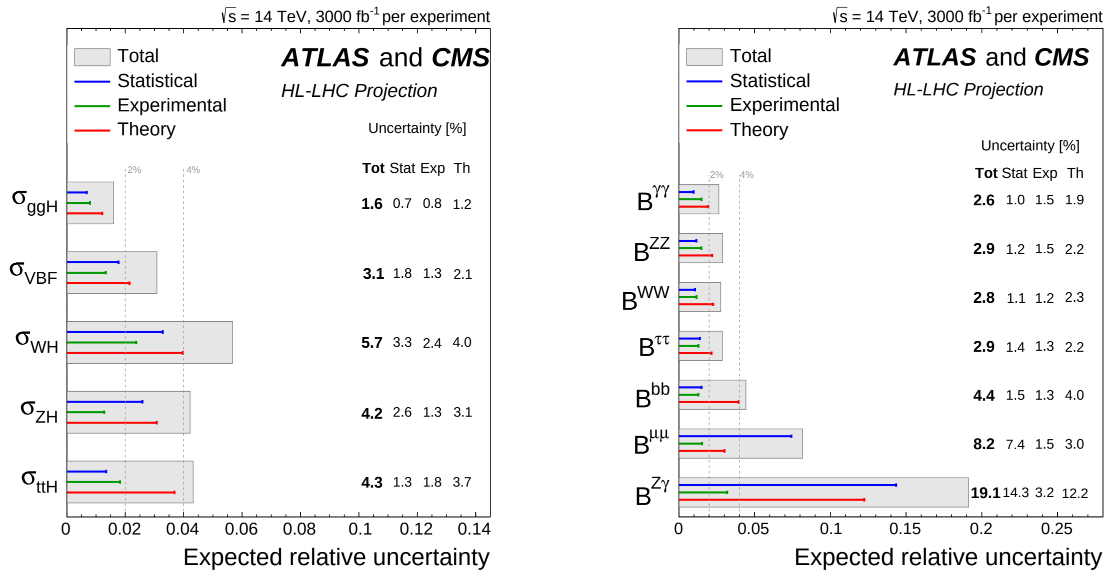

FIGURE 4: The expected 1σ uncertainties on the Higgs boson production cross sections (left panel) and branching ratios (right panel) normalised to the SM prediction at the HL-LHC. The projected statistical, experimental, and theory uncertainties are indicated by a blue, green, and red line, respectively. Figure reproduced from [77].

space. By studying, for example, high-p_T Higgs boson production, it will also be possible to probe the Higgs sector at higher energy than ever before.

In Figure 4, we show the projected uncertainties on the Higgs boson production cross sections and branching ratios normalised to the SM prediction. These projections are made assuming a reduction of the systematic uncertainties according to the improvements expected to be reached at the end of HL-LHC run, and theory uncertainties are halved compared to the current LHCHWG recommendations.

It is clear that an improved determination of α_s would have a significant impact on the precision of theoretical predictions for processes involving the Higgs boson. For a recent review containing a wish list of experimental and theoretical developments required to reduce the uncertainty on $\alpha_s(M_Z^2)$ to permille level in the next decade, see [169]. Improvements in the determination of PDFs and the production of N^{3}LO PDF sets would also have a significant impact on the theory uncertainty of Higgs boson production. For an overview of anticipated future developments for PDFs, see [170].

5. SUMMARY

In this review, we have summarised the immense progress made in computing higher-order QCD and EW corrections for Higgs boson production and decay. In the decade since the discovery of the Higgs boson, several landmark calculations have been completed that enable us to study the Higgs sector at unprecedented precision.

Not captured by this review, but underlying the incredible calculations we have discussed, are advances in our understanding of quantum field theory and the mathematical/computational techniques required to compute precise perturbative predictions, for a review of some of the developments; see, e.g., [171, 172]. These techniques have allowed us to connect the Lagrangian of the SM to concrete measurable observables at colliders and to experimentally test and refine our best understanding of three of the four known fundamental forces.

With the HL-LHC collider on the horizon, in order to best utilise the anticipated experimental results, the theory community now needs to achieve the very ambitious goal of halving the uncertainty on many key Higgs boson observables. The large increase in luminosity at the HL-LHC will also enable experiments to dramatically improve the precision of differential measurements and will require precise theoretical predictions differentially as well as for fiducial cross sections. Alongside this, it is essential to pursue improvements in the determination of PDFs and α_s as well as in the matching of fixed-order calculations to parton showers. Now, more than ever, the community needs to match the incredible performance of the last decade.

As a final note to this celebration of the discovery of the Higgs boson and the dawning of the Higgs precision era, we also look forward to the future. If we want to continue exploring fundamental particle physics and the Higgs sector in particular, it is crucial that we now commit to building high-energy colliders that will operate beyond the HL-LHC.

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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