Joule-Thomson Expansion of RN-AdS Black Holes in $f(R)$ gravity

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Abstract

In this paper, we study Joule-Thomson expansion for charged AdS black holes in $f(R)$ gravity. We obtain the inversion temperatures as well as the inversion curves, and investigate the similarities and differences between van der Waals fluids and charged AdS black holes in $f(R)$ gravity for this expansion. In addition, we determine the position of the inversion point versus different values of the mass M, the charge Q and the parameter b for such black hole. At this point, the Joule-Thomson coefficient μ vanishes, an import feature that we used to obtain the cooling-heating regions by scrutinizing the sign of the μ quantity.

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Nowadays, an $f(R)$ gravity is one of the important classes of the modified Einstein's gravity. In general, It is built through adding higher powers of the scalar curvature R, the Riemann and Ricci tensors, or their derivatives to the lagrangian description [1, 2, 3, 4, 5]. This kind of gravity mimics successfully the history of the universe, especially the current cosmic acceleration, the inflation and structure formation in the early Universe [2, 3], various extensions of $f(R)$ gravity theory have been elaborated ranging from three dimensional [6] and asymptotically Lifshitz black hole solutions [7] to $f(R)$ gravity's rainbow model [8].

Recently, a special attention has been devoted to the study of the black holes phase transition particularly after the introduction of the notion of the extended phase space via the identification of the cosmological constant with the pressure and its conjugate quantity with thermodynamic volume [9]. In this context, the black hole behaves like Van der Waals fluid [10, 11, 12] leading to a remarkable correspondence between the thermal physics of black holes and simple substances [13]. Numerous extensions of these works have been elaborated for rotating and hairy black hole [14, 15], high curvature theories of gravity and M-theory [16, 17, 18, 19]. More exotic results as holographic heat engine [20] as well as other technics ranging from the behaviour of the quasi-normal modes [21, 22] to AdS/CFT tools [23, 24] and chaos structure [25] have consolidated the similarity with the Van der Waals fluid.

Meanwhile, a considerable effort has been dedicated to explore the thermodynamics of the AdS black hole in the $f(R)$ gravity background with a constant curvature [26] where the essential of thermodynamical quantities like the entropy, heat capacity and the Helmholtz free energy are calculated, then the extended phase space and the critical Van der Waals-like behavior introduced in [27, 26] as well as the canonical ensembles in [28]. Here we would like to go further in the thermodynamical investigation and study the Joule-Thomson expansion for the charged-AdS black hole configuration in the $f(R)$ gravity background.

More recently, the authors of [29] have investigated the JT expansion for AdS charged black holes with the aim to confront the resulting features with those of Van der Waals fluids. The extension to rotating-AdS black hole [30] and the charged black hole solution in the presence of the quintessence field [31] have also been considered. The JT expansion of a gas generally takes place at a constant enthalpy, a quantity identified to a mass when the thermodynamical system is a black hole, more precisely its mass. JT expansion [32] is a convenient isoenthalpic tool that a thermal system exhibits with a thermal expansion, where the Joule-Thomson coefficient $\mu_{JT} = \left.\frac{\partial T}{\partial P}\right)_H$ is the main quantity to discriminate between the cooling and heating regimes of the system. It is worth noting that when expanding a thermal system with a temperature T, the pressure always decreases yielding a negative sign to ∂P. In this context, we can consider two different regimes with respect to the so-called the inversion temperature, defined as the temperature T_i at which the Joule-Thomson coefficient vanishes $\mu_{JT}(T_i) = 0$: If $T < T_i$ $(T > T_i)$, then the Joule-Thomson processus cools (warms) the system with $\partial T < 0$ and $\mu_{JT} > 0$ ($\partial T > 0$ and $\mu_{JT} < 0$) respectively. When the system temperature tends to T_i, its pressure is referred as the inversion pressure P_i, so defining a special point called the inversion point (T_i, P_i) at which the cooling-heating transition occurs.

The outline of this work is as follows: In the next section, we review briefly the essential of the thermodynamic properties and stability of the charged-AdS black hole solution in $f(R)$ background. In section 3, we study the JT expansion under constant mass, and derive the inversion temperature T_i as well as the corresponding inversion point of such a black hole. We also show when the cooling phase is changed to heating phase at a particular (inversion) pressure P_i. The last section is devoted to our conclusion.

We briefly review the main features of the four-dimensional charged AdS black hole corresponding in the $f(R)$ gravity background with a constant Ricci scalar curvature [26]. The action is given by

$$S = \int_{\mathcal{M}} d^4x\sqrt{-g}[R + f(R) - F_{\mu\nu}F^{\mu\nu}]. \tag{1}$$

Here, R denotes the Ricci scalar curvature while $f(R)$ is an arbitrary function of R. In addition $F_{\mu\nu}$ stands for electromagnetic field tensor given by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu \partial_\mu$, where A_μ is the electromagnetic potential. From the action (1), the equations of motion for gravitational field $g_{\mu\nu}$ and the gauge field A_μ are,

$$R_{\mu\nu}[1 + f'(R)] - \frac{1}{2}g_{\mu\nu}[R + f(R)] + (g_{\mu\nu}\nabla^2 - \nabla_\mu\nabla_\nu)f'(R) = T_{\mu\nu}, \tag{2}$$

$$\partial_\mu(\sqrt{-g}F^{\mu\nu}) = 0. \tag{3}$$

The analytic solution of equation (2) has been determined in [26] when the Ricci scalar curvature is constant $R = R_0 = const$, where in this simple case, (2) simplifies to:

$$R_{\mu\nu}[1 + f'(R_0)] - \frac{g_{\mu\nu}}{4} R_0[1 + f'(R_0)] = T_{\mu\nu}, \tag{4}$$

The charged static spherical black hole solution of (2) in 4d gravity model takes the form, [26]

$$ds^2 = -N(r)dt^2 + \frac{dr^2}{N(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{5}$$

where the metric function $N(r)$ is given by,

$$N(r) = 1 - \frac{2m}{r} + \frac{q^2}{br^2} - \frac{R_0}{12} r^2 \tag{6}$$

with $b = [1 + f'(R_0)]$ while the two parameters m and q are proportional to the black hole mass and charge respectively [33]

$$M = mb, \quad Q = \frac{q}{\sqrt{b}}. \tag{7}$$

In the next relevant step, we briefly discuss the physical properties of the solutions. First, we consider the curvature ′ singularities which yield a diverging Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ at $r = 0$ but finite elsewhere [34]. Regarding Eq.(6), first note that a vanishing value of the parameter b leads to an ill-defined solution. If b has negative values, with $R_0 < 0$, then the signature of the spacetime changes and the conserved quantities such as mass become negative. However, when b gets positive values, we see from Fig.1 that three black hole solutions may show up: a black hole solution with an inner and outer horizons (green line), an extremal black hole (orange line) and a naked singularity (red line).

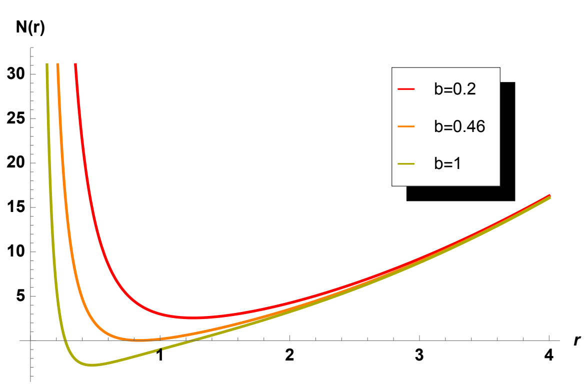

FIGURE 1: The function $N(r)$ versus r for $m = 2$, $q = 1$ and $R_0 = -12$

In this background, the electric potential Φ can be evaluated as

$$\Phi = \frac{\sqrt{b}q}{r_+}. \tag{8}$$

where the black hole event horizon r_+ denotes the largest root of the equation $N(r_+) = 0$. At the event horizon, one can also derive the Hawking temperature as well as the entropy of this kind of black hole [26, 33],

$$T = \frac{N'(r_+)}{4\pi}\bigg|_{r=r_+} = \frac{1}{4\pi r_+}\left(1 - \frac{q^2}{r_+^2 b} - \frac{R_0 r_+^2}{4}\right), \tag{9}$$

$$S = \pi r_+^2 b. \tag{10}$$

Recalling the analogy between the cosmological constant and the thermodynamics pressure, while its corresponding conjugate quantity is identified to the volume [35, 9, 36], one can deduce the following relations,

$$P = -\frac{bR_0}{32\pi}, \quad \text{with } R_0 = -\frac{12}{l^2} = -4\Lambda, \tag{11}$$

and,

$$V = \frac{4\pi r_+^3}{3}. \tag{12}$$

At this stage, it is straightforward to see that the above black hole quantities satisfy to the following Smarr relation:

$$M = 2TS + \Phi Q - 2PV. \tag{13}$$

Furthermore, by taking into account the $f(R)$ corrections with a constant Ricci scalar, the first law of thermodynamics is written as:

$$d\left(\frac{M}{b}\right) = Td\left(\frac{S}{b}\right) + \left(\frac{\Phi}{b}\right)dQ + Vd\left(\frac{P}{b}\right). \tag{14}$$

At last, from the equations of the Hawking temperature (9) and the pressure (11) of such a black hole, one can easily derive the corresponding equation of state, $P = P(T, r_+)$,

$$P = \frac{bT}{2r_+} - \frac{b}{8\pi r_+^2} + \frac{q^2}{8\pi r_+^4}. \tag{15}$$

It is worth noting here that the $f(R)$ background induced corrections to the pressure (11) and to the subsequent formulas can bring to light new possible feature which might be revealed through the phase transition structure of the charged-AdS black holes. The next section will be devoted to verify this proposal by means of Joule-Thomson expansion.

Applying method similar to the one used in [37], we consider Joule-Thomson expansion for charged-AdS black holes in $f(R)$ gravity. For a fixed charge the Joule-Thomson coefficient is given by,

$$\mu = \left(\frac{\partial T}{\partial P}\right)_M = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right], \tag{16}$$

besides, the equation of the state of such a black hole is provided in terms of thermodynamic volume by substituting (12) in (15), the equation (9) transforms to:

$$T = \frac{1}{12V}\left(\frac{12\sqrt[3]{\frac{6}{\pi}}PV^{4/3}}{b} - 4Q^2 + \left(\frac{6}{\pi}\right)^{2/3} V^{2/3}\right), \tag{17}$$

by using Eq.(17) this into the right hand side of Eq.(16), one can derive the temperature corresponding to a vanishing Joule-Thomson coefficient, dubbed inversion temperature T_i:

$$\begin{aligned} T_i &= V\left(\frac{\partial T}{\partial V}\right)_P \\ &= \frac{1}{36V}\left(\frac{12\sqrt[3]{\frac{6}{\pi}}V^{4/3}P_i}{b}+12Q^2-\left(\frac{6}{\pi}\right)^{2/3}V^{2/3}\right) \\ &= \frac{1}{12\pi b r_+^3}\left(-br_+^2+3bQ^2+8\pi r_+^4P_i\right), \end{aligned} \tag{18}$$

T_i can also be rewritten in terms of its corresponding pressure:

$$\begin{aligned} T_i &= \frac{1}{12V}\left(\frac{12\sqrt[3]{\frac{6}{\pi}}V^{4/3}P_i}{b}-4Q^2+\left(\frac{6}{\pi}\right)^{2/3}V^{2/3}\right) \\ &= \frac{1}{4\pi b r_+^3}\left(b\left(r_+^2-Q^2\right)+8\pi r_+^4P_i\right). \end{aligned} \tag{19}$$

By subtracting Eq. (18) from Eq. (19), we get the following polynomial equation,

$$-2br_+^2+3bQ^2-8\pi r_+^4P_i=0, \tag{20}$$

which possesses four roots. Here, we only consider the real positive root given by,

$$r_+=\frac{\sqrt{\frac{\sqrt{b(b+24\pi Q^2P_i)}}{\pi P_i}-\frac{b}{\pi P_i}}}{2\sqrt{2}}. \tag{21}$$

Once this root is substituted into Eq.(19), the inversion temperature becomes,

$$T_i=\frac{\sqrt{P_i}\left(-\sqrt{b\left(b+24\pi Q^2P_i\right)}+b+16\pi Q^2P_i\right)}{\sqrt{2\pi}\left(\sqrt{b\left(b+24\pi Q^2P_i\right)}-b\right)^{3/2}}, \tag{22}$$

so in the limit where the inversion pressure P_i vanishes, the inversion temperature reaches its minimum at:

$$T_i^{min}=\frac{1}{6\sqrt{6}\pi Q}. \tag{23}$$

Consequently, the critical temperature is just twice the value of the inversion temperature,

$$\frac{T_i^{min}}{T_c}=\frac{1}{2}, \tag{24}$$

in perfect agreement with the result of [29].

In Fig.2 we plot the inversion curves for charged AdS black hole for different values of charge Q. We can see that, in contrast to Van der Waals fluids, there is only a lower inversion curve which does not terminate at any point, since the expression inside the square root of Eq.(22) is always positive. This feature has also been observed in [29] with charged AdS black holes as well as in [30] with the rotating AdS black holes.

Next, thanks to Eq.(7) and Eq.(15), we also illustrate in Fig.3 the isenthalpic curve corresponding to a constant mass in the (T,P)-diagram.

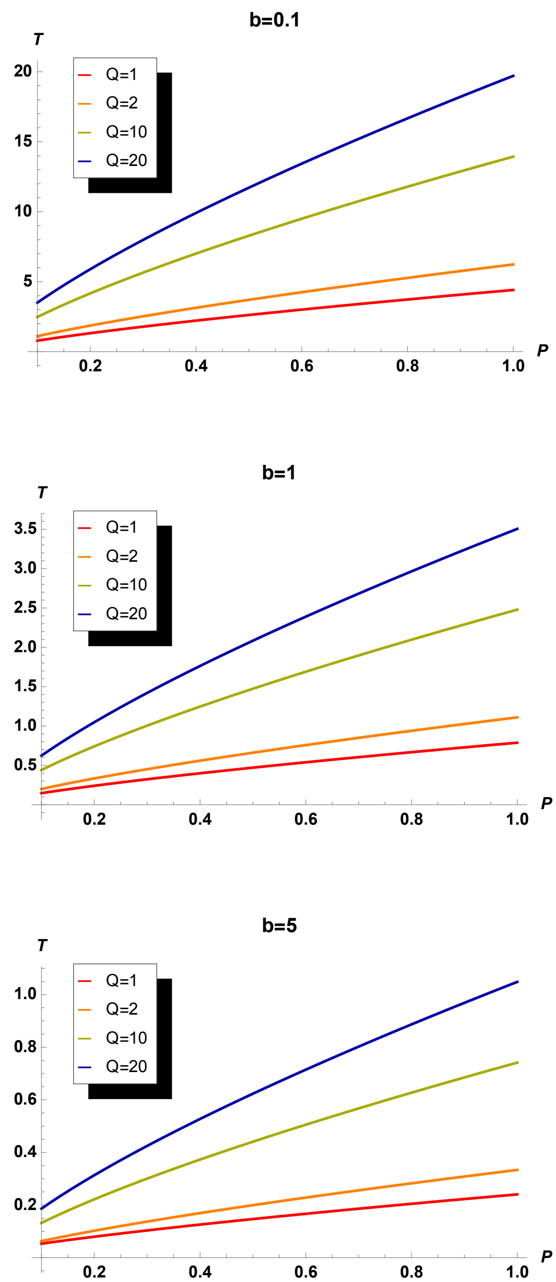

FIGURE 2: Inversion curves. From bottom to top, the curves correspond to $Q=1;2;10;20$.

From Fig.3, we see that the inversion curves divide the space (T,P) into two separated regions: The region above the inversion curves corresponds to the cooling region, while the region under the inversion curves corresponds to the heating one. Note that one can discriminate between the cooling / heating regions just by checking the sign of the isenthalpic curves slope. The sign of the slope is positive in the cooling region and it flips to negative in the heating region. The cooling / heating phenomenon never takes place on the inversion curve, which

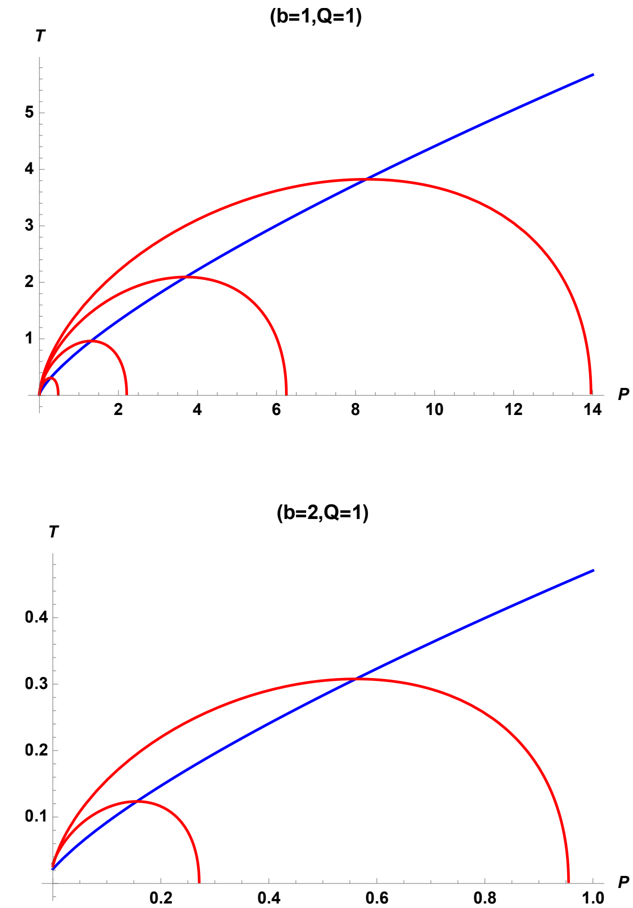

FIGURE 3: Inversion and isenthalpic curves for charged AdS black hole. From bottom to top, the isenthalpic curves correspond to increasing values of M. Red and blue lines are isenthalpic and inversion curves, respectively. Our inputs are: Q = 1, M = 1.5; 2; 2.5; 3 and b=1; 2.

plays the role of a separating boundary between the two regions.

In this paper, we have studied the Joule-Thomson expansion for RN-AdS black hole in $f(R)$ gravity background in the context of the extended phase space, where the cosmological constant is identified with the pressure. Here, the black hole mass is interpreted as an enthalpy, so we assume that mass does not change during the Joule-Thomson expansion. Our analysis has shown that the inversion curve always corresponds to the lower curve. This means that the black hole always cools above the inversion curve during the expansion. At last, we have identified the cooling and heating regions for different values of the parameter b and the black hole mass M.

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