Dark SU(2) Antecedents of the $U(1)$ Higgs Model

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Abstract

The original spontaneously broken $U(1)$ gauge model with one complex Higgs scalar field has been known in recent years as a possible prototype dark-matter model. Its antecedents in the context of $SU(2)$ are discussed. Three specific examples are described, with one dubbed “quantum scotodynamics”.

DOI: 10.31526/LHEP.2018.02

Consider the addition of the $U(1)_D$ Higgs model [1] to the standard $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge model (SM) of quarks and leptons. The former may be used for dark matter [2, 3, 4, 5, 6, 7, 8] because it has the built-in $Z_2$ symmetry where the massive gauge boson $Z_D$ after spontaneous symmetry breaking is odd and the one physical real scalar boson $h_D$ is even. However, $U(1)_D$ may mix kinetically [9] with $U(1)_Y$, in which case the above $Z_2$ symmetry would be violated. To avoid this problem, it is suggested here that $U(1)_D$ be replaced with an $SU(2)$ antecedent, with an enriched dark-matter sector. Three explicit examples will be discussed. Note that this version of dark $SU(2)$ requires that it be broken to $U(1)$, in contrast to the case where a local or global $SU(2)$ dark symmetry remains [10].

To break $SU(2)_D$ to $U(1)_D$, the simplest choice is a real scalar triplet

\[ \chi = (\chi_1, \chi_2, \chi_3) \]  
(1)

with $\langle \chi_3 \rangle = v_3$. In that case, the vector gauge bosons

\[ W^\pm_D = \frac{D_1 \pm i D_2}{\sqrt{2}} \]  
(2)

acquire mass given by $m^2_{W_D} = 2g^2 v_3^3$. Note that the superscript $\pm$ refers to dark charge, the details of which will be discussed later.

To break $U(1)_D$ in the context of $SU(2)_D$ so that $D_3 = Z_D$ acquires mass, a complex scalar doublet

\[ \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \]  
(3)

is used. Moreover, a global $U(1)_\Phi$ symmetry is imposed, i.e.

\[ \Phi \rightarrow e^{i \theta} \Phi, \]  
(4)

which prevents the coupling of $\tilde{\chi}$ to the triplet $\Phi$. The scalar potential consisting of $\chi$ and $\Phi$ is then given by

\[ V = m_2^2 \Phi^\dagger \Phi + \frac{1}{2} m_3^2 (\vec{\chi} \cdot \vec{\chi}) + \mu_0 \Phi^\dagger (\vec{\sigma} \cdot \vec{\chi}) \Phi + \frac{1}{2} \lambda_2 (\Phi^\dagger \Phi)^2 + \frac{1}{2} \lambda_3 (\vec{\chi} \cdot \vec{\chi})^2 + \lambda_4 (\Phi^\dagger \Phi)(\vec{\chi} \cdot \vec{\chi}). \]  
(5)

Note that the triplet combination of two identical real scalar triplets is zero. The minimum of $V$ admits a solution

\[ \langle \chi_{1,2} \rangle = 0, \quad \langle \chi_3 \rangle = v_3, \quad \langle \phi_1 \rangle = 0, \quad \langle \phi_2 \rangle = v_2 / \sqrt{2}, \]  
(6)

where $v_2$ is assumed real without any loss of generality, and

\[ 0 = v_3 [m_3^2 + 2 \lambda_3 v_3^2 + \lambda_4 v_2^2] - \mu_0 v_2^2 / 2, \]  
(7)

\[ 0 = v_2 [m_2^2 + 2 \lambda_2 v_2^2 / 2 + \lambda_4 v_3^2 - \mu_0 v_3], \]  
(8)

provided that

\[ m_2^2 + \lambda_4 v_3^2 + \mu_0 v_3 > 0, \]  
(9)

\[ m_2^2 + \lambda_4 v_3^2 - \mu_0 v_3 < 0. \]  
(10)

As a result

\[ m^2_{W_D} = 2g^2 v_3^3 + \frac{1}{4} g^2 D_3 v_3^2, \quad m^2_{Z_D} = \frac{1}{4} g^2 D_3 v_3^2, \quad m^2_{\Phi} = 2\mu_0 v_3, \]  
(11)

and the $2 \times 2$ mass-squared matrix spanning $h_D = \sqrt{2} Re(\phi_2) - v_2$ and $H_D = \chi_3 - v_3$ is given by

\[ M^2_{h_D, H_D} = \begin{pmatrix} \lambda_2 v_2^2 & v_2 (2 \lambda_4 v_3 - \mu_0) \\ v_2 (2 \lambda_4 v_3 - \mu_0) & 4 \lambda_3 v_3^2 + \mu_0 v_3^2 / 2 v_2^2 \end{pmatrix}. \]  
(12)

A global residual symmetry remains, under which

\[ W^+_D, \phi_1 \sim +1, \quad W^-_D, \phi_1^* \sim -1, \quad Z_D, h_D, H_D \sim 0. \]  
(13)

This comes from $I_{3D} + S_\Phi$, where $S_\Phi = 1/2$ for $\Phi$ and zero for all other fields. It is possible because of the imposed global $U(1)_\Phi$ symmetry. Whereas $\langle \phi_2 \rangle = v_2 / \sqrt{2}$ breaks both $I_{3D}$ and $S_\Phi$, the linear combination $I_{3D} + S_\Phi$ is zero for $\phi_2$, so it remains as a residual dark symmetry.

An important consequence of this structure is the emergence of a dark charge conjugation symmetry as in the original Higgs model [11], i.e.

\[ W^+_D \leftrightarrow W^-_D (D_2 \leftrightarrow -D_2), \quad \phi_1 \leftrightarrow \phi_1^*, \quad Z_D (D_3) \leftrightarrow -Z_D (D_3). \]  
(14)
This comes from the gauge-invariant terms
\[ -\frac{1}{4} \left( \partial_{\mu} \tilde{D}_{\nu} - \partial_{\nu} \tilde{D}_{\mu} + g_{D} D_{\mu} \times D_{\nu} \right)^{2} + |D_{\mu} \Phi - i \frac{g_{D}}{2} \bar{\sigma} \cdot D_{\mu} \Phi|^{2}. \tag{15} \]

It means that $Z_{D}$ is stable if its mass is less than twice that of $\phi_{1}$, in complete analogy to the $U(1)_{D}$ model of Ref. [8]. This makes it possible in principle to implement the inclusion of self-interacting dark matter, i.e. $\phi_{1}$ or $W_{D}$ of order 100 GeV with $Z_{D}$ as the light stable mediator of order 10 to 100 MeV, to explain [11] the observed core-cusp anomaly in dwarf galaxies [12]. If $Z_{D}$ is unstable and decays to SM particles, as is the case for the light mediator proposed in most models, then very strong constraints exist [13] from the cosmic microwave background (CMB) which basically rule out [14] this scenario. On the other hand, $h_{D}$ must also be light and decay quickly within its mixing with the SM Higgs boson $h$ before big bang nucleosynthesis (BBN). In that case, the elastic scattering of $W_{D}$ or $\phi_{1}$ off nuclei through $h_{D}$ exchange is much too large to be acceptable with the present data. In Ref. [8], this is not a problem because the dark matter is a Dirac fermion which couples to $Z_{D}$ but not $h_{D}$.

As it is, this specific $SU(2)_{D}$ antecedent of the $U(1)$ Higgs model may still be a model of dark matter without addressing the core-cusp anomaly in dwarf galaxies. Assuming that $W_{D}$ is heavy enough to decay into $\phi_{1} h_{D}$ and $Z_{D}$ heavy enough to decay into $\phi_{1} \phi_{1}^{*}$, then the complex scalar $\phi_{1}$ may be considered dark matter. Assuming that $h_{D}$ is lighter than $\phi_{1}$, the annihilation cross section of $\phi_{1} \phi_{1}^{*}$ at rest $\times$ relative velocity is given by
\[ \sigma(\phi_{1} \phi_{1}^{*} \rightarrow h_{D} h_{D}) \nu_{rel} = \frac{\lambda_{2}^{2} \sqrt{1 - r_{1}}}{64 \pi m_{\phi_{1}}^{2}} \left[ 1 + \frac{r_{1}(2 + r_{1})}{(2 - r_{1})(4 - r_{1})} \right]^{2}, \tag{16} \]
where $r_{1} = m_{h_{D}}^{2} / m_{\phi_{1}}^{2}$. Assuming as an example $m_{h_{D}} = 150$ GeV and $m_{h_{D}} = 100$ GeV, the above may be set such to 4.4 $\times$ 10$^{-26}$ cm$^{3}$/s for $\lambda_{2} = 0.126$.

There is always the allowed quartic $\lambda_{2 h}$ coupling between the $SU(2)_{D}$ Higgs doublet and the $SU(2)_{L} \times U(1)_{Y}$ Higgs doublet of the SM, so that $\phi_{1}$ interacts through the SM Higgs boson $h$ in direct-search experiments. Using present data [15], it has been shown [16] that $\lambda_{2 h} < 4.4 \times 10^{-4}$. This is also the mixing between $h_{D}$ and $h$. Even with this limit on $\lambda_{2 h}$, it can still be large enough so that $h_{D}$ decays promptly to $h h$ in the early Universe. This interaction [8] also keeps $h_{D}$ in thermal equilibrium with the particles of the SM.

Consider the addition of a fermion doublet
\[ \Psi = \begin{pmatrix} \psi_{1} \\ \psi_{2} \end{pmatrix}_{L}, \tag{17} \]
to the $SU(2)_{D}$ model discussed in the previous section. It has the allowed interactions
\[ i \bar{\Psi} \gamma^{\mu} (\partial_{\mu} - i \frac{g_{D}}{2} \bar{\sigma} \cdot D_{\mu}) \Psi + [f \bar{\Psi} (\bar{\sigma} \cdot \vec{q}) \Psi + H.c.], \tag{18} \]
where $\bar{\Psi} = (\psi_{2} - \psi_{1})_{L}$. Since $\langle \chi_{3} \rangle = v_{3}$, this shows that $\psi_{1,2}$ combine to form a Dirac fermion of mass $f v_{3}$. To be specific, let $\psi_{1}$ be the left-handed component of the Dirac fermion $\psi$, and $\psi_{2}$ redefine as the conjugate of its right-handed component, i.e. $\psi_{2}L \sim \bar{\psi}_{R}$. Now $\psi_{1}$ has dark charge 1/2 and $\psi_{2}$ has dark charge $-1/2$. Together they form a Dirac fermion $\psi$ of charge 1/2, which interacts vectorially with $D_{3} = Z_{D}$. Note that $\bar{\psi}_{R} \psi$ is odd under dark charge conjugation as expected. Note also that $\psi$ has no direct coupling to $h_{D}$ because of $SU(2)_{D}$ gauge invariance. This allows the inclusion of self-interacting dark matter as described below.

Consider the elastic scattering of $\psi$ with $\bar{\psi}$ through the exchange of the light mediator $Z_{D}$. Its cross section in the limit of zero momentum is
\[ \sigma(\psi \bar{\psi} \rightarrow \psi \bar{\psi}) = \frac{\frac{g_{D}^{4} m_{\phi_{1}}^{2}}{256 \pi m_{\psi}^{2}}}{m_{\psi}^{2}} = \frac{m_{h}^{2}}{4 \pi v_{2}^{2}}. \tag{19} \]
For the benchmark value of $\sigma / m_{\phi} \sim 1 \text{ cm}^{2} / \text{g}$ for self-interacting matter, this is satisfied for example
\[ m_{\psi} = 100 \text{ GeV}, \quad v_{2} = 200 \text{ MeV}. \tag{20} \]
This low-energy effective theory consisting of $\psi$, $Z_{D}$ and $h_{D}$ may be dubbed quantum scotodynamics, from the Greek 'scotos' meaning darkness.

Consider now the annihilation of $\psi \bar{\psi} \rightarrow Z_{D} Z_{D}$. Since $Z_{D}$ is much lighter than $\psi$, this cross section $\times$ relative velocity is given by
\[ \sigma(\psi \bar{\psi} \rightarrow Z_{D} Z_{D}) \nu_{rel} = \frac{g_{D}^{4} \sqrt{1 - r}}{256 \pi m_{\psi}^{2}}. \tag{21} \]
For $m_{\psi} = 100$ GeV, and setting $\sigma \nu_{rel} = 4.4 \times 10^{-26} \text{ cm}^{3} / \text{s}$
\[ g_{D} = 0.42 \tag{22} \]
is obtained, which implies from Eq. (11) that
\[ m_{Z_{D}} = 42 \text{ MeV}. \tag{23} \]
As shown in Ref. [8], the light mediator $Z_{D}$ is stable but annihilates quickly to $h_{D}$ which decays. The cross section $\times$ relative velocity is given by
\[ \sigma(Z_{D} Z_{D} \rightarrow h_{D} h_{D}) \nu_{rel} = \frac{g_{D}^{4} \sqrt{1 - r}}{256 \pi m_{\psi}^{2}} \times \left[ \frac{4[r^{2} + 4(2 - r)^{2}]}{(4 - r)^{2}} - \frac{24r(2 + r)}{9(2 - r)(4 - r)} + \frac{8(2 + r)^{2}}{9(2 - r)^{2}} \right]. \tag{24} \]
where $r = m_{h_{D}}^{2} / m_{Z_{D}}^{2}$. Assuming $m_{h_{D}} = 21$ MeV as an example so that $r = 0.25$, the above is equal to $4 \times 10^{-18} \text{ cm}^{3} / \text{s}$, which is orders of magnitude greater than what is required for $Z_{D}$ to be a significant component of dark matter. It may re-emerge at late times by $\phi_{1} \phi_{1}^{*}$ annihilation through Sommerfeld enhancement, but its fraction as dark matter remains negligible. Since $Z_{D}$ is
stable, it would also not disturb the cosmic microwave background (CMB).

As for \( h_D \), it is allowed to mix with the SM Higgs boson \( h \) in the \( 2 \times 2 \) mass-squared matrix

\[
M^2_{h_Dh} = \begin{pmatrix}
\lambda_2 v^2_\phi & \lambda_2 v_\phi v_h \\
\lambda_2 v_\phi v_h & m_h^2
\end{pmatrix},
\]

where \( v_h = 246 \text{ GeV} \) and \( m_h = 125 \text{ GeV} \). For \( m_{h_D} < m_h \), the \( h_D - h \) mixing is \( \theta_{2h} = \frac{\lambda_2 v_\phi v_h}{m_h^2} \). Assuming \( \lambda_2 = 0.01 \),

\[
\theta_{2h} = 3.15 \times 10^{-5} \quad \text{and the } h_D \text{ lifetime for } e^- e^+ \text{ decay is given by}
\]

\[
\Gamma^{-1}(h_D \rightarrow e^- e^+) = \frac{8\pi v^2_\phi}{m_{h_D} m^2_{2h}} = 0.184 \text{ s},
\]

which is short enough not to affect big bang nucleosynthesis (BBN). The decay of the SM Higgs boson to \( h_D h_D \) is given by

\[
\Gamma(h \rightarrow h_D h_D) = \frac{\lambda_3^2 v_\phi^2}{16 \pi m_D^2} = 0.963 \text{ MeV},
\]

which is less than 25% of the SM width of 4.12 MeV and allowed by present data. Note that \( \lambda_3 = 0.0114 \) in Eq. (25) for \( m_{h_D} = 21 \text{ MeV} \). Note also the important fact that \( \psi \) does not couple directly to \( h_D \), otherwise Eq. (26) would be impossible, as discussed in the previous section.

In summary, a successful description of self-interacting fermion dark matter (\( \psi \) with \( m_\psi = 100 \text{ GeV} \)) through a stable light vector gauge boson \( (Z_D \text{ with } m_{Z_D} = 42 \text{ MeV}) \) in an \( SU(2)_D \) gauge model has been rendered. The Higgs scalar \( h_D \) associated with \( Z_D \) is also light (21 MeV), but it decays away quickly before the onset of BBN. Other heavier particles in the dark sector are \( W^\pm_D \) (which decays to \( \psi_1 \psi_1 / \psi_2 \psi_2 \) \( \phi_1 \) (which decays to \( W^\pm_D \)), and \( H_D \) which mixes slightly with \( h \) and \( h_D \).

In the previous two examples, an imposed symmetry of the \( SU(2)_D \) scalar doublet \( \Phi \), i.e. Eq. (4), is necessary for obtaining a dark symmetry. Hence the latter is not predestined, i.e. not the automatic consequence of gauge symmetry and particle content. To have a predestined dark \( Z_2 \) symmetry, the simpler scalar triplet is now replaced with a scalar quintet. This is analogous to having a fermion quintet in the SM for minimal dark matter, i.e. for simplicity.

Consider thereby the real scalar quintet

\[
\zeta = (\zeta^+, \zeta^0, \zeta^-, \zeta^-)
\]

with \( \langle \zeta^0 \rangle = v_\zeta \), then \( W^\pm_D \) obtains a mass given by \( m^2_{W^\pm_D} = 6g^2_D v^2_\zeta \) from absorbing \( \zeta^- \). This leaves \( \zeta^{\pm, 0} \) as physical scalar bosons with two units of dark charge, interacting with \( Z_D \). The scalar potential consisting of \( \zeta \) and \( \Phi \) is then given by

\[
V = \frac{m^2_\Phi}{2} \Phi^\dagger \Phi + \frac{m^2_\zeta}{2} \zeta^0 + \frac{1}{2} \lambda_2 (\Phi^\dagger \Phi)^2 + \lambda_5 (\Phi^\dagger \Phi) (\zeta^0) + V_3 + V_4,
\]

where \( V_3 \) contains the one cubic invariant formed out of 3 scalar quintets and \( V_4 \) contains two quartic invariants. To show this explicitly, consider first the decomposition \( 5 \times 5 = 1 + 3 + 5 + 7 + 9 \). Since 5 is assumed real, only the symmetric combinations of 1, 5, and 9 are possible. Now the product \( (5 \times 5) \times (5 \times 5) \) contains 5, 5 × 5, and 9 × 5. Only \( 5 \times 5 \) contains 1, hence there is just one cubic invariant. The product \( (5 \times 5) \times (5 \times 5) \) contains 1 × 1, 5 × 5, and 9 × 9, which all contain 1, but only two are independent, resulting thus in two quartic invariants. As for a possible term connecting \( \zeta \) with \( \Phi \), consider the triplet \( \phi_1 \psi_1 \bar{\psi}_1 \) pointed out earlier. Whereas it is obvious that \( \zeta \) cannot couple to it because it is a quintet, but if the product \( \zeta \times \zeta \) contains a triplet, then a quartic term would exist which violates \( U(1)_\Phi \). As it is, such a triplet is identically zero as explained in the above. Note that if a quartet were used, \( 4 \times 4 \) would contain a triplett, because 4 is necessarily complex. Similarly, a sextet would not work. A real septet is possible as well as any real odd-dimensional representation higher than 5. Thus the scalar potential of Eq. (30) has automatically the necessary extra \( U(1)_\Phi \) symmetry, so that \( I_3 + S_\Phi \) remains unbroken as \( \phi_2 \) acquires a vacuum expectation value \( v_2 / \sqrt{2} \) as explained previously.

Assuming that

\[
m_\zeta < 2m_\Phi < m_D < m_{W_D},
\]

then \( W^\pm_D \) decays to \( \phi_1 h_D, Z_D \) decays to \( \phi_3 \phi_4 \), but both \( \phi_1 \) and \( \zeta \) are stable. Hence this is an explicit example of two-component dark matter under one dark \( U(1) \) symmetry. Let

\[
m_\zeta = 200 \text{ GeV}, \quad m_\Phi = 150 \text{ GeV}, \quad m_{W_D} = 100 \text{ GeV},
\]

then using Eq. (16) for \( v_1 (\phi_1 \phi_1^* \rightarrow h_D h_D) v_{rel} \) and the analogous

\[
v_2 (\zeta \zeta^* \rightarrow h_D h_D, \phi_1 \phi_1^* v_{rel} = \frac{\lambda_3^2 \sqrt{1 - r_2}}{64 \pi m_D^2} \times \left[ 1 + \frac{2 \lambda_3 \lambda_2}{2 - r_2} - \frac{3r_2}{4 - r_2} \right]^2 + \frac{\lambda_5^2 \sqrt{1 - r_2}}{32 \pi m_D^2} \left[ 1 - \frac{r_2}{4 - r_2} \right]^2,
\]

where \( r_2 = m^2_{W_D} / m^2_\zeta \) and \( r_3 = m^2_\Phi / m^2_\zeta \), the condition for the correct relic abundance is roughly given by

\[
\langle v_1 v_{rel} \rangle^{-1} + \langle v_2 v_{rel} \rangle^{-1} = (4.4 \times 10^{-26} \text{ cm}^3 / \text{s})^{-1}.
\]

It has for example the reasonable solution \( \lambda_3 = \lambda_2 = 0.173 \), in which case \( \phi_1 \) is 53% and \( \zeta \) 47% of dark matter. Again the mixing of \( \zeta \) with the SM Higgs boson \( h \) must be small as it is for \( \phi_1 \) to satisfy direct-search limits as discussed previously.

In this scenario, the addition of the fermion doublet of Eq. (17) could also provide a low-energy effective theory of quantum scotodynamics with light \( Z_D \) and \( h_D \). In that case, \( \zeta^{\pm, 0} \) would decay into \( W^\pm_D W^\pm_D, \phi_1 \) would decay into \( W^D_D h_D \), and \( W^\pm_D \) would decay into \( \psi_1 \psi_1 / \psi_2 \psi_2 \).
Exploring the possible $SU(2)$ antecedents of the famous $U(1)$ Higgs model for a nontrivial application to dark matter, three interesting examples have been identified and discussed. The minimal version with one real scalar triplet $\chi$ and one complex scalar doublet $\Phi$ admits $\phi_1$ as dark matter, but a global $U(1)$ symmetry has to be imposed. With the addition of a fermion doublet $\psi$, the inception of self-interacting dark matter may be implemented successfully, avoiding all potential astrophysical and laboratory constraints. A third example replaces $\chi$ with the real scalar quintet $\zeta$, in which case the dark $U(1)$ symmetry becomes predestined, i.e. automatic from the gauge symmetry and particle content.

Since all these scenarios are based on an extended dark $SU(2)_D$ sector, they are only confirmed if the heavier gauge bosons $W_D^{\pm}$ and heavier scalar bosons $H_D$ or $\zeta^{\pm\pm}$ could be observed. This is a very challenging phenomenological question because the only connection between the dark sector and the SM is through the SM Higgs boson. It is unfortunately, a generic feature of all dark-matter proposals using this so-called Higgs portal. The focus here is rather on a possible theoretical understanding of where dark matter may come from.

**ACKNOWLEDGEMENT**

This work was supported in part by the U. S. Department of Energy Grant No. DE-SC0008541.

**References**