Dark Matter and Muon $(g-2)$ from a Discrete Z_4 Symmetric Model

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Abstract

The nonzero neutrino mass and nature of Dark Matter (DM) is still unknown within the Standard Model (SM). In 2021, there was a 4.2σ discrepancy with SM results in the measurement of muon magnetic moment reported by Fermilab. Recently, Fermilab released its precise results for muon's magnetic moment, and it shows a 5.1σ discrepancy. In this work, we study the corelation between neutrino masses, muon $(g-2)$ anomaly, and Dark Matter within a framework based on the Z_4 extension of the scotogenic model, in which the neutrino masses are generated at one loop level. We extend the model with a vector-like lepton (VLL) triplet in order to explain muon $(g-2)$. Here, the coupling of VLL triplet ψ_T to inert doublet η provides a positive contribution to muon anomalous magnetic moment. We also studied the DM phenomenology of ψ_T by considering the neutral component of ψ_T as the lightest DM candidate. We show that, for the mass of the VLL triplet M_ψ in TeV scale, the model can well explain muon $(g-2)$ anomaly and also gives required relic density.

Keywords: Z_4 symmetry, Dark Matter, muon $(g-2)$, VLL triplet.
DOI: 10.31526/LHEP.2024.512

1. INTRODUCTION

The astrophysical and cosmological experiments suggest that ∼24% energy density of the Universe is in the form of dark matter. However, we still do not know the exact nature of it. The nonavailability of potential DM candidates within the SM suggests a search for physics beyond the SM (BSM). Among many proposed candidates, WIMPs (Weakly Interacting Massive Particles) have been studied intensively as suitable DM candidates. They have mass and interaction strength around the electroweak scale and are produced through a freeze-out mechanism after decoupling from the thermal bath. Therefore, they can account for the observed relic density. At present, the dark matter relic density as per Planck data is [1]

$$\Omega h^2 = 0.12 \pm 0.001. \tag{1}$$

Another long-standing anomaly that questions the successfulness of SM is the muon's anomalous magnetic moment (Δa_μ). Fermilab in 2021 released its results for muon's magnetic moment, and it was in a 4.2σ discrepancy with SM results [2]. Recently, Fermilab again released the data for runs II and III which confirms the discrepancy measured before. This has updated the world average to $a_\mu^{\text{Exp}} = 116592059(22) \times 10^{-11}$. The new measurements show a 5.1σ discrepancy with SM results as [3]

$$\Delta a_\mu \equiv a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = (24.5 - 4.9) \times 10^{-10}. \tag{2}$$

This also hints at physics beyond the SM. Several theoretical models have been put forth to explain this discrepancy along with dark matter. The readers can refer to [4, 5, 6, 7] for detailed study.

Symmetries play an important role in model building in particle physics. See [8, 9, 10, 11] for further reading. In this paper, we consider the Z_4 symmetric scotogenic model in order to explain dark matter and muon $(g-2)$ in light of neutrino phenomenology. The charge assignments of fields under Z_4 symmetry help in particular structure of neutrino mass matrix and also the coupling of VLL triplet with muon that provides additional contribution to muon $(g-2)$.

The paper is structured as follows. In Section 2, we discuss the scotogenic model. In Section 3, we analyze the results and then we conclude in Section 4.

2. THE MODEL

This work is an extension of our previous work [12] where DM candidate was considered the real component of inert doublet η. Here, we extend that work to study the dark matter phenomenology of VLL triplet. The SM gauge symmetry is extended by a discrete Z_4 symmetric group. To obtain small neutrino masses, the scotogenic model is considered that includes three right-handed neutrinos and a scalar doublet with an imposed Z_2 symmetry that provides a DM candidate also. The particle content of the model is given in Table 1. Here, N_k $(k = 1, 2, 3)$ are three right-handed neutrinos, ψ_T is the vector-like lepton triplet, S is the singlet scalar, and η is the inert scalar doublet.

The relavant terms in the Yukawa Lagrangian are

$$\begin{aligned}
-\mathcal{L} \supseteq\ & \frac{M_{11}}{2} N_1 N_1 + M_{23} N_2 N_3 + y_{\eta 1} \bar{L}_e \tilde{\eta} N_1 + y_{\eta 2} \bar{L}_\mu \tilde{\eta} N_2 \\
& + y_{\eta 3} \bar{L}_\tau \tilde{\eta} N_3 + y_{12} S N_1 N_2 + y_{13} S^* N_1 N_3 + y_\psi \eta^\dagger \bar{\psi}_{T,R} L_\mu \\
& + M_\psi \bar{\psi_T} \psi_T + H.c.,
\end{aligned} \tag{3}$$

where $\tilde{\eta} = i\sigma_2 \eta^*$ and L_α $(\alpha = e, \mu, \tau)$ are SM left-handed lepton doublets. The scalar potential is written as

$$\begin{aligned}
V(H, S, \eta) =\ & -\mu_H^2 \left(H^\dagger H\right) + \lambda_1 \left(H^\dagger H\right)^2 - \mu_S^2 \left(S^\dagger S\right) \\
& + \lambda_S \left(S^\dagger S\right)^2 + \lambda_{HS} \left(H^\dagger H\right)\left(S^\dagger S\right) + \mu_\eta^2 \left(\eta^\dagger \eta\right) \\
& + \lambda_2 \left(\eta^\dagger \eta\right)^2 + \lambda_3 \left(\eta^\dagger \eta\right)\left(H^\dagger H\right) \\
& + \lambda_4 \left(\eta^\dagger H\right)\left(H^\dagger \eta\right) + \frac{\lambda_5}{2}\left[\left(H^\dagger \eta\right)^2 + \left(\eta^\dagger H\right)^2\right] \\
& + \lambda_{\eta S} \left(\eta^\dagger \eta\right)\left(S^\dagger S\right) + H.c.
\end{aligned} \tag{4}$$

Symmetry Group	N_1, N_2, N_3	ψ_T	S	η
$SU(2)_L \times U(1)_Y$	$(1, 0)$	$(3, -1)$	$(1, 0)$	$(2, 1/2)$
Z_4	$(1, i, -i)$	i	$-i$	1
Z_2	$-$	$-$	$+$	$-$

TABLE 1: The field content and respective charge assignments of the model under $SU(2)_L \times U(1)_Y \times Z_4 \times Z_2$.

The neutral component of Higgs doublet H breaks the SM gauge symmetry, and the Z_4 symmetry is spontaneously broken by the nonzero vacuum expectation value (vev) of the scalar singlet S. Also, $\mu_\eta^2 > 0$ is assumed for η to not acquire vev. Using equation (3), the forms of charged lepton mass matrix M_l, Dirac Yukawa matrix y_D, and right-handed neutrino mass matrix M_R are given by

$$M_l = \frac{1}{\sqrt{2}} \begin{bmatrix} y_e v & 0 & 0 \\ 0 & y_\mu v & 0 \\ 0 & 0 & y_\tau v \end{bmatrix}, \quad y_D = \begin{bmatrix} y_{\eta 1} & 0 & 0 \\ 0 & y_{\eta 2} & 0 \\ 0 & 0 & y_{\eta 3} \end{bmatrix}, \tag{5}$$

$$M_R = \begin{bmatrix} M_{11} & y_{12} v_S/\sqrt{2} & y_{13} v_S/\sqrt{2} \\ y_{12} v_S/\sqrt{2} & 0 & M_{23} e^{i\delta} \\ y_{13} v_S/\sqrt{2} & M_{23} e^{i\delta} & 0 \end{bmatrix}. \tag{6}$$

Here, v and v_S are vevs of the Higgs boson H and scalar singlet S, respectively, and δ is the phase remained after redefining the fields.

3. RESULTS AND DISCUSSION

3.1. Neutrino Mass

Neutrino mass can be generated at one loop level by Scotogenic mechanism. The neutrino mass matrix can be written as [13, 14]

$$M_{ij}^\nu = \sum_k \frac{y_{ik} y_{jk} M_k}{32\pi^2} \left[L_K\left(M_{\eta_R}^2\right) - L_K\left(M_{\eta_I}^2\right) \right], \tag{7}$$

where

$$L_K\left(M^2\right) = \frac{M^2}{M^2 - M_K^2} \ln \frac{M^2}{M_K^2}. \tag{8}$$

Here, M_k is the mass of k^{th} right-handed neutrino, and M_{η_R, η_I} are the masses of real and imaginary parts of inert doublet η. We have used Cassas-Ibarra (CI) [15] parametrization for radiative seesaw mechanism [16] through which the Yukawa coupling matrix that satisfies neutrino data is written as

$$y_{\alpha k} = \left(U D_\nu^{1/2} R^\dagger \Lambda^{1/2} \right)_{\alpha k}. \tag{9}$$

Here, R is the complex orthogonal matrix that satisfies $RR^T = 1$, $D_\nu = \text{diag}(m_1, m_2, m_3)$ is the diagonal light neutrino mass matrix, and the form of diagonal matrix Λ is given by

$$\Lambda_K = \frac{2\pi^2}{\lambda_5} \zeta_K \frac{2M_K}{v^2}, \tag{10}$$

where

$$\zeta_K = \left(\frac{M_K^2}{8\left(M_{\eta_R^2} - M_{\eta_I^2}\right)} \left[L_K\left(M_{\eta_R}^2\right) - L_K\left(M_{\eta_I}^2\right) \right] \right). \tag{11}$$

3.2. Dark Matter

The VLL triplet ψ_T is considered a dark matter candidate in the model. In order to do DM phenomenology, we firstly implemented our model in FeynRule [18] and then calculated the relic density using the package MicrOmegas [19]. Variation of relic density as a function of dark matter mass M_ψ is shown in Figure 1. The data points displayed in blue color are well within the experimental range for relic density. As can be seen from Figure 1, correct relic density can be generated when the mass of the fermion triplet is around 700 GeV to 1.5 TeV. Figure 2 shows the variation of Yukawa coupling y_ψ as a function of the mass of dark matter M_ψ. The data points in blue color represent the value of Yukawa coupling ~0.02 to 1 for which M_ψ (in the range of 700 GeV to 1.5 TeV) gives the correct relic density.

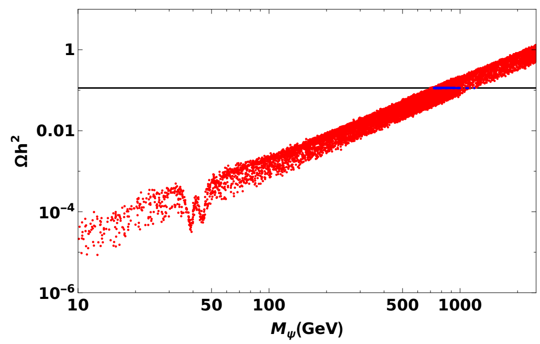

FIGURE 1: Relic density of dark matter as a function of mass of VLL triplet M_ψ.

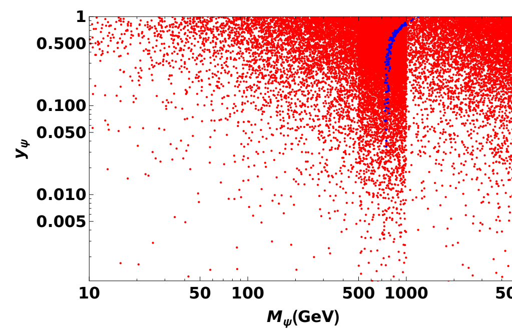

FIGURE 2: Yukawa coupling y_ψ as a function of mass of VLL triplet M_ψ.

3.3. Muon $(g - 2)$

The magnetic moment of muon in terms of its spin ($\overrightarrow{S}$) is given as

$$\overrightarrow{M} = g_\mu \frac{e}{2m_\mu} \overrightarrow{S}, \tag{12}$$

where m_μ is the mass of muon and g_μ is the gyromagnetic ratio. According to Dirac, the value of g_μ is exactly 2, but due to quantum corrections, the value slightly differs from 2. The deviation of g_μ is defined in terms of quantity a_μ as

$$a_\mu = \frac{g_\mu - 2}{2}. \tag{13}$$

In this model, the coupling of the vector-like lepton triplet ψ_T with SM muon through inert doublet η provides an additional contribution to muon $(g-2)$ as [17]

$$\Delta a_\mu = \frac{m_\mu^2 y_\psi^2}{32\pi^2 M_\eta^2}\left[5F_{\rm FFS}\left(M_\psi^2/M_\eta^2\right) - 2F_{\rm SSF}\left(M_\psi^2/M_\eta^2\right)\right], \quad (14)$$

where m_μ, M_ψ, and M_η are the masses of the muon, VLL triplet ψ_T, and inert scalar doublet η, respectively, and y_ψ is the coupling constant. Also,

$$F_{\rm FFS}(x) = \frac{1}{6(x-1)^4}\left[x^3 - 6x^2 + 3x + 2 + 6x\ln x\right], \quad (15)$$

$$F_{\rm SSF}(x) = \frac{1}{6(x-1)^4}\left[-2x^3 - 3x^2 + 6x - 1 + 6x^2\ln x\right], \quad (16)$$

where $x = \frac{M_\psi^2}{M_\eta^2}$ and $M_\eta \approx M_{\eta_R}$.

The mass of the VLL triplet and inert doublet, i.e., M_ψ and M_η, respectively, is varied in the range of 0.1 TeV to 5 TeV and the Yukawa couplings are kept within the perturbative limit. Figure 3 shows the variation of muon's anomalous magnetic moment Δa_μ as a function of inert doublet mass M_η. Figure 4 shows the variation of the anomalous magnetic moment of muon Δa_μ as a function of VLL triplet mass M_ψ. The horizontal lines represent the experimental range of Δa_μ.

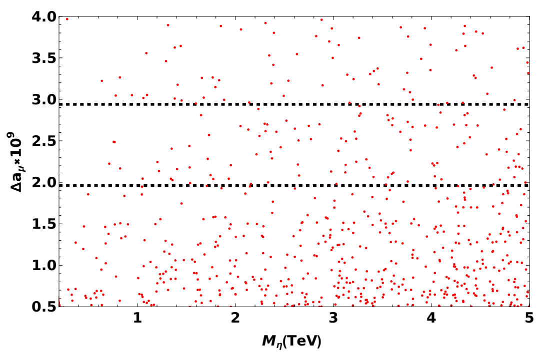

FIGURE 3: Muon anomalous magnetic moment Δa_μ as a function of mass of inert doublet M_η. The experimental range of Δa_μ is depicted by horizontal lines.

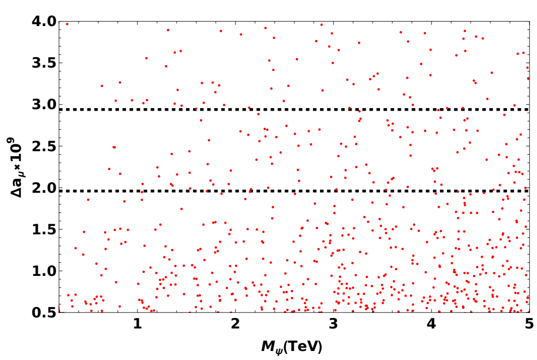

FIGURE 4: Muon anomalous magnetic moment Δa_μ as a function of VLL triplet mass M_ψ. The experimental range of Δa_μ is depicted by horizontal lines.

4. CONCLUSION

Motivated by the recent results of Fermilab that hints at physics beyond SM, we considered a Scotogenic model based on Z_4 discrete symmetry. We extended the model with a vector-like lepton triplet in order to explain muon $(g-2)$. The VLL triplet is assumed odd under the discrete Z_2 symmetry. As such, it is also assumed the dark matter candidate within the model. The mass of the VLL triplet that gives correct relic density lies in the range of around 700 GeV to 1.5 TeV. In this mass scale, the VLL triplet also explains muon $(g-2)$. Hence, the model simultaneously explains muon $(g-2)$ and dark matter while neutrino mass is generated at one loop level.

CONFLICTS OF INTEREST

The authors declare that there are no conflicts of interest regarding the publication of this paper.

ACKNOWLEDGMENTS

S. Arora acknowledges the financial support provided by the Central University of Himachal Pradesh. B. C. Chauhan is thankful to the Inter-University Centre for Astronomy and Astrophysics (IUCAA) for providing necessary facilities during the completion of this work.

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