A Particle Physics Model without Higgs

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Abstract

In this work, we describe the construction of a particle physics model where chiral symmetry, broken at the UV scale by "irrelevant" $d > 4$ operators, is recovered at low energy. In the critical, chiral symmetric theory, masses of elementary degrees of freedom are generated by a peculiar nonperturbative field-theoretical mechanism and not as in the Higgs scenario. Consistency of mass formulae with phenomenology requires the existence of a new sector of superstrongly interacting particles (denoted by Tera-particles), gauge invariantly coupled to Standard Model matter, living at an energy scale, Λ_T, of the order of a few TeVs. We give the expression of the full Lagrangian of a model encompassing quarks, Tera-quarks and W, as well as leptons, Tera-leptons, and B gauge bosons when, besides strong, there are Tera-strong and weak interactions, and also hypercharge is included. We prove that, upon integrating out the (heavy) Tera-DoFs, the resulting low-energy effective Lagrangian of the critical model essentially coincides with the Standard Model Lagrangian. This implies that the present model passes all the precision tests that the Standard Model is able to pass. There are a number of good reasons for considering speculative and unorthodox theories of this kind. First of all, unlike the Standard Model, in this scenario masses are not free parameters but are determined by the dynamics of the theory. Secondly, we have a physical understanding of the origin of the electroweak scale as the scale of a new interaction. Thirdly, we envisage a solution to the strong CP problem, and last but not least, the Higgs mass tuning problem does not even arise because there is no fundamental Higgs.

Keywords: beyond the standard model, Higgs, new particles, naturalness, strong CP problem
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1. INTRODUCTION

In this letter, we outline the construction of a beyond-the-Standard-Model model (bSMm) where chiral symmetry, broken at the UV scale by "irrelevant" $d > 4$ operators, is recovered at low energy. Elementary particle masses are not generated via the Higgs mechanism, but emerge as a sort of nonperturbative (NP) anomalies coming from a delicate interplay between residual chiral breaking effects surviving after chiral symmetry restoration, and IR features originating from the spontaneous breaking of the (recovered) chiral symmetry, which occurs in any strongly interacting theory.

The model is built by extending results derived in [1]. In particular, since NP elementary particle masses are proportional to the RGI scale of the theory (see equation (16)), we are lead to predict the existence of a superstrongly interacting sector of particles (denoted by Tera-particles as suggested in [2]) so that the full theory encompassing SM matter and Tera-particles will have an RGI scale $\Lambda_{\rm RGI} \equiv \Lambda_T \gg \Lambda_{\rm QCD}$ in the few TeV region, thus yielding top and electroweak (EW) boson masses of the correct order of magnitude.

In this work, we give the expression of the Lagrangian of this putative bSMm. We provide parametric formulae for the masses of elementary standard and Tera-particles from which interesting estimates of mass ratios and Λ_T can be extracted. A first account of these results is given in [3, 4].

Models of the kind developed here have a number of appealing theoretical features that allow solving some of the puzzling problems left open by the present formulation of the SM. We list them below.

(1) Elementary particle masses are not unconstrained Lagrangian parameters, like in the SM, but are dynamically determined. They are proportional to the RGI scale of the theory, times coefficient functions depending on the gauge couplings.

(2) There is no longer a Higgs mass tuning problem [5, 6] as there is no fundamental Higgs.

(3) The EW scale is naturally interpreted as (a fraction of) the scale of the new interaction, Λ_T.

(4) One can get a cheap solution (i.e., without axions) for the strong CP problem.

(5) It was proven in [7] that with a reasonable choice of spectrum and hypercharges of Tera-particles, a model extending the SM degrees of freedom (DoFs) with the inclusion of the Tera-sector leads to a theory with the unification of the running of the $U(1)_Y$, $SU(2)_L$, and $SU(N_c = 3)$ gauge couplings at a scale $\Lambda_{\rm GUT} \sim 10^{18}$ GeV. Overall unification of SM and Tera-gauge couplings can also be achieved by suitably tuning the Tera matter content.

We end this Introduction with two observations. First of all, as in [1, 3, 4], for now we mostly ignore weak isospin splitting and the existence of flavor and it is intended that our more phenomenological considerations refer to the heaviest of the flavor families. Secondly, we observe that the low-energy effective Lagrangian (LEEL) of the model (see Section 6) valid for (momenta)$^2 \ll \Lambda_T^2$ looks like the SM Lagrangian [3]. Indeed, in our scheme, the 125 GeV state detected at LHC, which we suggestively denote by h, is interpreted as a composite W^+W^-/ZZ scalar resonance, bound by Tera-strong exchanges, left behind after integrating out the heavy Tera-DoFs. As m_h is very small compared to the conjectured value of Λ_T, the existence of h needs to be accounted for in describing low-energy physics. Ignoring EW perturbative corrections, differences between the LEEL of this model and the SM Lagrangian may only appear in the effective trilinear and quadrilinear couplings of the h self-interactions. Support for the interpretation of the 125 GeV boson as a W^+W^-/ZZ bound state emerges both from some nonrelativistic calculations (that we do not repeat here [4]), as well as from an approximate Bethe-Salpeter-like approach. From the two computations we get consistent estimates of the WW binding energy of the order of the W mass itself, in line with the large experimental value $E_b = 2M_W - m_h \sim 160 - 125 = 35\,\mathrm{GeV}$.

1.1. *Plan of the Paper*

The plan of the paper is as follows. In Section 2, we provide the expression of the Lagrangian of a putative bSMm including SM as well as Tera-particles. Following the discussion in [1, 3, 4], we prove that in the Nambu-Goldstone (NG) phase of the chirally symmetric (critical) theory all elementary particles acquire an NP mass from the interplay between residual chiral breaking effects at the UV scale and IR features coming from the phenomenon of spontaneous chiral symmetry breaking. As a result masses, unlike in the SM, will be proportional to the RGI scale of the theory, Λ_T. In Section 3, we provide the leading loop order parametric expression of the masses of all the elementary particles. In Section 4, we illustrate a possible solution to the strong CP problem. In Section 5, we present an approximate Bethe-Salpeter-like calculation of the binding energy of the W^+W^-/ZZ composite state. In Section 6, we give the expression of the $d \leq 4$ quantum effective Lagrangian (QEL), which yields the full quantum information of the physics of the model, showing that it essentially coincides with the Lagrangian of the SM. In Section 7, we derive crude estimates of Λ_T and the mass of Tera-fermions and of the heaviest family of SM fermions in units of M_W, and we discuss the kind of assumptions and approximations entering our phenomenological mass estimates. Conclusions and a brief outlook of future lines of investigation can be found in Section 8.

2. THE bSMm LAGRANGIAN

The full bSMm Lagrangian can be obtained from equations (3.1)–(3.5) of [3] by adding kinetic, Yukawa and Wilson-like terms for the new particles (leptons and Tera-leptons), while at the same time appropriately extending the expression of the covariant derivatives to encompass $U(1)_Y$ interactions. As we said, we restrict ourselves to the one-family case and drop flavor indices. Since we lack at the moment a mechanism to remove family degeneracy, appending a flavor index to quarks and leptons would not add anything useful while making notations more clumsy. Even restricting to the one-family case and in the limit of unbroken weak isospin, one finds a rather lengthy expression (for preliminary results see [8]):

$$\begin{aligned}&\mathcal{L}_{\rm bSMm}(q,\ell,Q,L;\Phi;A,G,W,B)\\&\quad=\mathcal{L}_{\rm kin}(q,\ell,Q,L;\Phi;A,G,W,B)+\mathcal{V}(\Phi)\\&\qquad+\mathcal{L}_{\rm Yuk}(q,\ell,Q,L;\Phi)+\mathcal{L}_{\rm Wil}(q,\ell,Q,L;\Phi;A,G,W,B),\end{aligned}\tag{1}$$

where

$$\begin{aligned}&\mathcal{L}_{\rm kin}(q,\ell,Q,L;\Phi;A,G,W,B)\\&\quad=\frac{1}{4}\left(F^A\cdot F^A+F^G\cdot F^G+F^W\cdot F^W+F^B F^B\right)\\&\qquad+\left[\bar q_L\,\not{\mathcal{D}}^{BWA}q_L+\bar q_R\,\not{\mathcal{D}}^{BA}q_R+\bar\ell_L\,\not{\mathcal{D}}^{BW}\ell_L+\bar\ell_R\,\not{\mathcal{D}}^{B}\ell_R\right]\\&\qquad+\Big[\bar Q_L\,\not{\mathcal{D}}^{BWAG}Q_L+\bar Q_R\,\not{\mathcal{D}}^{BAG}Q_R\\&\qquad\qquad+\bar L_L\,\not{\mathcal{D}}^{BWG}L_L+\bar L_R\,\not{\mathcal{D}}^{BG}L_R\Big]\\&\qquad+\frac{k_b}{2}{\rm Tr}\left[\left(\mathcal{D}^{WB}_\mu\Phi\right)^\dagger\mathcal{D}^{WB}_\mu\Phi\right],\end{aligned}\tag{2}$$

$$\mathcal{V}(\Phi)=\frac{\mu_0^2}{2}k_b{\rm Tr}\left[\Phi^\dagger\Phi\right]+\frac{\lambda_0}{4}\left(k_b{\rm Tr}\left[\Phi^\dagger\Phi\right]\right)^2,\tag{3}$$

$$\mathcal{L}_{\rm Yuk}(q,\ell,Q,L;\Phi)=\sum_{f=q,\ell,Q,L}\eta_f\left(\bar f_L\Phi f_R+{\rm hc}\right),\tag{4}$$

$$\begin{aligned}&\mathcal{L}_{\rm Wil}(q,\ell,Q,L;\Phi;A,G,W,B)\\&\quad=\frac{b^2}{2}\rho_q\left(\bar q_L\overleftarrow{\mathcal{D}}^{BWA}_\mu\Phi\mathcal{D}^{BA}_\mu q_R+{\rm hc}\right)\\&\qquad+\frac{b^2}{2}\rho_\ell\left(\bar\ell_L\overleftarrow{\mathcal{D}}^{BW}_\mu\Phi\mathcal{D}^{B}_\mu\ell_R+{\rm hc}\right)\\&\qquad+\frac{b^2}{2}\rho_Q\left(\bar Q_L\overleftarrow{\mathcal{D}}^{BWAG}_\mu\Phi\mathcal{D}^{BAG}_\mu Q_R+{\rm hc}\right)\\&\qquad+\frac{b^2}{2}\rho_L\left(\bar L_L\overleftarrow{\mathcal{D}}^{BWG}_\mu\Phi\mathcal{D}^{BG}_\mu L_R+{\rm hc}\right).\end{aligned}\tag{5}$$

Following the notations of [1, 3, 4], we have indicated by $\mathcal{D}^X_\mu$ the covariant derivative with respect to the group of transformations of which $\{X\}$ are the associated gauge bosons. The most general expression of the covariant derivative is

$$\mathcal{D}^{BWAG}_\mu=\partial_\mu-iYg_YB_\mu-ig_w\tau^rW^r_\mu-ig_s\frac{\lambda^a}{2}A^a_\mu-ig_T\frac{\lambda^\alpha_T}{2}G^\alpha_\mu,\tag{6}$$

where Y, τ^r $(r=1,2,3)$, λ^a $(a=1,2,\ldots,N_c^2-1)$, and λ^α_T $(\alpha=1,2,\ldots,N_T^2-1)$ are, respectively, the $U(1)_Y$ hypercharge and the generators of the $SU(2)_L$, $SU(N_c=3)$, and $SU(N_T=3)$ groups with g_Y, g_w, g_s, and g_T being the corresponding gauge couplings. The scalar field, Φ, is a 2×2 matrix with $\Phi=(\phi\mid\tilde\phi)$, $\tilde\phi=-i\tau^2\phi^\star$ and ϕ an iso-doublet of complex scalar fields, which feels $U(1)_Y$ and $SU(2)_L$, but neither $SU(N_c=3)$ nor $SU(N_T=3)$ gauge interactions. We immediately note that, despite the appearances, Φ is not the Higgs boson, but rather an effective way to describe an $\chi_L\times\chi_R$-symmetric UV completion of the model (see Section 2.1).

For the $SU(2)_L$ SM matter doublets we use the notation $q_L=(u_L,d_L)^T$ and $\ell_L=(\nu_L,e_L)^T$. Right-handed components (q^u_R, q^d_R, and $\ell^u_R\equiv\nu_R$, $\ell^d_R\equiv e_R$) are $SU(2)_L$ singlets. However, since we do not remove the *up-down* weak isospin degeneracy, in equation (1), we have used for all fermions (including Tera-fermions) a “doublet-like” notation also for Right-handed components.

The distinctive feature of the Lagrangian (1) is the presence of the "irrelevant" (chiral breaking) $d = 6$ Wilson-like operator $\mathcal{L}_{\rm Wil}$, which occurs multiplied by two inverse powers of the UV cut-off, $b^2 \sim \Lambda_{UV}^{-2}$. This makes $\mathcal{L}_{\rm bSMm}$ power counting renormalizable, similar to what happens in the case of Wilson lattice QCD (WLQCD), despite the presence of the (chiral breaking) $d = 5$ Wilson term. Here, it is like if in $\mathcal{L}_{\rm bSMm}$ the Wilson r parameter was lifted to a field, i.e., $r \to b\Phi$.

2.1. Symmetries of the Lagrangian

Among other obvious symmetries, the Lagrangian (1) is exactly invariant under the (global) transformations $\chi_L \times \chi_R$, involving all fermions ($f = q, \ell, Q, L$), the W bosons, and the scalar, Φ, given by the formulae ($\Omega_{L/R} \in SU(2)$):

$$\chi_L \times \chi_R = [\tilde{\chi}_L \times (\Phi \to \Omega_L \Phi)] \times \left[\tilde{\chi}_R \times \left(\Phi \to \Phi\Omega_R^\dagger\right)\right], \quad (7)$$

$$\tilde{\chi}_L : \begin{cases} f_L \to \Omega_L f_L \quad \bar{f}_L \to \bar{f}_L \Omega_L^\dagger \\ W_\mu \to \Omega_L W_\mu \Omega_L^\dagger, \end{cases} \quad (8)$$

$$\tilde{\chi}_R : \quad f_R \to \Omega_R f_R, \quad \bar{f}_R \to \bar{f}_R \Omega_R^\dagger. \quad (9)$$

A key consequence of the $\chi_L \times \chi_R$ invariance is that the chiral noninvariant operators $m\bar{\psi}\psi$ can never be quantum-mechanically generated with either a finite or a linearly divergent coefficient. Besides, we remark that Tera-interactions have nothing to do with Technicolor. No mass can, in fact, be ever generated via the chiral Tera-condensates since the latter vanish as a consequence of the exact $\chi_L \times \chi_R$ symmetry.

For generic η_f and k_b, $\mathcal{L}_{\rm bSMm}$ is not invariant under the "chiral" transformations $\tilde{\chi}_L \times \tilde{\chi}_R$ (8)-(9) owing to the presence of $\mathcal{L}_{\rm Wil}$, $\mathcal{L}_{\rm Yuk}$, and the scalar kinetic term. Invariance under $\tilde{\chi}_L \times \tilde{\chi}_R$ can be recovered (up to $O(b^2)$ terms) enforcing the conservation of the $\tilde{\chi}_L \times \tilde{\chi}_R$ currents by appropriately tuning η_f, $f = q, \ell, Q, L$, and k_b. According to the strategy suggested in [9, 10] and adapted to the present situation in [1], this occurs at the "critical" values of η_f and k_b that solve the tuning equations:

$$\begin{aligned} \eta_f &= \bar{\eta}_f^L\left(\{\eta\}, k_b; \{g\}, \{\rho\}, \mu_0, \lambda_0\right), \\ \eta_f &= \bar{\eta}_f^R\left(\{\eta\}, k_b; \{g\}, \{\rho\}, \mu_0, \lambda_0\right), \\ k_b &= \bar{k}_b^L\left(\{\eta\}, k_b; \{g\}, \{\rho\}, \mu_0, \lambda_0\right) \end{aligned} \quad (10)$$

with $\{g = g_Y, g_w, g_s, g_T\}$, $\{\eta\}$ and $\{\rho\}$ denoting the set of gauge couplings, Yukawa, and ρ coefficients, respectively. The superscript L (R) refers to the condition derived by enforcing the conservation of the $\tilde{\chi}_L$ ($\tilde{\chi}_R$) current; see equation (8) (equation (9)). The function $\bar{\eta}_f^L$ ($\bar{\eta}_f^R$) is the mixing coefficient between the $\tilde{\chi}_L$ ($\tilde{\chi}_R$) rotations of the $d = 6$ Wilson-like operator of the f fermion and the $\tilde{\chi}_L$ ($\tilde{\chi}_R$) rotation of the corresponding $d = 4$ Yukawa term, while $\bar{k}_b^L$ is the mixing coefficient with the $\tilde{\chi}_L$ rotation of the scalar kinetic operator. The derivation of the above tuning conditions is given in Appendix A of [3].

According to the 't Hooft criterion [6], the tuning of η_f and k_b at their critical values is a costless and "natural" step as it leads to an enlargement of the theory symmetries.

In the next two subsections, we illustrate the nature and the implications of the critical conditions (10) in the Wigner phase, where the scalar potential $\mathcal{V}$ has a single minimum, and in the NG phase, where $\mathcal{V}$ has a typical Mexican hat shape. We limit for simplicity the discussion to the quark and Tera-quark sectors. Similar considerations would apply to the lepton sector.

2.2. The Critical Conditions in the Wigner Phase

In the upper and middle panels of Figure 1, we illustrate at the lowest 1-loop order the mechanism that in the Wigner phase leads to the cancellation of the chiral breaking quark and Tera-quark Yukawa terms, respectively. The figures show the mixing between the quark (Tera-quark) Yukawa operator (grey disks) and the quark (Tera-quark) Wilson-like operator (grey box). In the lowest panel, we report the leading 1-loop order diagrams yielding the cancellation between the scalar kinetic term (grey disk) and the contribution from the sum of the quark and Tera-quark Wilson-like operators (grey boxes). The empty boxes represent the insertion of Wilson-like vertices from the Lagrangian necessary to close the fermion loops.

The key observation is that, a part from a quadratically divergent scalar mass counter-term, the loop diagrams in Figure 1 yield finite results because loop UV power divergencies are exactly compensated by b^2 factors coming from Wilson-like vertices.

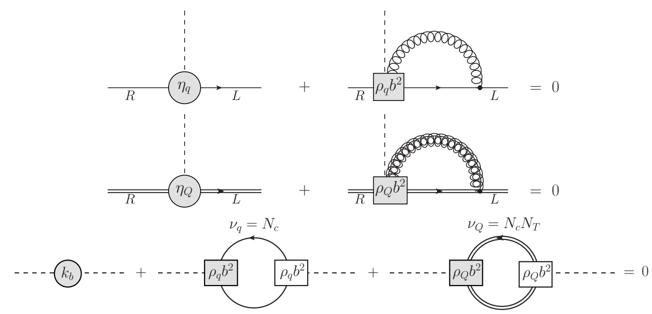

FIGURE 1: Upper (middle) panel: the cancellation mechanism of the quark (Tera-quark) Yukawa vertex implied by the tuning conditions determining $\eta_{q\,cr}$ ($\eta_{Q\,cr}$). The grey box, labeled by ρ_q (ρ_Q), represents the quark (Tera-quark) Wilson-like vertex. Lower panel: the cancellation mechanism of the scalar kinetic term implied by the tuning condition determining $k_{b\,cr}$. The integers ν_q and ν_Q are the multiplicities of quarks and Tera-quarks running in the loops with N_c and N_T being the number of colors and Tera-colors. The grey disc, labeled by k_b, represents the insertion of the scalar kinetic term. The empty boxes represent the insertion of Wilson-like vertices from the Lagrangian. Single lines represent particles and double lines Tera-particles. The figures show the lowest loop order Wigner phase diagrams.

2.3. The Critical Conditions in the Nambu-Goldstone Phase

In the NG phase, the criticality conditions for η_q, η_Q, and k_b imply the cancellations shown in Figure 2. We see that enforcing invariance under $\tilde{\chi}_L \times \tilde{\chi}_R$ implies that in the QEL of the critical theory, precisely the Higgs-like masses of quarks, Tera-quarks, and W's are killed. The diagrams in the upper and middle panels of Figure 2 are directly obtained from the corresponding diagrams in Figure 1 by setting the scalar field equal to its vev. The diagrams in the lowest panel of Figure 2 follow from the corresponding ones in Figure 1 using $SU(2)_L$ gauge invariance and setting the scalar at its vev.

Again all the loop diagrams in Figure 2 yield finite results because, as before, the loop UV power divergencies are exactly compensated by the b^2 factors coming from the insertion of Wilson-like vertices.

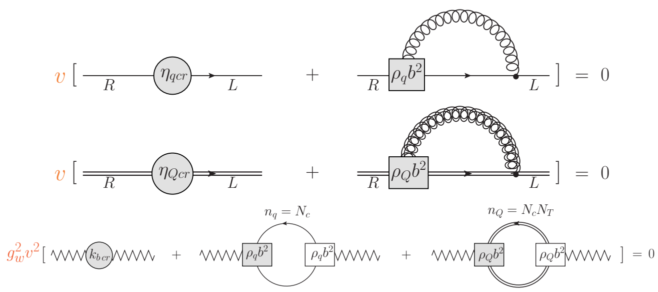

FIGURE 2: Upper (middle) panel: the cancellation mechanism of the Higgs-like mass term of quarks (Tera-quarks). Lower panel: the cancellation mechanism of the Higgs-like W mass term. Wiggly lines are W's. The rest of the notations are as in Figure 1. The figures show the lowest loop order NG phase diagrams.

3. NONPERTURBATIVE EFFECTS

Following the NP diagrammatic analysis developed in [1, 3, 4], one can derive the list of the formally $O(b^2)$ Symanzik operators that need to be taken into account to describe the NP effects coming from the spontaneous breaking of the recovered $\tilde{\chi}_L \times \tilde{\chi}_R$ (chiral) symmetry. The list in equations (3.11)–(3.13) of [3] must be extended to include the extra $\chi_L \times \chi_R$ invariant $d = 6$ operators involving leptons, Tera-leptons, and the B boson. Limiting to the operators entering the NP diagrams of Figure 3, one finds the CP-invariant operators

$$O^T_{6,\bar{Q}Q} \propto b^2 \Lambda_T \alpha_T |\Phi| \left[\bar{Q}_L \,\tilde{\mathcal{D}}^{BWAG} Q_L + \bar{Q}_R \,\tilde{\mathcal{D}}^{BAG} Q_R \right], \quad (11)$$

$$O^T_{6,\bar{L}L} \propto b^2 \Lambda_T \alpha_T |\Phi| \left[\bar{L}_L \,\tilde{\mathcal{D}}^{BWG} L_L + \bar{L}_R \,\tilde{\mathcal{D}}^{BG} L_R \right], \quad (12)$$

$$O_{6,AA} \propto b^2 \Lambda_T g_s^2 |\Phi| F^A \cdot F^A, \quad (13)$$

$$O^T_{6,GG} \propto b^2 \Lambda_T g_T^2 |\Phi| F^G \cdot F^G, \quad (14)$$

$$O^T_{6,BB} \propto b^2 \Lambda_T g_Y^2 |\Phi| F^B \cdot F^B. \quad (15)$$

A physical interpretation of the NP origin of the Symanzik operators (11)–(15), as well as a derivation of the ρ, N_c, and N_T dependence of the proportionality factors in front of them, is provided in Appendix B of [4]. In Section 4, we prove that NP effects can only generate CP-invariant Symanzik operators.

We stress that NP operators like those in equations (11)–(15), though formally of $O(b^2)$, cannot be neglected in the limit $b \to 0$ if we want to get the correct description of the physics of the model, including features of NP origin. In fact, as we have repeatedly remarked, there is an exact compensation between the IR b^2-behavior coming from the insertion of the NP Symanzik operators and/or the Wilson-like vertices and the UV power divergencies of loop integrals, eventually yielding nonvanishing, finite contributions [1, 3, 4].

According to [11], the proper way to keep all these NP effects correctly into account is to construct an "augmented Lagrangian" by adding to the fundamental Lagrangian a linear combination of the NP operators (11)–(15).

3.1. Mass Generation

The "augmented Lagrangian" allows constructing a completely new class of diagrams. Among them, at the lowest loop order, one finds the typical (amputated) vertices shown in the six panels of Figure 3 where the dandling double dotted lines are the U factors which remain after $|\Phi|$ contraction. From top to bottom, these diagrams provide NP mass terms to leptons, quarks, Tera-leptons, Tera-quarks, charged W's, and neutral EW bosons (see equation (27)). Blobs represent the appropriate Symanzik operators among those in equations (11)–(15) and boxes the Wilson-like vertex insertions necessary to close the loops.

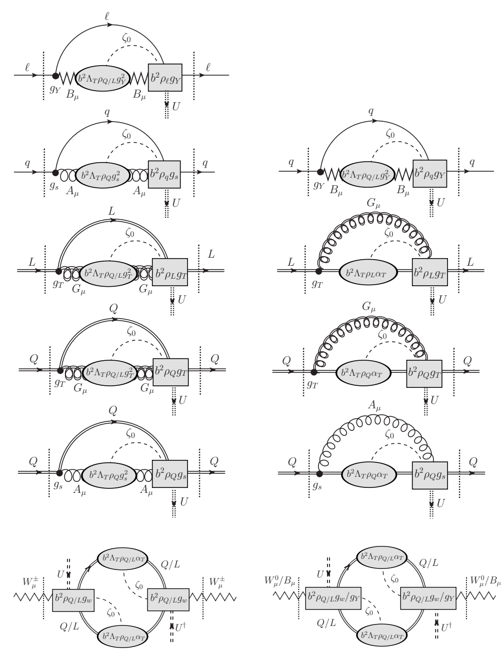

FIGURE 3: Examples of lowest order loop NP diagrams contributing (from top to bottom) to the mass terms of leptons, quarks, Tera-leptons, Tera-quarks and EW bosons. The symbol Q/L means that either hidden Tera-quarks or Tera-leptons run inside the blob, as explained in [4]. The notation W^0_μ / B_μ means that either a neutral W or a $U(1)_Y$ boson is emitted/absorbed, accompanied by a g_w or g_Y gauge factor. Vertical dotted lines mean external leg amputation. The rest of the notation is as explained in the text and Figures 1 and 2.

Again all these diagrams are finite, because of the exact matching between the UV power divergency of multiloop integrals and the b^2-factors brought in by the insertion of (11)–(15) operators and Wilson-like terms. As this compensation is essentially based on dimensional arguments, we expect it to hold at all loops. A careful analysis of the expression of the building blocks making up the diagrams in Figure 3 yields the following parametric mass formulae:

$$\begin{aligned}
m_\ell &= C_\ell \Lambda_T, \quad C_\ell = c_\ell O\left(\alpha_Y^2\right), \\
m_q &= C_q \Lambda_T, \quad C_q = c_q O\left(\alpha_s^2\right), \\
m_L &= C_L \Lambda_T, \quad C_L = c_L O\left(\alpha_T^2\right), \\
m_Q &= C_Q \Lambda_T, \quad C_Q = c_Q O\left(\alpha_T^2\right), \\
M_{W^\pm} &= C_{W^\pm} \Lambda_T, \quad C_{W^\pm} = g_w c_w,\ c_w = k_w O\left(\alpha_T\right), \\
M_Z &= C_Z \Lambda_T, \quad C_Z = \sqrt{g_w^2 + g_Y^2}\, c_w, \\
M_{A^0} &= 0.
\end{aligned} \quad (16)$$

Two important remarks are in order here. (1) Since the kinematical region responsible for the emergence of nonvanishing masses is where all loop momenta are $O(b^{-1})$, the running masses produced by the diagrams of Figure 3 should then be viewed as evaluated at an asymptotically large UV cut-off, or more realistically, if the theory unifies, at the unification scale, Λ_{GUT}. In Section 7, exploiting this observation, we work out from equation (16) some crude phenomenological estimates of Λ_T and elementary particle mass ratios. (2) We stress that in the model described by the Lagrangian (1), elementary particle masses are independent of the vev of Φ. They are, instead, proportional to Λ_T, times a power expansion in the gauge couplings. Naturally, the EW boson masses carry explicit g_w, g_Y factors. Owing to custodial symmetry, which is unbroken at the leading order in the EW interactions, the diagonalization of the self-energy matrix represented by the bottom-right diagrams of Figure 3 yields, just like in the SM, a massive Z boson and a massless photon.

3.2. ρ Dependence, Universality, and Predictive Power

In the mass formulae (16), we have not indicated the dependence on the ρ parameters present in the Lagrangian (1). Actually, one can prove [4] that all physical observables are only functions of ρ ratios. Symmetries can put constraints on the ρ values, possibly mitigating the dependence upon their ratios. For instance, if $\rho_f = \rho, f = q, \ell, Q, L$, then the ρ dependence completely drops out from all physical quantities.

The issue of the ρ dependence of the NP-ly generated masses impacts the problem of universality because it looks that generically masses depend on the values we assign to their ratios. Actually, the situation is somewhat more complicated than that. In particular, one can see that the mass of the fermion f depends on the dimension of the associated Wilson-like term. One can show, in fact, that generically the larger the dimension of the Wilson-like operators, the higher the power of the gauge coupling controlling the leading behavior of the coefficients C_f ($f = q, Q, \ell, L$) and c_w. However, rather than a problem for universality, this feature might provide a handle to understand family mass hierarchy (from heavier to lighter) as associated with Wilson-like terms of different dimensions (from smaller to larger) and perhaps also weak isospin splitting. The use of this observation is made in Section 7.

4. THE STRONG AND TERA-STRONG CP PROBLEM

The cancellation mechanism entailed by the criticality conditions outlined in Sections 2.2 and 2.3 offers an interesting and cheap (i.e., without axions) solution to the strong CP problem which in the present case is apparently even more severe than in QCD because in equation (1) we have two kinds of nonabelian strong gauge bosons, namely, gluons and Tera-gluons.

We want to argue that actually in the critical model (1) there is neither a strong nor a Tera-strong CP problem. The reason can be traced back to the structure of the criticality conditions illustrated in the first two panels of Figure 1 (and Figure 2). In fact, suppose we are in a situation where terms like $i\theta_s \widetilde{F}F$ and/or $i\theta_T \widetilde{G}G$ are presented in equation (1). Let us imagine performing anomalous $U(1)^q_A$ and/or $U(1)^Q_A$ axial phase rotations on quarks and/or Tera-quarks with angles that would cancel the corresponding topological density terms in the Lagrangian. Naturally, these phases (obviously depending on θ_s and/or θ_T) would appear in the Yukawa and Wilson-like terms. The key observation is that the dependence on such $U(1)^q_A$ and/or $U(1)^Q_A$ phases only shows up in overall factors in front of the sums of the fermionic diagrams of Figure 1 (and for that matter also of Figure 2). Thus, at the critical point, where these sums vanish there is no longer a dependence on the $U(1)^q_A$ and/or $U(1)^Q_A$ rotation angles, hence on θ_s and/or θ_T. The argument shows that the CP breaking terms $i\theta_s \widetilde{F}F$ and/or $i\theta_T \widetilde{G}G$ cannot appear in the QEL of the critical theory (see equation (27)).

One might suspect that CP-violating effects can be resurrected by some CP-violating $O(b^2)$ NP Symanzik operator, just like it happens for the CP-conserving operators (11)–(15) relevant for mass terms generation. Actually, this is not so because the possible NP candidates, namely, $\widetilde{O}_{6,AA} \propto b^2 \Lambda_T g_s^2 |\Phi| F^A \cdot \widetilde{F}^A$ or $\widetilde{O}_{6,GG} \propto b^2 \Lambda_T g_T^2 |\Phi| F^G \cdot \widetilde{F}^G$, cannot occur. The reason is that Tera-fermions only have CP-invariant interactions with gluons and Tera-gluons and thus, only CP-invariant $O(b^2)$ NP Symanzik operators can be generated (see Appendix B in [4]).

5. THE W^+W^-/ZZ COMPOSITE STATE

In this section, we want to provide support for the conjecture that the 125 GeV resonance is a W^+W^-/ZZ state bound by Tera-strong exchanges. A theoretically sound approach to the calculation of the binding energy, E_{bind}, of the composite $h = W^+W^-/ZZ$ state is represented by the use of the Bethe-Salpeter (BS) equation. The latter requires the knowledge of the effective WW-WW coupling which can be extracted from the four-point amputated correlator[1]

$$G_4(p_1, p_2, p_3, p_4) = \left\langle \widehat{W}(p_1) \widehat{W}(p_2) \widehat{W}(p_3) \widehat{W}(p_4) \right\rangle_{\mathrm{amp}} \quad (17)$$

$$= g_w^4 \left\langle \widehat{J}^L(p_1) \widehat{J}^L(p_2) \widehat{J}^L(p_3) \widehat{J}^L(p_4) \right\rangle, \quad (18)$$

where $\widehat{W}(p)$ and $\widehat{J}^L(p)$ are the Fourier transforms of $W(x)$ and $J^L(x)$. G_4 is dominated by the sum of Tera-strong exchanges.

An estimate of the WW binding energy, E_{bind}, due to Tera-exchanges, can be obtained under the key assumption that E_{bind} is (parametrically) small compared to $2M_W$. As we see from equation (18), E_{bind} is, indeed, expected to be proportional to g_w^4. In this situation, the dominant contribution in the BS equation is the iteration of the amputated kernel (see upper panel in Figure 4)

$$\Delta\left(s = 4E^2, t = 0\right) \delta(\vec{p} - \vec{p}\,')\delta(E - E') \quad (19)$$

$$= g_w^4 \left\langle \widehat{J}^L(\vec{p}, E) \widehat{J}^L(-\vec{p}, E) \widehat{J}^L(\vec{p}\,', E') \widehat{J}^L(-\vec{p}\,', E') \right\rangle \quad (20)$$

with attached (almost) on-shell W legs.

Δ is directly related to the energy shift of the initial free state due to the interaction. In the case at hand, in fact, the BS iteration can be cast in the approximated form (see the lower

[1] We do not display Lorentz and weak isospin indices in the formulae below, because at the crude level of this discussion, we are unable to assess their role.

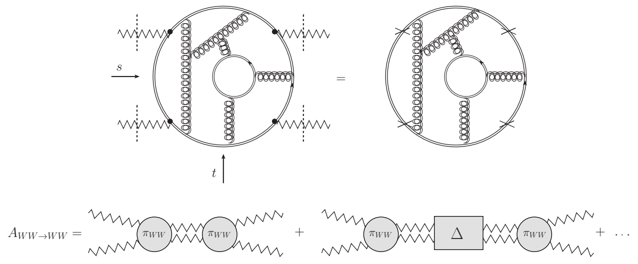

FIGURE 4: Upper panel, the amputated kernel $\Delta(s,t)$. Lower panel, the iteration of equation (21). Notations are as in Figure 3.

panel of Figure 4)

$$
\begin{aligned}
A_{WW\to WW}(s) \Rightarrow & \frac{(\pi_{WW}(s))^2}{s+4M_W^2}\left[1-\frac{\Delta(s,0)}{s+4M_W^2}+\cdots\right] \\
& = \frac{(\pi_{WW}(s))^2}{s+4M_W^2+\Delta(s,0)} \xrightarrow{s\to s_{\rm pole}} \frac{g_w^2 M_W^2}{s+m_h^2},
\end{aligned}
\tag{21}
$$

where

$$
\pi_{WW}(s)|_{s_{\rm pole}} = O\left(g_w^2\Lambda_T\right) = O\left(g_w M_W\right) \tag{22}
$$

is the price one needs to pay to have two W sufficiently close to each other to be able to feel Tera-interactions. The resummation in equation (21) is very similar to the one that it is encountered in the Witten-Veneziano derivation of the η' mass formula [13, 14].

The obvious difference is that here the iterated, exchanged intermediate state is not the pion but the state of two loosely interacting W's, whose invariant mass we approximate by simply setting $\text{mass}^2_{WW} \sim 4M_W^2$. The role of the topological susceptibility is here played by Δ. From the second line of equation (21), one concludes that $\Delta(s,0)|_{s_{\rm pole}}$ is the WW binding energy.

Owing to dimensional arguments and g_w^2 counting, we can write for the value of $\Delta(s)$ at the pole the parametric expression $\Delta(s_{\rm pole},0) = c g_w^4 4M_W^2$. The crucial assumption we make in this formula is the sign of c that we conjecture to be negative if it has to give rise to binding. We remark that the g_w^2 dependence of the pole residue in equation (21) correctly fits with the g_w^2 dependence of the SM WW-WW scattering amplitude for which one gets

$$
A_4^{\rm SM}(s)\Big|_{\rm pole} = \frac{g^2_{hWW}}{s+m_h^2} = \frac{4g_w^2 M_W^2}{s+m_h^2}. \tag{23}
$$

This consistency supports the correctness of the g_w dependence of the coefficient $\pi_{WW}(s)|_{s_{\rm pole}}$ assumed in equation (22). In conclusion, from (21), one has for the h-(mass)^2

$$
m_h^2 = 4M_W^2 + c g_w^4 4M_W^2. \tag{24}
$$

Taking equation (24) at face value with $g_w \sim 0.62$ and c negative and $O(1)$, one obtains from $m_h = 2M_W\sqrt{1+cg_w^4} = 2M_W - E_{\rm bind}$ the interesting estimate

$$
E_{\rm bind} = -c g_w^4 M_W\left[1+O\left(g_w^4\right)\right] \sim 12\,\text{GeV}. \tag{25}
$$

We have found a value of $E_{\rm bind}$ parametrically of the order of the W mass itself times four powers of the weak coupling, like in the nonrelativistic approximation described in Appendix D of [4]. The precise numerical magnitude of the estimate (25) depends on an "at this moment unknown" coefficient c in equation (24) whose size and sign, however, could be extracted from unquenched LQCD-like simulations of the four-point correlator (20).

It is interesting to note, in fact, that, neglecting weak and strong loop corrections, compared to Tera-strong effects (as well as the impact of Tera-leptons), Δ could be extracted from lattice simulations of the QCD type. The idea is that, pretending that the Tera-particles in the diagrams in the upper panel of Figure 4 are quarks and gluons, one can obtain the physical value of Δ by rescaling the four-current amplitude evaluated in unquenched LQCD simulations by the ratio $\Lambda_T/\Lambda_{\rm QCD}$. In the approximation we are working on, one can take for J^L the naive, continuum-like expression of the *Left*-handed weak current. We can limit to simulate the amplitude Δ in QCD, and not in the much more costly toy-model introduced in [1], because NP effects do not appear to be relevant in the formation of the h-bound state. It will be enough to introduce by hand quark masses of appropriate values.

6. THE QEL OF THE CRITICAL THEORY

The form of the QEL, $\Gamma^{\rm NG}_{cr}$, which describes the physics of the critical theory in the NG phase, including the NP mass terms identified in Section 3, is essentially dictated by geometrical considerations. It is, in fact, highly constrained by dimensional and symmetry arguments, in particular by the exact invariance under $\chi_L\times\chi_R$ and the observation that at the critical point neither the scalar field kinetic term nor the Yukawa terms, which both would break $\tilde\chi_L\times\tilde\chi_R$, should be present in the QEL, $\Gamma^{\rm NG}_{cr}$, of the theory [1, 3]. Focusing on the $d\le 4$ part, the expression of the $\Gamma^{\rm NG}_{4\,cr}$ is obtained by including all the operators of dimension $d\le 4$, invariant under $\chi_L\times\chi_R$ that can be constructed in terms of the matter and gauge fields of the theory and the nonanalytic field U, defined by the standard polar decomposition

$$
\Phi = RU, \quad R = v+\zeta_0, \quad U = \exp\left[i\vec\tau\,\vec\zeta/c_w\Lambda_T\right]. \tag{26}
$$

The choice of the (arbitrary) mass scale in the exponent has been conveniently made with an eye to equation (27), i.e., so as to have the NG bosons, ζ^i, $i=1,2,3$, canonically normalized. From the constraints imposed by the above considerations, one obtains

$$
\begin{aligned}
\Gamma^{\rm NG}_{4\,cr} = & \frac14\left(F^A\cdot F^A + F^G\cdot F^G + F^W\cdot F^W + F^B\cdot F^B\right) \\
& + \left[\bar q_L \,\mathcal{D}^{BWA} q_L + \bar q_R\,\mathcal{D}^{BA} q_R\right] + C_q\Lambda_T\left(\bar q_L U q_R + \bar q_R U^\dagger q_L\right) \\
& + \left[\bar\ell_L\,\mathcal{D}^{BW}\ell_L + \bar\ell_R\,\mathcal{D}^{B}\ell_R\right] + C_\ell\Lambda_T\left(\bar\ell_L U\ell_R + \bar\ell_R U^\dagger \ell_L\right) \\
& + \left[\bar Q_L\,\mathcal{D}^{BWAG} Q_L + \bar Q_R\,\mathcal{D}^{BAG} Q_R\right] \\
& + C_Q\Lambda_T\left(\bar Q_L U Q_R + \bar Q_R U^\dagger Q_L\right) \\
& + \left[\bar L_L\,\mathcal{D}^{BWA} L_L + \bar L_R\,\mathcal{D}^{BA} L_R\right] \\
& + C_L\Lambda_T\left(\bar L_L U L_R + \bar L_R U^\dagger L_L\right) \\
& + \frac12 c_w^2\Lambda_T^2 \,\text{Tr}\left[\left(\mathcal{D}_\mu^{BW}U\right)^\dagger \mathcal{D}_\mu^{BW} U\right].
\end{aligned}
\tag{27}
$$

We explicitly note that in equation (27) only U appears and not Φ. Expanding U around the unit, we get the mass identifications (16). We stress that the "mass terms" in equation (27) do not depend on v. They have a somewhat unusual expression and a peculiar conceptual status. They appear, in fact, as a sort of NP anomalies preventing the full recovery of $\tilde{\chi}_L \times \tilde{\chi}_R$ invariance. Like in the SM, the NG bosons ζ^i, $i = 1, 2, 3$ become the longitudinal W DoFs, as explicitly demonstrated in [3].

6.1. The LEEL of the Critical Model and the SM

We show in this subsection that, upon integrating out the (heavy) Tera-DoFs, the resulting LEEL of the critical model (1) in the NG phase closely resembles the SM Lagrangian. The argument rests on the conjecture (see Section 5) that the 125 GeV resonance detected at LHC is a W^+W^-/ZZ composite state bound by Tera-particle exchanges and not an elementary particle.

This state, denoted by h, is a singlet under all the symmetries of the theory. As its mass is 125 GeV $\ll \Lambda_T$, it must be included in the LEEL, valid for (momenta)$^2 \ll \Lambda_T^2$. Ignoring for simplicity families, $U(1)_Y$ interactions, leptons, and weak isospin splitting (extending the argument poses no problem), in these kinematical conditions the most general LEEL invariant under the symmetries of the critical model (1), including h, takes the form

$$\begin{aligned}
&\mathcal{L}^{\rm NG}_{LE}(q; A, W; U, h) \\
&\quad= \frac{1}{4}\left(F^A \cdot F^A + F^W \cdot F^W\right) + \left(\bar{q}_L\, \not{\mathcal{D}}^{AW} q_L + \bar{q}_R\, \not{\mathcal{D}}^{A} q_R\right) \\
&\qquad+ \left(y_q h + k_q k_v\right)\left(\bar{q}_L U q_R + \bar{q}_R U^\dagger q_L\right) + \frac{1}{2}\partial_\mu h \partial_\mu h \\
&\qquad+ \frac{1}{2}\left(k_v^2 + 2k_v k_1 h + k_2 h^2\right) \mathrm{Tr}\left[\left(\mathcal{D}^W_\mu U\right)^\dagger \mathcal{D}^W_\mu U\right] \\
&\qquad+ \widetilde{\mathcal{V}}(h) + \cdots,
\end{aligned} \tag{28}$$

where dots represent $\tilde{\chi}_L \times \tilde{\chi}_R$ violating operators with $d > 4$. The scalar potential $\widetilde{\mathcal{V}}(h)$ comprises the cubic and quartic self-interactions of the h field, as well as the h mass term, $m_h^2 h^2/2$. The k_v, k_1, k_2, y_q, and k_q coefficients and the $\widetilde{\mathcal{V}}$-couplings are parameters that need to be fixed by matching onto the underlying (renormalizable and unitary) fundamental critical theory (1).

Tree-level matching of $\mathcal{L}^{\rm NG}_{4,LE}$ in equation (28) with the QEL $\Gamma^{\rm NG}_{4\,cr}$ in equation (27) requires the identifications

$$m_q = C_q \Lambda_T = k_q k_v, \quad M_W = g_w\, c_w \Lambda_T = g_w k_v, \tag{29}$$

while the unitarity of the mother theory (1) implies [15, 16]

$$y_q = k_q,\; k_1 = k_2 = 1. \tag{30}$$

With these choices, neglecting small loop effects controlled by the couplings g_w and y_q, one recognizes that, with the exception of $\widetilde{\mathcal{V}}(h)$, it is just the combination $\Phi \equiv (k_v + h)U$ that enters the $d \leq 4$ part of $\mathcal{L}^{\rm NG}_{LE}$ (see equation (28)). The latter can thus be rewritten in the suggestive form

$$\begin{aligned}
&\mathcal{L}^{\rm NG}_{4,LE}(q; A, W; U, h) \\
&\quad= \frac{1}{4}\left(F^A \cdot F^A + F^W \cdot F^W\right) + \left(\bar{q}_L\, \not{\mathcal{D}}^{AW} q_L + \bar{q}_R\, \not{\mathcal{D}}^{A} q_R\right) \\
&\qquad+ y_q\left(\bar{q}_L \Phi q_R + \bar{q}_R \Phi^\dagger q_L\right) + \frac{1}{2}\mathrm{Tr}\left[\left(\mathcal{D}^W_\mu \Phi\right)^\dagger \mathcal{D}^W_\mu \Phi\right] + \widetilde{\mathcal{V}}(h).
\end{aligned} \tag{31}$$

We see that (up to $O(\alpha_w)$ or $O(y_q^2)$ corrections) $\mathcal{L}^{\rm NG}_{4,LE}$ looks like the SM Lagrangian. In particular, just like it happens in the case of the Higgs mechanism in the SM, the effective Yukawa coupling of h to fermions is given by

$$y_q = k_q = \frac{m_q}{k_v} = \frac{m_q}{c_w \Lambda_T}, \tag{32}$$

where $k_v = c_w \Lambda_T$ is what in the SM is the Higgs vev. Thus, also here Yukawa couplings are proportional to fermion masses.

There are, however, two key differences between the LEEL of our model (see equation (31)) and the SM Lagrangian. First of all, the proportionality factor between the Yukawa coupling (32) and the fermion mass is not in our hands but it is completely fixed by the NP dynamics. Secondly, since $\widetilde{\mathcal{V}}(h)$ describes, besides the mass, the self-interactions of the (composite) h state, there is no reason why it should have the same form as the SM Higgs potential. This implies that differences with the SM case may well appear in the trilinear and quadrilinear h self-couplings.

7. A BIT OF PHENOMENOLOGY

The key questions we need to address in order to put the present model on a solid basis and make it useful for phenomenology are (1) how we can make contact between the theoretical mass estimates provided by equation (16) and the phenomenological values of elementary particle masses and (2) whether we can access the value of the scale of the new interaction from the knowledge of the available low energy physics data. In the next subsections, we will show that it is possible to get from equation (16) crude, but encouraging, phenomenological estimates of Λ_T and masses of the fermions of the heaviest family in the unit of the W mass.

7.1. Mass Estimate Strategy

As we already observed, the values of the masses given by equation (16) refer to asymptotically large UV cut-off, or more realistically, if the theory unifies, to the unification scale, $\Lambda_{\rm GUT}$. Consequently, if the mass generation mechanism is realized in a model that yields gauge coupling unification (like the one discussed in [7] that we are focusing on here), then at the unification scale, the fermion masses given by equation (16) will be close to each other, owing to the fact that the gauge couplings are evaluated at the same scale $\Lambda_{\rm GUT}$ where they are essentially all equal. As we shall see, the phenomenological situation is, however, a bit more complicated than that.

The running of masses from $\Lambda_{\rm GUT}$ down to lower energy scales will be different for different fermion species reflecting the RGI evolution of the fermion bilinear operator from which each mass is factorized out and the different running of the coupling of the gauge interactions each fermion is subjected to. One can hope to arrive in this way at the correct order of magnitude of the phenomenological values of elementary particle masses.

The formula describing the fermion mass running from, say, $\Lambda_1 = \Lambda_{\rm GUT}$ down to a scale Λ_2, that we take somewhat larger than Λ_T (see equation (48)), and equal, as a try, to 5 TeV, at leading order reads

$$m_f(5\,\mathrm{TeV}) = m_f\left(\Lambda_{\rm GUT}\right) \prod_p \left[\frac{\alpha_p(5\,\mathrm{TeV})}{\alpha_p\left(\Lambda_{\rm GUT}\right)}\right]^{\gamma^f_{0p}/2\beta_{0p}}, \tag{33}$$

where the product is over all the interactions felt by the fermion f. In [7], it is shown that the unifying couplings g_n, $n = 1, 2, 3, 4$ are related to the physical ones by the relations

$$g_1^2 = \frac{4}{3} g_Y^2, \quad g_2^2 = g_w^2, \quad g_3^2 = g_s^2, \quad g_4^2 = \frac{8 + N_S}{12} g_T^2. \tag{34}$$

For the first coefficient of the β functions one finds [7]

$$\beta_{0T} = \frac{11}{3} N_T - \frac{4}{3} \left(N_c + 1 \right), \tag{35}$$

$$\beta_{0s} = \frac{11}{3} N_c - \frac{4}{3} \left(N_T + N_f \right), \tag{36}$$

$$\beta_{0w} = 2 \frac{11}{3} - \frac{1}{3} N_f \left(N_c + 1 \right) - \frac{1}{3} N_T \left(N_c + 1 \right), \tag{37}$$

$$\beta_{0Y} = -\frac{2}{3} \left[\left(\frac{22}{36} N_c + \frac{3}{2} \right) N_f + \frac{1}{2} N_T \left(N_c + 1 \right) \right]. \tag{38}$$

Unification of $U(1)_Y$, $SU_L(2)$, and $SU(N_c)$ occurs with the (essentially unique) choice $N_T = N_c = N_f = 3$. To get unification with also Tera-strong interactions, the best choice is to take $N_S = 5$, as the number of extra Tera-particles endowed with only Tera-strong interactions [7]. The estimates that follow have a mild dependence on N_S. One should also notice that in the present unifying scheme β_{0w} is negative so, unlike the case of the SM, weak interactions are not asymptotically free. From equation (37), one finds $\beta_{0w} = -2/3$ leading to a very slow and flat g_w^2 running.

The running of masses under EW interactions can be estimated to give some $10 \div 20\%$ contribution in equation (33). We neglect these corrections as they are well below the accuracy of our approximations. Focusing on strong and Tera-strong running, we get from equations (35)-(36) and the PDG quark mass review

$$\beta_{0s} = 3, \quad \beta_{0T} = \frac{17}{3}, \quad \gamma_{0s}^Q = \gamma_{0s}^L = \gamma_{0s}^q = 8, \quad \gamma_{0T}^Q = \gamma_{0T}^L = 8. \tag{39}$$

We determine the values of the gauge couplings at the scale of 5 TeV in the mass-independent $\overline{\text{MS}}$ scheme from the plot reported in Figure 6 of [7]. We find $\alpha_s(5\,\text{TeV}) \sim 1/13$ and $\alpha_T(5\,\text{TeV}) \sim 2$. With $N_S = 5$ we obtain from equation (34)

$$\alpha_3(5\,\text{TeV}) = \alpha_s(5\,\text{TeV}) \sim \frac{1}{13}, \tag{40}$$

$$\alpha_4(5\,\text{TeV}) = \frac{8 + N_S}{12} \alpha_T(5\,\text{TeV}) = \frac{13}{12} \alpha_T(5\,\text{TeV}) \sim \frac{13}{12} \cdot 2. \tag{41}$$

On the basis of the previous considerations, we propose to estimate the value of Tera-fermion masses and the masses of the fermions of the heaviest of the SM families in the $\overline{\text{MS}}$ scheme, starting from the asymptotic formulae

$$m_f \left(\Lambda_{\text{GUT}} \right) = C_f \bar{\alpha}_s^{u_f} \bar{g}_s^2 \Lambda_T, \quad f = t, b \tag{42}$$

$$m_\tau \left(\Lambda_{\text{GUT}} \right) = C_\tau \bar{\alpha}_Y^{u_\tau} \bar{g}_Y^2 \Lambda_T, \tag{43}$$

$$m_{Q/L} \left(\Lambda_{\text{GUT}} \right) = C_{Q/L} \bar{\alpha}_T^{u_{Q/L}} \bar{g}_T^2 \Lambda_T, \tag{44}$$

where overlining means that the gauge couplings are taken at the scale Λ_{GUT}, where g_1^2, g_2^2, g_3^2, and g_4^2 (see equation (34)) unify. The exponents u are integers for which we make the "ad hoc" choice, $u_t = u_Q = u_L = 1$ for top, Tera-quarks, and Tera-leptons, respectively, as implied by the mass formula (16). We recall that the choice $u_f = 1$ means that Wilson-like operator of the fermion f has $d = 6$. For the τ lepton we take instead $u_\tau = 2$ which means that Wilson-like operator of the τ lepton has $d = 8$ [4]. As we shall see in Section 7.2.4, the similar choice $u_b = 2$ is necessary if one wants to reproduce the mass splitting of the top-bottom weak iso-doublet.

We note [3, 4] that the factors $\bar{\alpha}$ come either from the insertion of the operators (11) and (12) or from the loop integration involving the Wilson-like term associated with the fermion in consideration, while $\bar{g}^2$ factors (with no $(4\pi)^{-1}$ rescaling) come from the insertion of the operators (13)–(15). For the numerical values of the physical $\bar{\alpha}$ couplings at the GUT scale, we get

$$\begin{aligned} \bar{\alpha}_Y &\sim \frac{3}{4} \cdot \frac{1}{28} = \frac{3}{112}, \\ \bar{\alpha}_w &\sim \frac{1}{28}, \\ \bar{\alpha}_s &\sim \frac{1}{28}, \\ \bar{\alpha}_T &\sim \frac{12}{13} \cdot \frac{1}{28} = \frac{12}{364}. \end{aligned} \tag{45}$$

These numbers are extracted from the unifying behavior displayed in Figure 6 of [7] with $N_S = 5$, recalling the prefactors in equation (34). For the corresponding gauge couplings we find

$$\bar{g}_Y \sim 0.58, \quad \bar{g}_w \sim 0.67, \quad \bar{g}_s \sim 0.67, \quad \bar{g}_T \sim 0.64. \tag{46}$$

7.2. Some Numerics

To get estimates of Λ_T and masses we need to fix a scale. We decided to express everything in terms of M_W because the latter turns out to be given by the most robust of our mass formulae. First of all, M_W only depends upon the Tera-sector Wilson-like operators that we take as $d = 6$ operators. Secondly, one can recognize (see Appendix C of [4]) that M_W has a weak dependence on N_c, N_T, and ρ ratios which actually disappears for large N_c and N_T.

7.2.1. M_W Mass

Using the fifth relation in equation (16) where, we recall, the factor $\bar{\alpha}_T$ comes from the insertion of the operators (11) and (12) and the factor $\bar{g}_w$ from the external weak coupling to which the W is attached, our theoretical estimate for the W pole mass is

$$M_W = k_w \bar{\alpha}_T \bar{g}_w \Lambda_T, \tag{47}$$

where, as we mentioned above, k_w is an (almost) N_c, N_T, and ρ independent coefficient. Using the numerical estimates in equations (45)-(46) and $M_W \sim 80\,\text{GeV}$ as an input, we can extract the value of the product $k_w \Lambda_T$ finding

$$80\,\text{GeV} = k_w \bar{\alpha}_T \bar{g}_w \Lambda_T = k_w \frac{12}{364} 0.67 \Lambda_T \to k_w \Lambda_T \sim 3.6\,\text{TeV}. \tag{48}$$

In fixing the W mass, we have ignored the effects of the loops that, just like in the SM, also here would give corrections to the M_W tree level value. Although these corrections are very small, they are crucial for SM precision tests. In the present context, we ignore them as we are only interested in the order of magnitude of the bulk contribution to the W mass, i.e., of the term that in our approach replaces the SM formula $M_W \sim g_w v$.

7.2.2. Tera-Fermion Mass Running

Using equation (33), we obtain for the running of the Tera-quark mass

$$\begin{aligned} m_Q(5\,\text{TeV}) &= C_Q \overline{\alpha}_T \overline{g}_T^2 \Lambda_T \left(\frac{2}{12/364}\right)^{8\frac{3}{17}\frac{1}{2}} \left(\frac{1/13}{1/28}\right)^{\frac{8}{3}\frac{1}{2}} \\ &\sim \frac{C_Q}{k_w}\frac{12}{364}(0.64)^2 \cdot 18.15 \cdot 2.78 \cdot 3600 \\ &\sim \frac{C_Q}{k_w} 2500\,\text{GeV}. \end{aligned} \tag{49}$$

This formula is obtained by multiplying equation (44) by the factor in equation (33) representing the running of the fermion mass under Tera-strong and strong interactions.

By dropping the last factor in the first line of equation (49), which represents the contribution from strong interaction running, we find from the running of the Tera-lepton mass

$$m_L(5\,\text{TeV}) = C_L \overline{\alpha}_T \overline{g}_T^2 \Lambda_T \left(\frac{2}{12/364}\right)^{8\frac{3}{17}\frac{1}{2}} \sim \frac{C_L}{k_w} 900\,\text{GeV}. \tag{50}$$

7.2.3. m_t running

Similarly, by dropping the first factor, for the top mass, we get

$$\begin{aligned} m_t(5\,\text{TeV}) &= C_t \overline{\alpha}_s \overline{g}_s^2 \Lambda_T \left(\frac{1/13}{1/28}\right)^{\frac{8}{3}\frac{1}{2}} \\ &\sim \frac{C_t}{k_w}\frac{1}{28}(0.67)^2 \cdot 2.78 \cdot 3600 \sim \frac{C_t}{k_w} 160\,\text{GeV}. \end{aligned} \tag{51}$$

7.2.4. m_b Running

To describe the top-bottom weak isospin breaking we conjecture that the Wilson-like operator associated with the b quark is of dimension 8 giving, as we observed below equation (46), $u_b = 2$. This leads in the b mass formula to the presence of an extra $\overline{\alpha}$ factor with respect to the top mass formula, which we take to be $\alpha_Y(\Lambda_{\text{GUT}}) = \overline{\alpha}_Y$, with the idea that weak isospin splitting is related to EW effects. We thus obtain

$$\begin{aligned} m_b(5\,\text{TeV}) &= C_b \overline{\alpha}_s \overline{g}_s^2 \overline{\alpha}_Y \Lambda_T \left(\frac{1/13}{1/28}\right)^{\frac{8}{3}\frac{1}{2}} \\ &\sim \frac{C_b}{k_w}\frac{1}{28}(0.67)^2 \frac{3}{112} \cdot 2.78 \cdot 3600 \sim \frac{C_b}{k_w} 4.3\,\text{GeV}. \end{aligned} \tag{52}$$

7.2.5. m_τ Mass

For the calculation of the τ lepton pole mass we will assume that the associated Wilson-like operator is of dimension 8, leading, as we said below equation (46), to $u_\tau = 2$. Thus, also in the τ mass formula we have to include an extra $\overline{\alpha}$ factor with respect to the top mass formula, wich again we take to be $\alpha_Y(\Lambda_{\text{GUT}}) = \overline{\alpha}_Y$, with the idea that leptons are lighter than quarks because they are only charged under EW interactions. From (43), we obtain

$$m_\tau \sim C_\tau \overline{\alpha}_Y^2 \overline{g}_Y^2 \Lambda_T \sim \frac{C_\tau}{k_w}\left(\frac{3}{112}\right)^2 (0.58)^2 \cdot 3600 \sim \frac{C_\tau}{k_w} 0.87\,\text{GeV}. \tag{53}$$

Given the level of the approximations we had to inject in these estimates (we ignore diagram multiplicities and $O(1)$ factors), if one imagines taking all constant ratios, C_f/k_w, equal to the unit, the numbers we are getting are even too good. Perhaps only the lepton masses are a bit too low. Naturally, there would be no problem in bringing the running fermion masses down to their respective self-consistent mass scale.

7.3. Comments

We conclude Section 7 by summarizing the simplifying assumptions and approximations we have made in the mass estimates we have presented above.

(1) First of all, for each particle, the starting point mass formula was the lowest loop order expression of the amputated self-energy diagrams displayed in Figure 3. As we observed previously, this calculation yields the value of the running masses at the GUT scale.

(2) We set to unit all multiplying factors, like C_f/k_w, occurring in the above numerical estimates.

(3) We set to unit the factors associated with the EW mass running.

(4) We assumed that the family mass hierarchy (from heavier to lighter) is induced by Wilson-like operators of increasing dimensions from $d_t = 6$, yielding $u_t = 1$ to larger dimensions. We neglect possible mixing effects if more than one Wilson-like operator with $d \geq 6$ is present.

(5) As for the quark weak isospin breaking, we assumed that the Wilson-like operators associated with the lightest partner of the doublet have dimensions of two units larger than the one of the heavier partner. This means $d_b = 8$, yielding $u_b = 2$. We attribute weak isospin splitting to EW interactions.

(6) Similarly, we assumed that the dimension of Wilson-like operators associated with leptons has dimensions larger than two units compared to the one of the corresponding quark, thus $d_\tau = 8$, yielding $u_\tau = 2$.

8. CONCLUSIONS AND OUTLOOK

In this paper, we have constructed a putative bSMm where all elementary particles get a dynamical mass from a unique NP field-theoretical feature, an alternative to the Higgs mechanism.

Masses for elementary fermions and EW bosons are generated in the NG phase of the theory, where the exact $\chi_L \times \chi_R$ symmetry (7) is spontaneously broken. The resulting mass terms appear in the QEL of the critical chiral invariant theory as a sort of NP anomalies preventing the full restoration of $\tilde{\chi}_L \times \tilde{\chi}_R$ invariance. This peculiar NP scenario was confirmed in [12] by direct numerical investigations based on lattice simulations of the model identified in [1] which is the simplest field-theoretical model displaying this NP mass generation mechanism.

Since, as shown in (16), all masses are proportional to the RGI scale of the theory, we have argued that to get the top quark and W, Z mass in the right ball-park, there must exist, besides SM matter, a superstrongly interacting sector of particles, gauge invariantly coupled to SM DoFs, so that the RGI scale of the whole theory, Λ_T, will lie in the few TeV region. From equation (16), encouraging estimates of Λ_T and elementary particle mass ratios can be obtained in the unit of M_W.

In the present formulation of the model, neutrinos are massless. In fact, with the SM hypercharge assignment $y_{\nu_R} = 0$, $\nu_R = \ell_R^u$ is "sterile" and the Wilson-like term associated with it in (1) is not able to give the neutrino a mass. Though we do not discuss here the crucial question of how to lift the neutrino mass, we note that in the present scenario there is a natural seesaw-like scale for neutrino masses, namely, $\Lambda_T^2/\Lambda_{\rm GUT} \sim [(1 \div 10)\,\mathrm{TeV}]^2/10^{17\div18}\,\mathrm{TeV} \sim 10^{-5} \div 10^{-3}\,\mathrm{eV}$, with $\Lambda_T \sim (1 \div 10)\,\mathrm{TeV}$ and $\Lambda_{\rm GUT} \sim 10^{17\div18}\,\mathrm{TeV}$ being reasonable estimates of the RGI scale of the theory (see Section 7) and unification scale of [7], respectively.

We have noted that, as a consequence of the structure of the criticality conditions, the present scheme offers a cheap (without axions) solution of the strong CP problem.

The 125 GeV resonance detected at LHC is interpreted as a W^+W^-/ZZ state bound by Tera-particle exchanges. This interpretation is supported by the BS-like analysis presented in Section 5 where an estimate of the $E_b = 2M_W - m_h$ binding energy is provided. Despite the crudeness of various approximations, a reasonable value of E_b is obtained.

Upon integrating out the heavy Tera-DoFs, one is left with SM matter DoFs, the U field plus this "light" (on the Λ_T scale) h boson. We have shown that including h and enforcing $\chi_L \times \chi_R$ invariance as well as unitarity leads to a LEEL valid for momenta$^2 \ll \Lambda_T^2$ that looks like the SM Lagrangian [3]. This means that the model we are advocating in this work passes all the precision tests that the SM is able to pass.

In conclusion, in this letter we have outlined the construction of an economic bSMm, in which (1) we can give a simple solution to the naturalness problem (lacking a Higgs field responsible for mass generation, the Higgs mass tuning problem does not even arise), (2) elementary fermion masses are not free parameters like in the SM (they are instead dynamically determined), (3) we get an understanding of the physical origin of the EW scale (as the scale of a new interaction), and (4) we obtain a solution of the strong CP problem (as a consequence of criticality).

Needless to say, the key issues of the origin of flavor and weak isospin splitting and of how to possibly compute the CKM matrix are open questions under active investigation from our side. In any case, the detection of Tera-hadrons would be an unmistakable sign of New Physics.

CONFLICTS OF INTEREST

The author declares that there are no conflicts of interest regarding the publication of this paper.

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